ASTROPARTICLES

Astroparticles and High Energy Physics Group

The Leptonic Sector in a Flavor Model with the S_3 Symmetry

Félix González-Canales

Instituto de Física Corpuscular

CSIC-Universidad de Valencia

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IFIC INSTITUT DE FÍSICA CORPUSCULAR

CSIC Consejo Superior de Investigaciones Científicas

Vniversitat de València

CONACYT Consejo Nacional de Ciencia y Tecnología

This talk is based on the works:

1. **The S_3 Flavour Symmetry: Neutrino Masses and Mixings,** FGC, A. Mondragón and M. Mondragón, Fortsch.Phys. 61 (2013) 546-570
2. **Quark sector of S_3 models: classification and comparison with experimental data,** FGC, A. Mondragón, M. Mondragón, U.J. Saldaña Salazar and L. Velasco-Sevilla, Phys. Rev. D 88 (2013) 096004
3. **Phenomenological effects of CP conserving Higgs bosons self couplings in SMxS_3,** E. Barradas Guevara, O. Félix-Beltrán, FGC and E. Rodríguez-Jáuregui, J. Phys. Conf. Ser. 492 (2014) 012015
4. **Trilinear self-couplings in an S_3 flavored Higgs model,** E. Barradas Guevara, O. Félix-Beltrán and E. Rodríguez-Jáuregui, Phys. Rev. D90 (2014) 095001
5. **CP breaking in S_3 flavored Higgs model,** E. Barradas Guevara, O. Félix-Beltrán and E. Rodríguez-Jáuregui, arXiv:1507.05180
6. **And work in progress...**

From Neutrino Oscillations

Very compelling evidence for finite masses and large neutrino mixing angles has been achieved from Solar, atmospheric, reactor and accelerator neutrino oscillation experiments.

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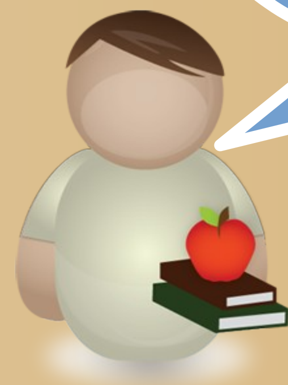

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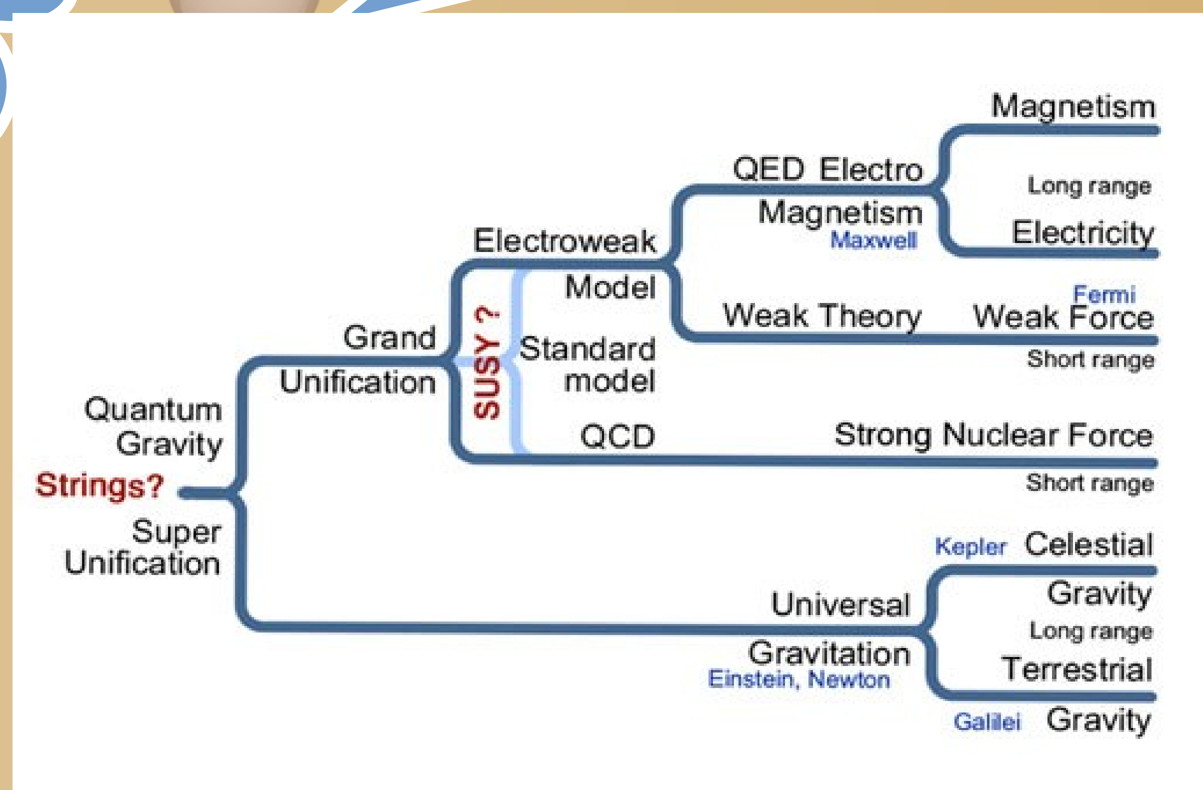

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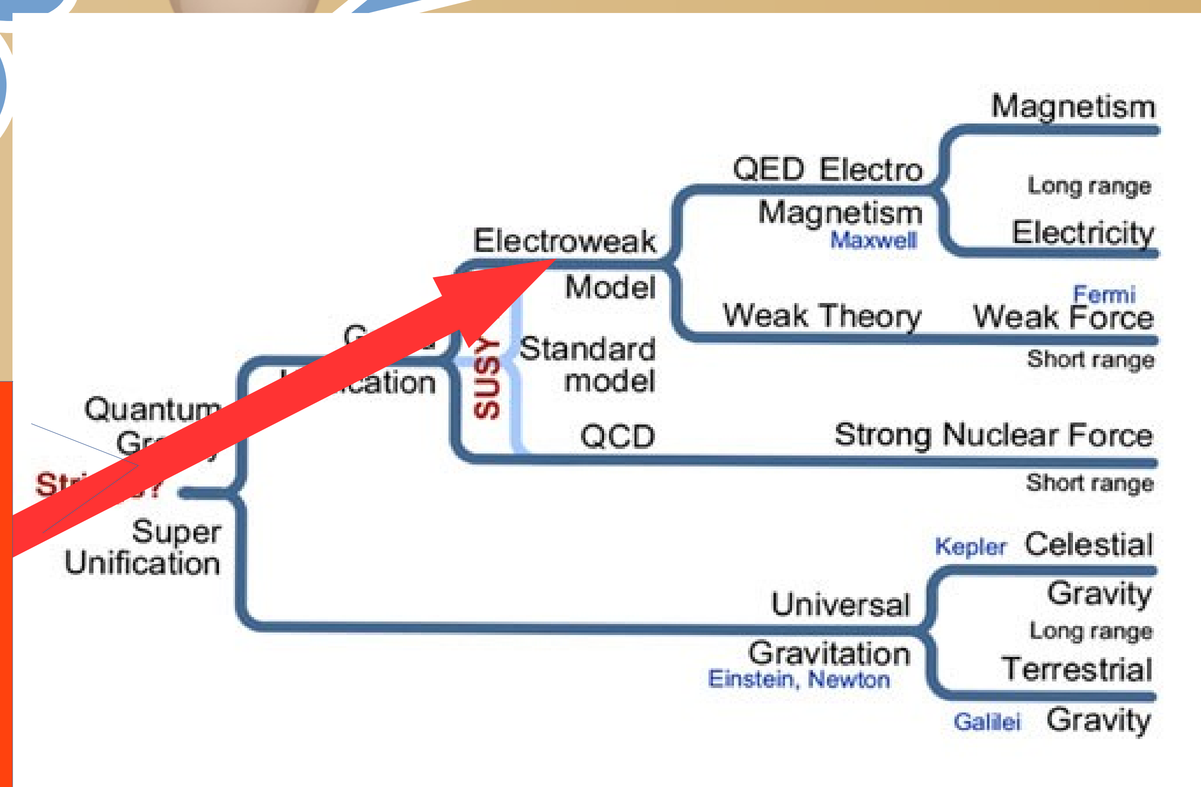
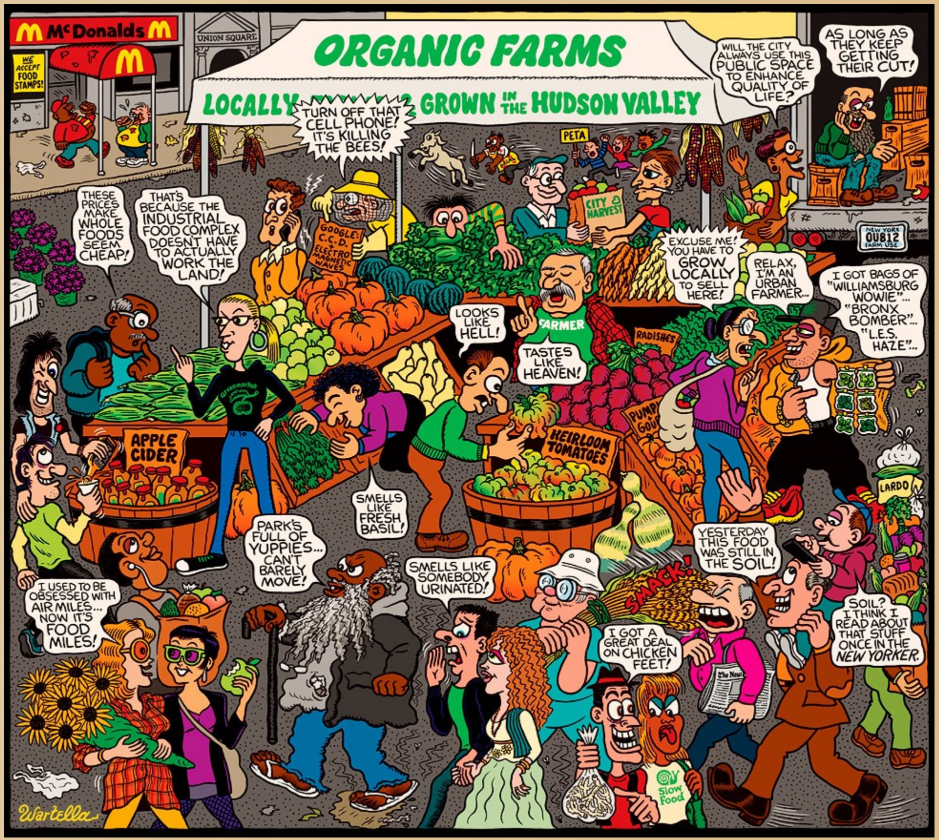
SUSY, 2HDM, Flavour Symmetries.....

We choose made a minimal extension of SM, we add a discrete flavor symmetry



Discrete Symmetries Market



http://www.ackxhpaez.com/runninscared5.html

- Abelian Symmetries

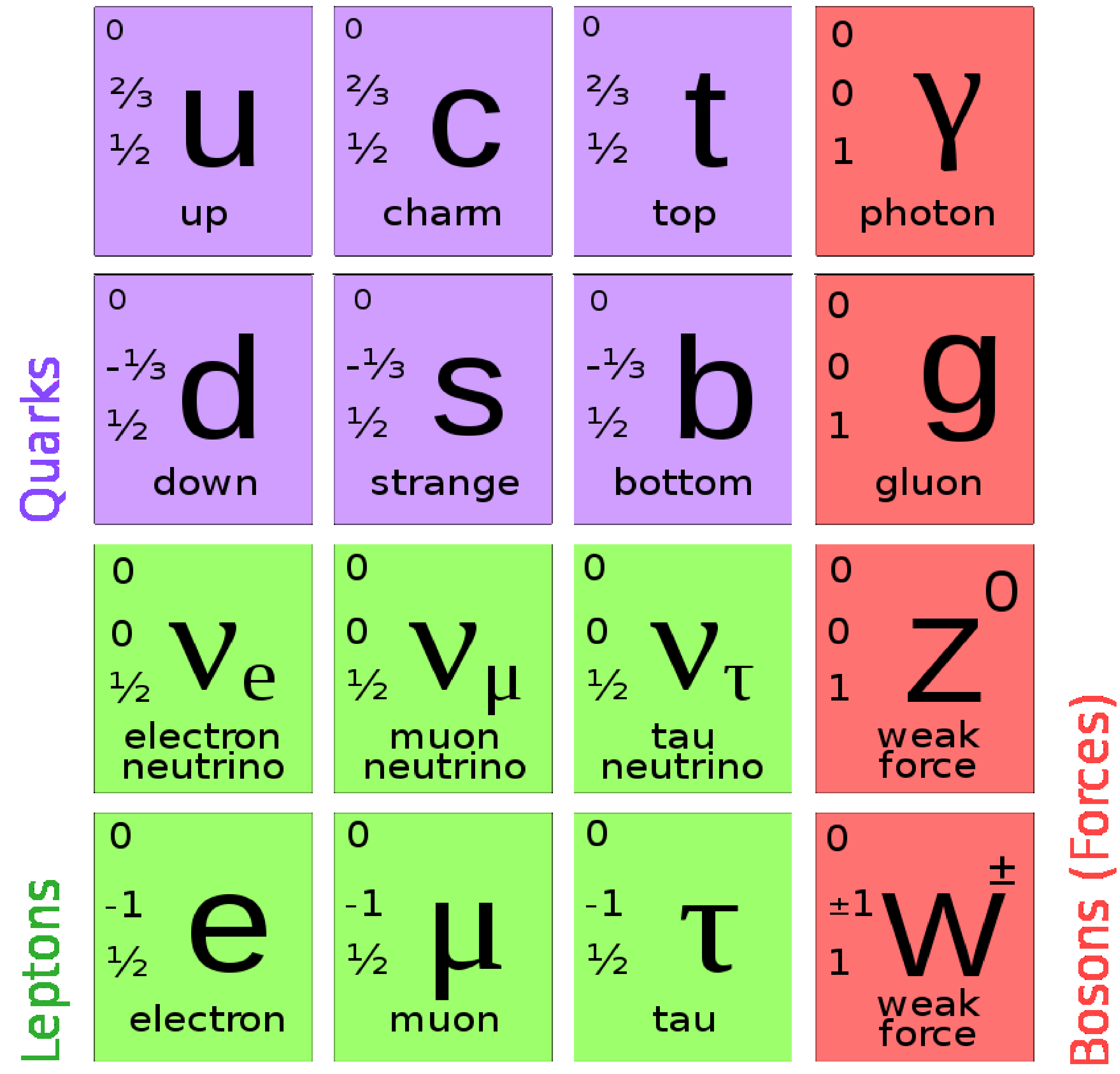
  Z_n

- Non Abelian Symmetries

  S_3, A_4, Q_6, ...

An Introduction to Non-Abelian Discrete Symmetries for Particle Physicists, Lect.Notes Phys. 858 (2012) 1-227

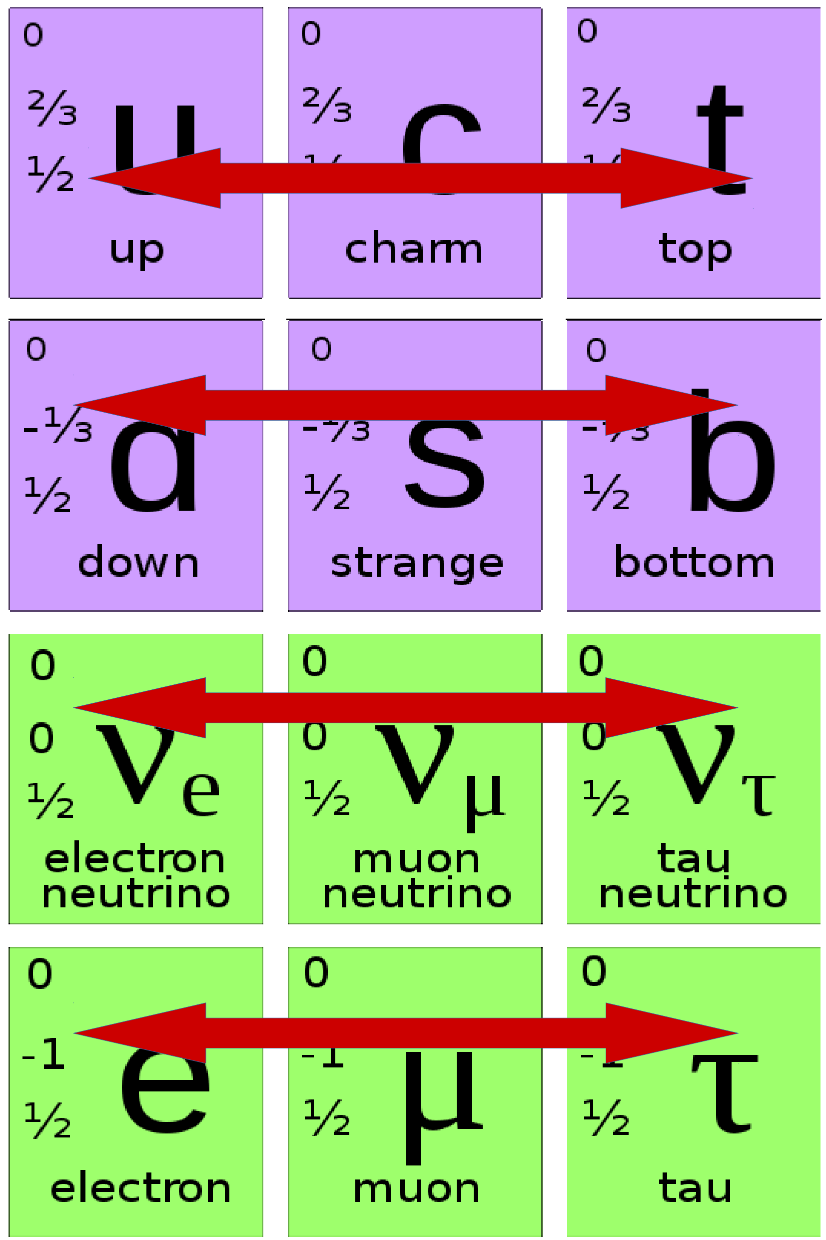
Horizontal Symmetry



Horizontal Symmetry

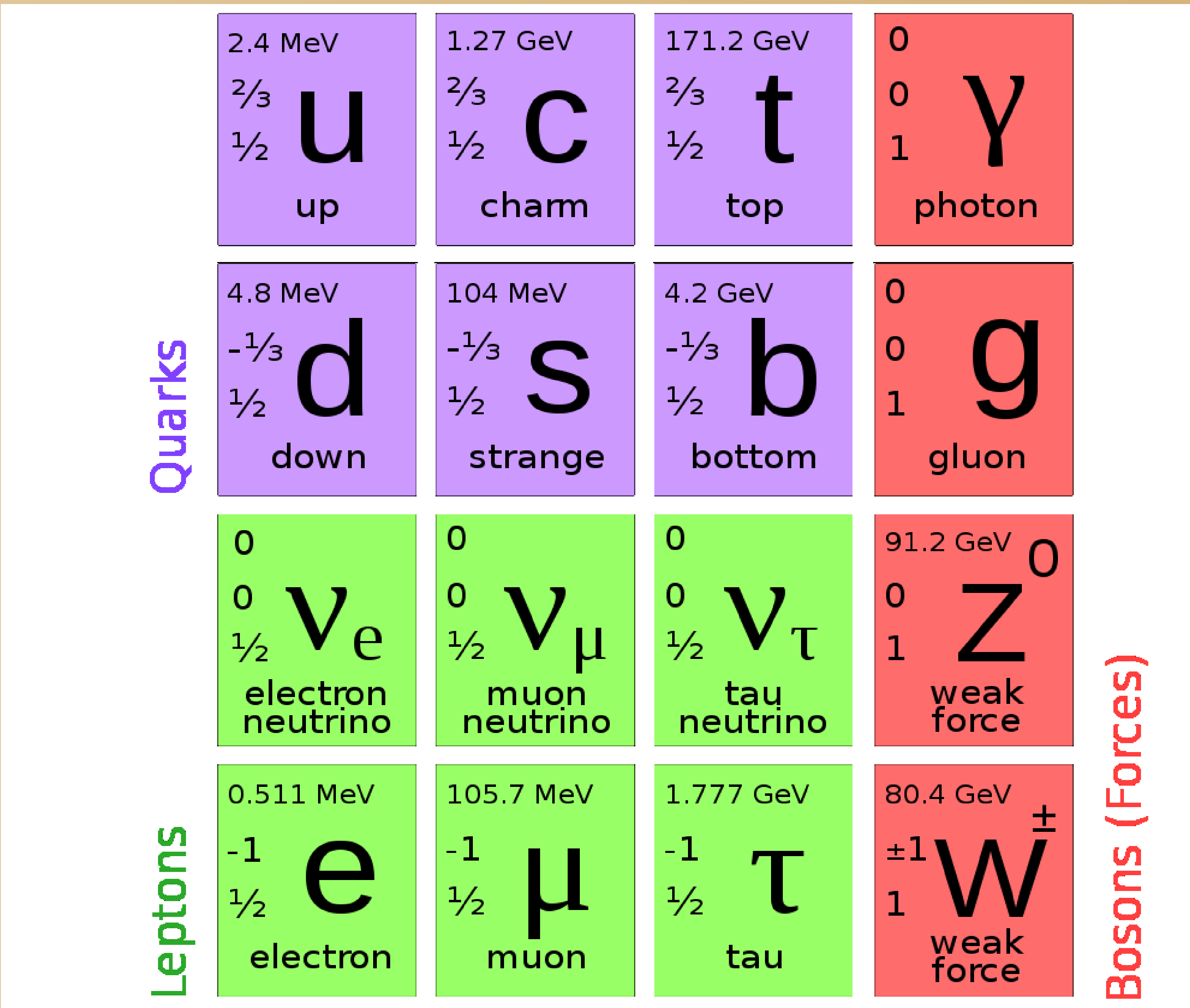
Permutation of three objects

we have the S_3 symmetry.



Before Electroweak symmetry breaking, the theory is quiral and all particles are Massless.

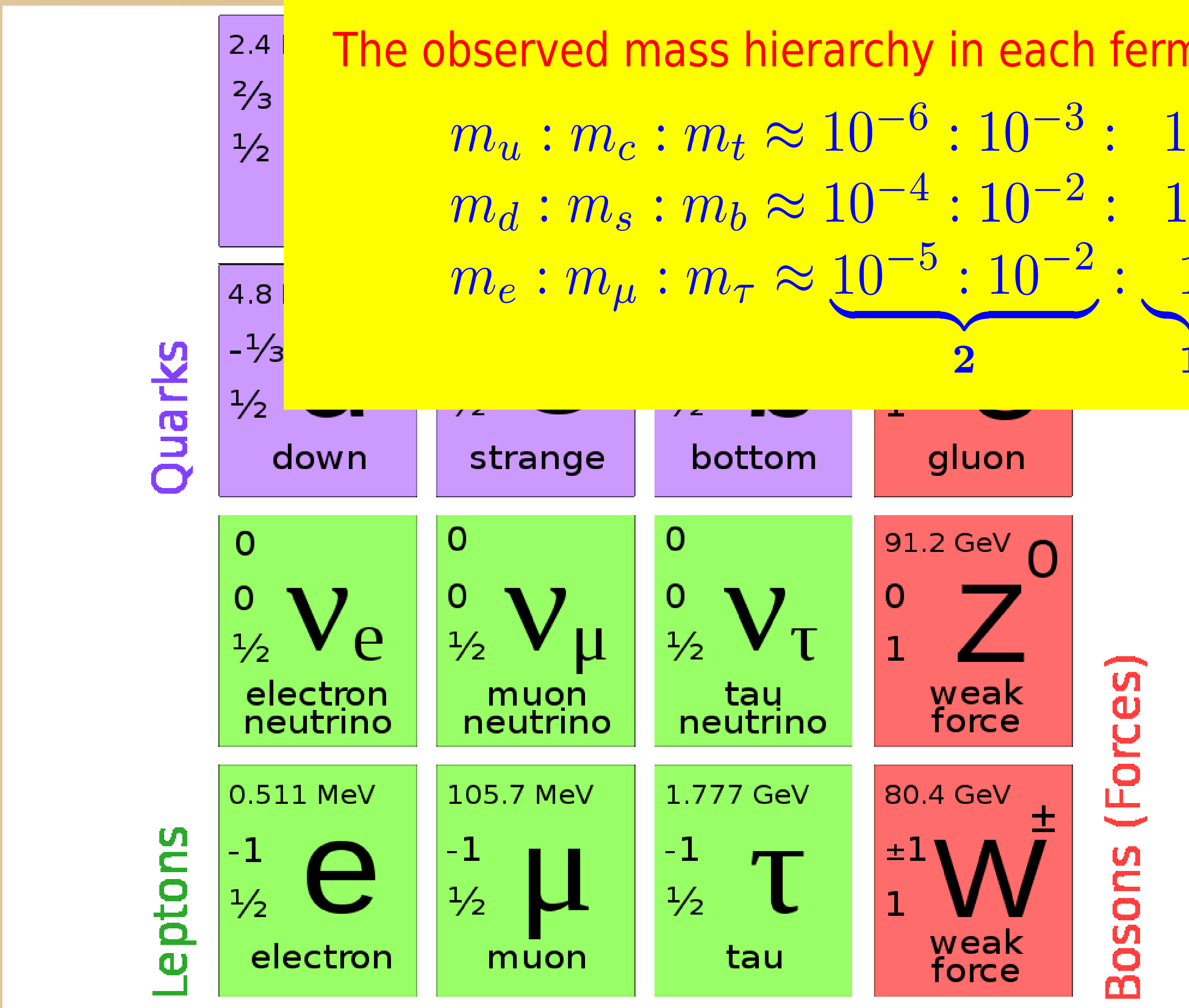
After electroweak symmetry breaking



After electroweak symmetry breaking

The observed mass hierarchy in each fermion sector

$$m_u : m_c : m_t \approx 10^{-6} : 10^{-3} : \ 1,$$
$$m_d : m_s : m_b \approx 10^{-4} : 10^{-2} : \ 1,$$
$$m_e : m_\mu : m_\tau \approx \underbrace{10^{-5} : 10^{-2}}_{\mathbf{2}} : \underbrace{1}_{\mathbf{1}} .$$



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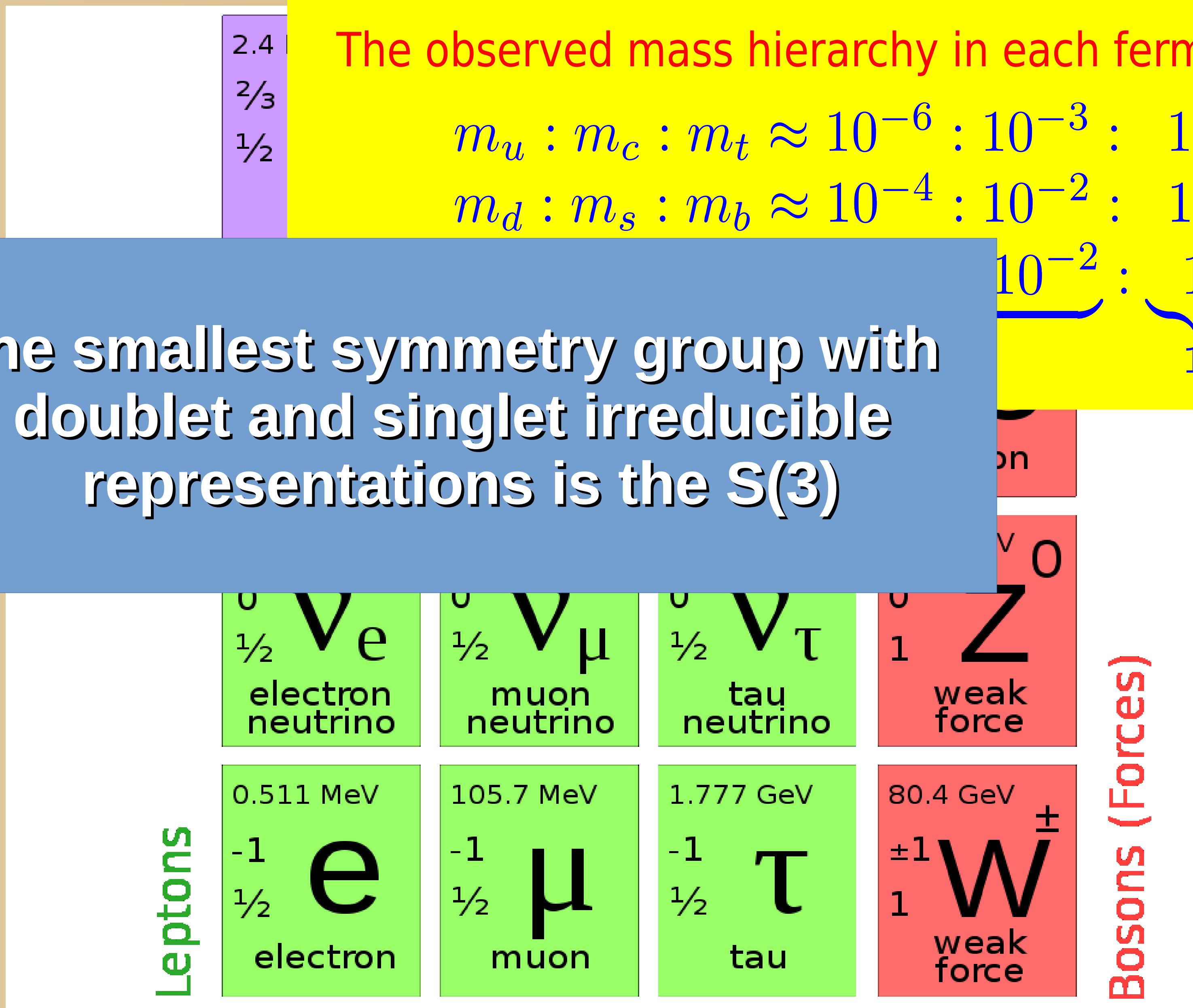
$$10^{-2} : \underbrace{1}_{\mathbf{1}} .$$

The smallest symmetry group with doublet and singlet irreducible representations is the S(3)



After electroweak symmetry breaking

2.4

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The observed mass hierarchy in each fermion sector

$$m_u : m_c : m_t \approx 10^{-6} : 10^{-3} : \; 1,$$

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$$\ldots 10^{-2} : 1 \, .$$

The sm…

dou…

re…

We propose...

- **The group S(3) of permutations of three objects as the flavour symmetry.**

- **We consider two ways to implement the symmetry:**

1) As a symmetry of the mass matrices.
1 Higgs Boson

2) As a symmetry of the Lagrangian.
3 or 4 Higgs Bosons

Matter Content of the Model

$$\underbrace{\mathbf{3} = \mathbf{2} \oplus \mathbf{1}_S}_{\text{Three dimensional representation}}$$

$$\left.\begin{array}{r} \psi^f_{D,(L,R)} = \begin{pmatrix} \psi^f_{1,(L,R)} \\ \psi^f_{2,(L,R)} \end{pmatrix} \\ H_D = \begin{pmatrix} H_1 \\ H_2 \end{pmatrix} \end{array}\right\} \mathbf{2},\ \text{doublet of } S_3 \qquad \left.\begin{array}{c} \psi^f_{S,(L,R)} \\ H_S \end{array}\right\} \mathbf{1},\ \text{symmetric singlet of } S_3$$

Matter Content of the Model

$$\underbrace{\mathbf{3} = \mathbf{2} \oplus \mathbf{1}_S}$$

S_3 Invariant Higgs Potential

$$\begin{aligned} V = {} & \mu_1^2 (x_1 + x_2) + \mu_0^2 x_3 + a x_3^2 + b (x_1 + x_2) x_3 + c (x_1 + x_2)^2 \\ & -4 d x_7^2 + 2e \left[(x_1 - x_2) x_6 + 2 x_4 x_5 \right] + f \left(x_5^2 + x_6^2 + x_8^2 + x_9^2 \right) \\ & + g \left[(x_1 - x_2)^2 + 4 x_4^2 \right] + 2h \left(x_5^2 + x_6^2 - x_8^2 - x_9^2 \right). \end{aligned}$$

$$x_1 = H_1^\dagger H_1, \quad x_4 = \mathcal{R}\left(H_1^\dagger H_2\right), \quad x_7 = \mathcal{I}\left(H_1^\dagger H_2\right),$$
$$x_2 = H_2^\dagger H_2, \quad x_5 = \mathcal{R}\left(H_1^\dagger H_S\right), \quad x_8 = \mathcal{I}\left(H_1^\dagger H_S\right),$$
$$x_3 = H_S^\dagger H_S, \quad x_6 = \mathcal{R}\left(H_2^\dagger H_S\right), \quad x_9 = \mathcal{I}\left(H_2^\dagger H_S\right).$$

$$H_1 = \begin{pmatrix} \phi_1 + i\phi_4 \\ \phi_7 + i\phi_{10} \end{pmatrix}, \quad H_2 = \begin{pmatrix} \phi_2 + i\phi_5 \\ \phi_8 + i\phi_{11} \end{pmatrix}, \quad H_S = \begin{pmatrix} \phi_3 + i\phi_6 \\ \phi_9 + i\phi_{12} \end{pmatrix}.$$

Matter Content of the Model

$$\underbrace{\mathbf{3} = \mathbf{2} \oplus \mathbf{1}_S}$$

S_3 Invariant Higgs Potential

$$\begin{aligned} V \;=\;& \mu_1^2\left(x_1+x_2\right)+\mu_0^2 x_3+a x_3^2+b\left(x_1+x_2\right)x_3+c\left(x_1+x_2\right)^2 \\ & -4d x_7^2+2e\left[\left(x_1-x_2\right)x_6+2x_4x_5\right]+f\left(x_5^2+x_6^2+x_8^2+x_9^2\right) \\ & +g\left[\left(x_1-x_2\right)^2+4x_4^2\right]+2h\left(x_5^2+x_6^2-x_8^2-x_9^2\right). \end{aligned}$$

- The parameters μ's have dimensions of mass and eight real couplings *a*...*h* are dimensionless.
- The Higgs potential has a minimum at

$$\phi_7 = v_1,\ \phi_8 = v_2,\ \phi_9 = v_3,\ \phi_i = 0,\ \ i \neq 7,8,9,$$

The minimization conditions give us three equations determined by demanding the vanishing of $\partial V/\partial \phi_i$.

$$\mu_1^2 = -\left(b+f+2h\right)v_s^2 - 2\left(c+g\right)\left(w_1^2+w_2^2\right) + \frac{3e\left(w_1^2-2w_1w_2-w_2^2\right)v_s}{w_1-w_2},$$

$$\mu_0^2 = -\left[2av_s^2 + \left(b+f+2h\right)\left(w_1^2 \text{ [redacted] }\right) - e\left(\frac{3w_1^2-w_2^2}{v_s}\right)w_2\right].$$

From these, the following relationship [redacted]g the Higgs vev's is obtained:

$$w_1 = \sqrt{3}\,\text{[redacted]}_2.$$

This generate the Nearest Neighbour Interaction (NNI) mass matrices

The Yukawa Lagrangian for Dirac fermion

$$\begin{aligned}\mathcal{L}_{Y_f} = \;& Y_1^f \left(\overline{\psi}^f_{S,L}\psi^f_{S,R}H_S\right) \\ &+\tfrac{1}{\sqrt{2}}Y_2^f \left(\overline{\psi}^f_{1,L}\psi^f_{1,R}+\overline{\psi}^f_{2,L}\psi^f_{2,R}\right) H_S \\ &+\tfrac{1}{2}Y_3^f \left[\left(\overline{\psi}^f_{1,L}H_2+\overline{\psi}^f_{2,L}H_1\right)\psi^f_{1,R}+\left(\overline{\psi}^f_{1,L}H_1-\overline{\psi}^f_{2,L}H_2\right)\psi^f_{2,R}\right] \\ &+\tfrac{1}{\sqrt{2}}Y_5^f \left(\overline{\psi}^f_{1,L}H_1+\overline{\psi}^f_{2,L}H_2\right)\psi^f_{S,R} \\ &+\tfrac{1}{\sqrt{2}}Y_6^f \left[\overline{\psi}^f_{S,L}\left(H_1\psi^f_{1,R}+H_2\psi^f_{2,R}\right)\right]+h.c., \quad f=d,\ e,\end{aligned}$$

For up quarks or Dirac neutrinos

$$H_i \to i\sigma_2 H_i^*.$$

The Generic Mass Matrix for the Dirac Fermions.

After electroweak symmetry breaking

$$w_1 \equiv \langle 0|H_1|0\rangle, \quad w_2 \equiv \langle 0|H_2|0\rangle, \quad v_S \equiv \langle 0|H_S|0\rangle,$$

$$\mathcal{M}^f_{S_3} = \begin{pmatrix} \mu_1+\mu_2 & \mu_4 & \mu_6 \\ \mu_4 & \mu_1-\mu_2 & \mu_7 \\ \mu_8 & \mu_9 & \mu_3 \end{pmatrix}$$

$$\mu_1 \equiv \sqrt{2}Y_2 v_S, \quad \mu_2 \equiv Y_3 w_2, \quad \mu_3 \equiv 2Y_1 v_S,$$
$$\mu_4 \equiv Y_3 w_1, \quad \mu_6 \equiv \sqrt{2}Y_5 w_1,$$
$$\mu_7 \equiv \sqrt{2}Y_5 w_2, \quad \mu_8 \equiv \sqrt{2}Y_6 w_1, \quad \mu_9 \equiv \sqrt{2}Y_6 w_2.$$

$$\mathcal{M}^f_{S_3} \longrightarrow \mathcal{M}^f \equiv \mathcal{R}(\theta)_{12}\mathcal{M}^f_{S_3}\mathcal{R}(\theta)^T_{12} \qquad \tan\theta = \frac{w_2}{w_1}$$

$$\mathcal{M}^f = \begin{pmatrix} \mu_1 + C_\theta^2\left(1-3T_\theta^2\right)\mu_2 & S_\theta C_\theta\left(3-T_\theta^2\right)\mu_2 & 0 \\ S_\theta C_\theta\left(3-T_\theta^2\right)\mu_2 & \mu_1 - C_\theta^2\left(1-3T_\theta^2\right)\mu_2 & \mu_7\sec(\theta) \\ 0 & \mu_9\sec(\theta) & \mu_3 \end{pmatrix}$$

The rotation is unobservable in quark mixings

$$V_{CKM} = U_u U_d^\dagger$$

NNI mass matrices for Dirac fermions

$$\mathcal{M}^f = \mu_1 \mathbb{I}_{3\times 3} + \mathbf{M}_{NNI}$$

$$\mathbf{M}_{NNI} = \begin{pmatrix} 0 & \frac{2}{\sqrt{3}}\mu_2 & 0 \\ \frac{2}{\sqrt{3}}\mu_2 & 0 & \frac{2}{\sqrt{3}}\mu_7 \\ 0 & \frac{2}{\sqrt{3}}\mu_9 & \mu_3 - \mu_1 \end{pmatrix}$$

The matrix has the shape NNI and eigenvalues σ_i^f

The physical masses are related to the shifted masses σ_i^f by:

$$m_i^f = \mu_1^f + \sigma_i^f$$

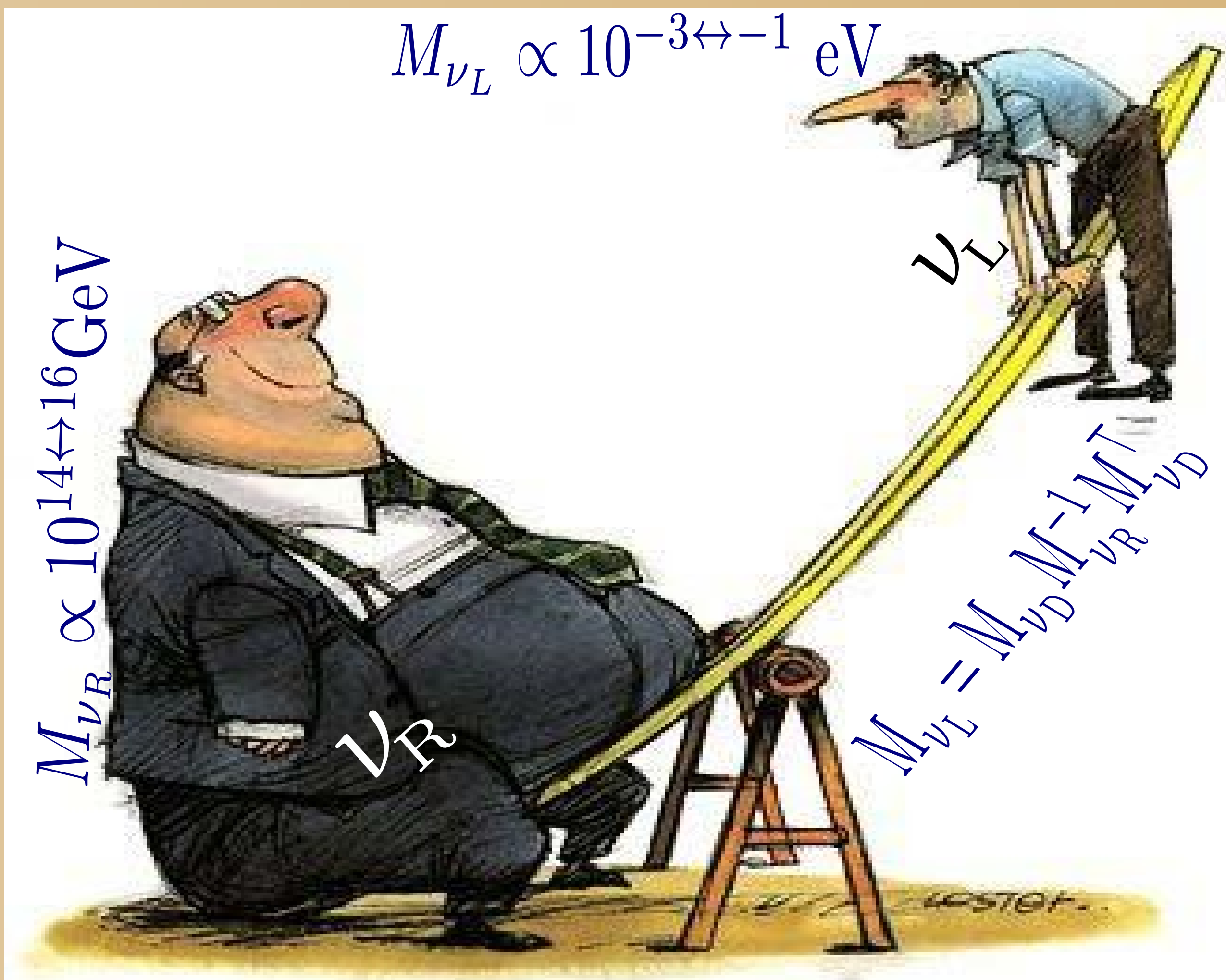
Numerical Results for Quarks

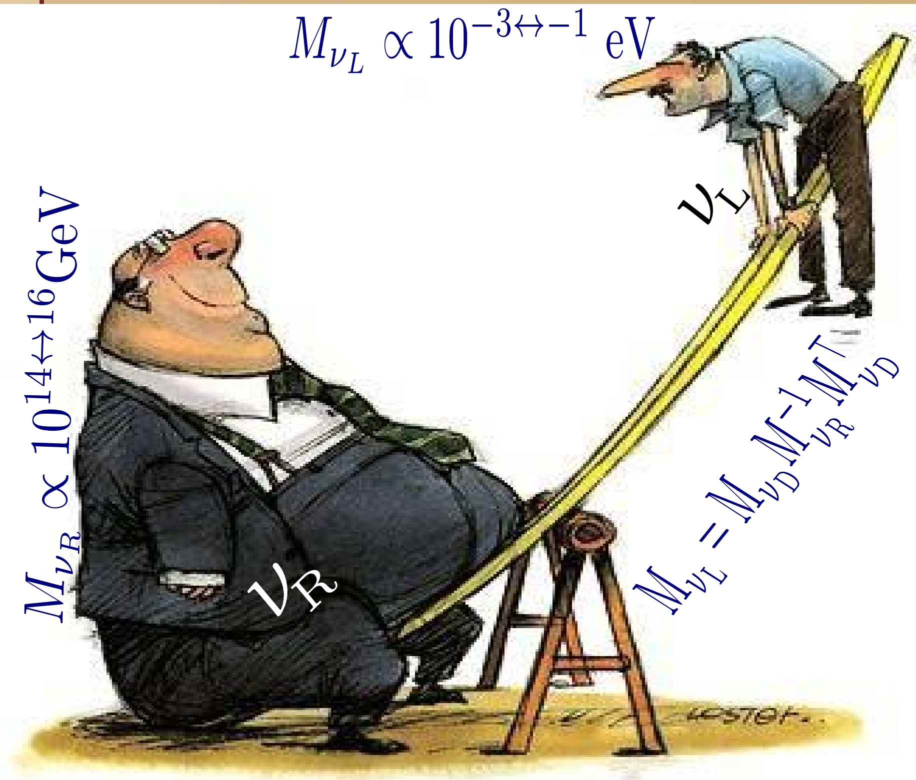
$$\chi^2 = \sum_{i=d,s,b} \frac{\left(\left|V_{ui}^{th}\right| - \left|V_{ui}^{ex}\right|\right)^2}{\sigma^2_{V_{ui}}} + \frac{\left(\left|V_{cb}^{th}\right| - \left|V_{cb}^{ex}\right|\right)^2}{\sigma^2_{V_{cb}}},$$

Quarks

Parameter	Central value	χ^2	Values with restricted precision	χ^2
Fit using $\lvert V_{cd}\rvert$ and the 2013 values of the parameters with $\tilde{m}_s^{th}$				
$\widetilde{\sigma_u}(M_Z)$	1.63792×10^{-6}		$(1.64 \pm 0.16) \times 10^{-6}$	
$\tilde{\sigma}_c(M_Z)$	3.58675×10^{-3}		$(3.59 \pm 0.78) \times 10^{-3}$	
$\widetilde{\sigma_d}(M_Z)$	1.21487×10^{-3}		$(1.21 \pm 0.88) \times 10^{-3}$	
$\tilde{\sigma}_s(M_Z)$	1.89366×10^{-2}		$(1.90 \pm 0.81) \times 10^{-3}$	
δ_u	9.98871×10^{-2}		$(1.00 \pm 0.5) \times 10^{-2}$	
δ_d	5.37158×10^{-2}		$(5.37 \pm 7.19 \times 10^{-2}$	
$\cos\phi_2$	7.82382×10^{-1}		$(7.82 \pm 9.35) \times 10^{-1}$	
		3.1×10^{-2}		4

Left handed Neutrinos and type-I seesaw





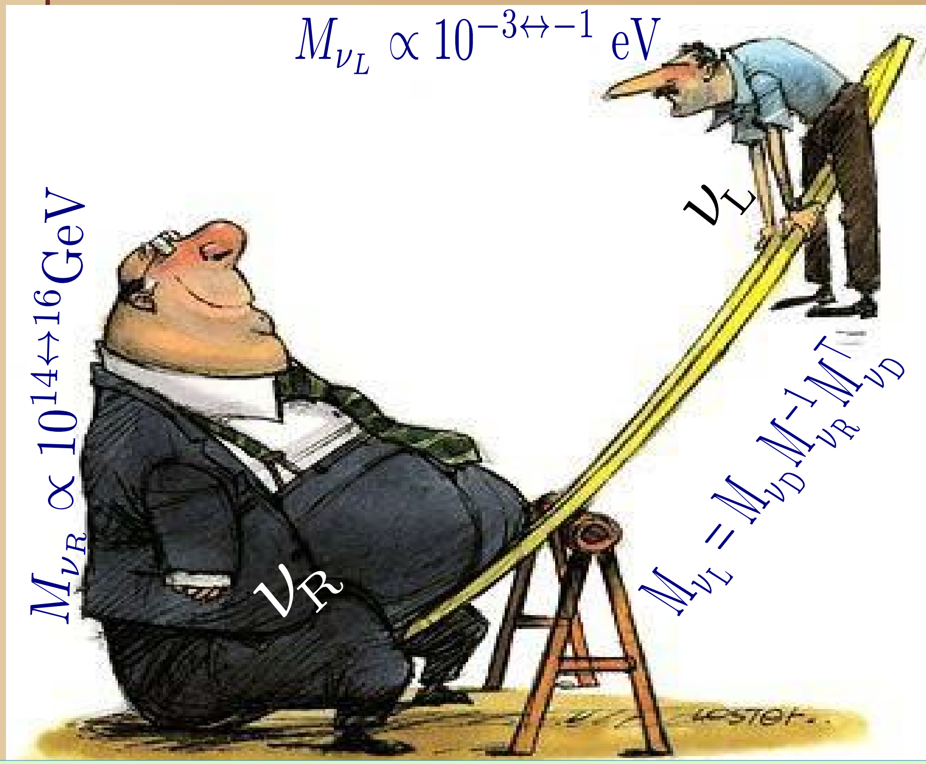
The left handed neutrino mass matrix:

$$\begin{pmatrix} \frac{2(\mu_2^\nu)^2}{\overline{M}} & \frac{2\lambda(\mu_2^\nu)^2}{\overline{M}} & \frac{2\mu_2^\nu\mu_4^\nu}{\overline{M}} \\ \frac{2\lambda(\mu_2^\nu)^2}{\overline{M}} & \frac{2(\mu_2^\nu)^2}{\overline{M}} & \frac{2\mu_2^\nu\mu_4^\nu\lambda}{\overline{M}} \\ \frac{2\mu_2^\nu\mu_4^\nu}{\overline{M}} & \frac{2\mu_2^\nu\mu_4^\nu\lambda}{\overline{M}} & \frac{2(\mu_4^\nu)^2}{\overline{M}} + \frac{(\mu_3^\nu)^2}{M_3} \end{pmatrix}$$

$$\lambda = \frac{1}{2}\left(\frac{M_2 - M_1}{M_1 + M_2}\right), \text{ and } \overline{M} = 2\frac{M_1 M_2}{M_2 + M_1}.$$

Texture zeroes in the fermionic matrices

$$\mathbf{M}_k = \mathbf{Q}_k \mathbf{U}_{\theta_k}\left(\mu_k^0 \mathbb{I}_{3\times 3} + \widehat{\mathbf{M}}_k\right)\mathbf{U}_{\theta_k}^\dagger \mathbf{Q}_k^{\dagger(\top)}$$



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$\mathbf{U}_{\theta_k}$ is a rotation matrix

$$\begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$\tan\theta = \frac{w_1}{w_2}$ for Dirac

$\tan\theta = 1$ for Neutrinos

CP violation phases

$$\mathbf{Q} \equiv \text{diag}\left(1, e^{i\phi_{1k}}, e^{i\phi_{2k}}\right)$$

$\widehat{\mathbf{M}}_k$ Matrix with two texture zeroes

$$\begin{pmatrix} 0 & \sqrt{\frac{\tilde\sigma_1^k \tilde\sigma_2^k}{1-\delta_k}} & 0 \\ \sqrt{\frac{\tilde\sigma_1^k \tilde\sigma_2^k}{1-\delta_k}} & \tilde\sigma_1^k - \tilde\sigma_2^k + \delta_k & \sqrt{\frac{\delta_k}{1-\delta_k}\xi_1^k \xi_2^k} \\ 0 & \sqrt{\frac{\delta_k}{1-\delta_k}\xi_1^k \xi_2^k} & 1-\delta_k \end{pmatrix}$$

$$\xi_1^k \equiv 1 - \tilde\sigma_1^k - \delta_k, \quad \xi_2^k \equiv 1 + \tilde\sigma_2^k - \delta_k ,$$

Numerical Results for Leptons

$$\chi^2 = \frac{\left(S_{12}^{2,ex} - S_{12}^{2,th}\right)^2}{\sigma^2_{S_{12}^{2,ex}}} + \frac{\left(S_{23}^{2,ex} - S_{23}^{2,th}\right)^2}{\sigma^2_{S_{23}^{2,ex}}} + \frac{\left(S_{13}^{2,ex} - S_{13}^{2,th}\right)^2}{\sigma^2_{S_{13}^{2,ex}}}$$

$$S_{ij}^2 \equiv \sin^2\theta_{ij}$$

$$\widetilde{m}_{\nu_1} = \frac{m_{\nu_1}}{m_{\nu_3}} = \sqrt{1 - \frac{\Delta m^2_{\rm ATM}}{m^2_{\nu_3}}}, \quad \widetilde{m}_{\nu_2} = \frac{m_{\nu_2}}{m_{\nu_3}} = \sqrt{1 - \frac{\Delta m^2_{32}}{m^2_{\nu_3}}} \quad ({\rm NH}),$$

$$\widetilde{m}_{\nu_3} = \frac{m_{\nu_3}}{m_{\nu_2}} = \sqrt{1 - \frac{\Delta m^2_{23}}{m_{\nu_2}}}, \quad \widetilde{m}_{\nu_1} = \frac{m_{\nu_1}}{m_{\nu_2}} = \sqrt{1 - \frac{\Delta m^2_{\odot}}{m^2_{\nu_2}}} \quad ({\rm IH}),$$

Numerical Results for Leptons

$$\chi^2 = \frac{\left(S_{12}^{2,ex} - S_{12}^{2,th}\right)^2}{\sigma^2_{S_{12}^{2,ex}}} + \frac{\left(S_{23}^{2,ex} - S_{23}^{2,th}\right)^2}{\sigma^2_{S_{23}^{2,ex}}} + \frac{\left(S_{13}^{2,ex} - S_{13}^{2,th}\right)^2}{\sigma^2_{S_{13}^{2,ex}}} \qquad S_{ij}^2 \equiv \sin^2\theta_{ij}$$

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$$\widetilde{m}_{\nu_3} = \frac{m_{\nu_3}}{m_{\nu_2}}$$

Best Fit point $\pm 1\sigma$, 2σ range and 3σ range (Phys Rev D90: 093006)

$$\Delta m^2_{21}\left[10^{-5}\ {\rm eV}^2\right] = 7.60^{+0.19}_{-0.18}\ (7.26 - 7.99)\ (7.11 - 8.18)\,,$$

$$\left|\Delta m^2_{31}\right|\left[10^{-3}\ {\rm eV}^2\right] = \begin{cases} 2.48^{+0.05}_{-0.07}\ (2.35 - 2.59)\ (2.30 - 2.65) & ({\rm NH}), \\ 2.38^{+0.05}_{-0.06}\ (2.26 - 2.48)\ (2.20 - 2.54) & ({\rm IH}), \end{cases}$$

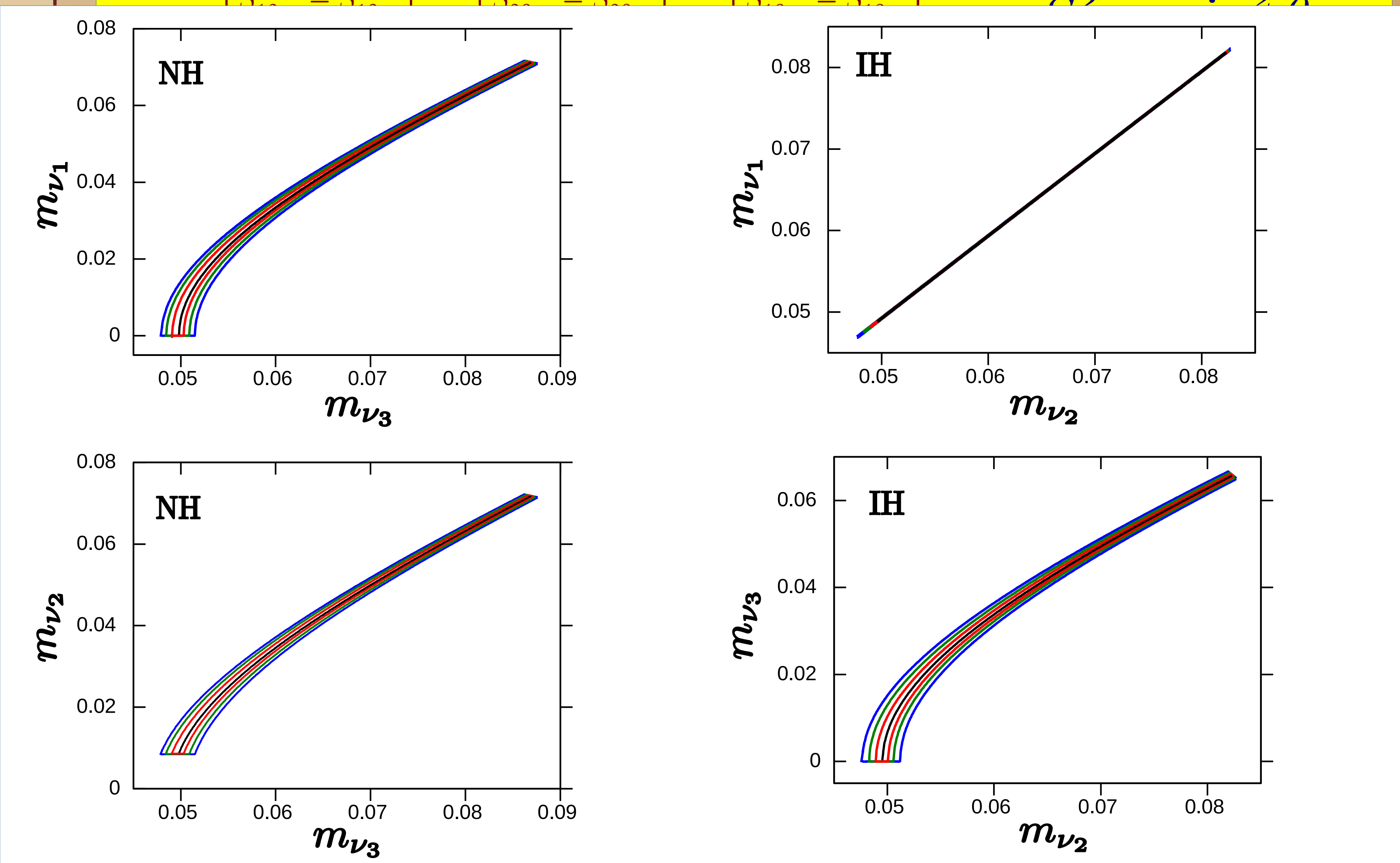
$$\sin^2\theta_{12}/10^{-1} = 3.23 \pm 0.16\ (2.92 - 3.57)\ (2.78 - 3.75)\,,$$

$$\sin^2\theta_{23}/10^{-1} = \begin{cases} 5.67^{+0.32}_{-1.24}\ (4.14 - 6.23)\ (3.93 - 6.43) & ({\rm NH}), \\ 5.73^{+0.25}_{-0.39}\ (4.35 - 6.21)\ (4.03 - 6.40) & ({\rm IH}), \end{cases}$$

$$\sin^2\theta_{13}/10^{-2} = \begin{cases} 2.26 \pm 0.12\ (2.02 - 2.50)\ (1.90 - 2.62) & ({\rm NH}), \\ 2.29 \pm 0.12\ (2.05 - 2.52)\ (1.93 - 2.65) & ({\rm IH}), \end{cases}$$

Numerical Results for Leptons

$\left(S^{2,ex} - S^{2,th}\right)^2 \quad \left(S^{2,ex} - S^{2,th}\right)^2 \quad \left(S^{2,ex} - S^{2,th}\right)^2 \quad S^2 = \sin^2\theta$



0: 093006)

),

2.65) (NH),

2.54) (IH),

3.75),

) (NH),

) (IH),

2) (NH),

2.29 ± 0.12 (2.05 – 2.52) (1.95 – 2.65) (IH),

Numerical Results for Leptons

$$\chi^2 = \frac{\left(S_{12}^{2,ex} - S_{12}^{2,th}\right)^2}{\sigma^2_{S_{12}^{2,ex}}} + \frac{\left(S_{23}^{2,ex} - S_{23}^{2,th}\right)^2}{\sigma^2_{S_{23}^{2,ex}}} + \frac{\left(S_{13}^{2,ex} - S_{13}^{2,th}\right)^2}{\sigma^2_{S_{13}^{2,ex}}} \qquad S_{ij}^2 \equiv \sin^2\theta_{ij}$$

$$\widetilde{m}_{\nu_1} = \frac{m_{\nu_1}}{m_{\nu_3}} = \sqrt{1 - \frac{\Delta m^2_{\rm ATM}}{m^2_{\nu_3}}}, \quad \widetilde{m}_{\nu_2} = \frac{m_{\nu_2}}{m_{\nu_3}} = \sqrt{1 - \frac{\Delta m^2_{32}}{m^2_{\nu_3}}} \quad ({\rm NH}),$$

$$\widetilde{m}_{\nu_3} = \frac{m_{\nu_3}}{m_{\nu_2}} = \sqrt{1 - \frac{\Delta m^2_{23}}{m_{\nu_2}}}, \quad \widetilde{m}_{\nu_1} = \frac{m_{\nu_1}}{m_{\nu_2}} = \sqrt{1 - \frac{\Delta m^2_{\odot}}{m^2_{\nu_2}}} \quad ({\rm IH}),$$

$$m_{\nu_3} = \begin{cases} \left(5.35^{+4.32}_{-1.73}\right) \times 10^{-2}{\rm eV} \\ \left(4.44^{+4.21}_{-3.87}\right) \times 10^{-2}{\rm eV} \end{cases}, \; m_{\nu_2} = \begin{cases} \left(2.01^{+6.42}_{-0.98}\right) \times 10^{-2}{\rm eV} \\ \left(6.71^{+3.25}_{-1.81}\right) \times 10^{-2}{\rm eV} \end{cases}, \; m_{\nu_1} = \begin{cases} \left(1.08^{+6.59}_{-1.30}\right) \times 10^{-2}{\rm eV} \\ \left(6.65^{+3.27}_{-1.83}\right) \times 10^{-2}{\rm eV} \end{cases}.$$

$$\delta_e = 0.8478^{+0,0045}_{-0.0046} \quad \widetilde{\mu}_0 = \begin{cases} 0.22^{+0.63}_{-0.20} \\ 0.56^{+0.29}_{-0.56} \end{cases} \quad \text{and} \quad \delta_\nu = \begin{cases} 0.75^{+0.24}_{-0.15} \\ 0.73^{+0.25}_{-0.09} \end{cases}.$$

Numerical Results for Leptons

$$\chi^2 = \frac{\left(S_{12}^{2,ex} - S_{12}^{2,th}\right)^2}{\cdots} + \frac{\left(S_{23}^{2,ex} - S_{23}^{2,th}\right)^2}{\cdots} + \frac{\left(S_{13}^{2,ex} - S_{13}^{2,th}\right)^2}{\cdots}$$

$$S_{ij}^2 \equiv \sin^2\theta_{ij}$$

$$\theta_{12}^{\ell th} = \begin{cases} \left(34.71^{+0.91}_{-0.98}\right)^\circ \\ \left(34.73^{+0.89}_{-1.11}\right)^\circ \end{cases}, \quad \theta_{23} = \begin{cases} \left(45.83^{+4.49}_{-3.98}\right)^\circ \\ \left(48.57^{+2.07}_{-2.76}\right)^\circ \end{cases}, \quad \theta_{13} = \begin{cases} \left(8.77^{+0.40}_{-0.32}\right)^\circ & \text{(NH)}, \\ \left(8.93^{+0.33}_{-0.39}\right)^\circ, & \text{(IH)}, \end{cases}$$

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Numerical Results for Leptons

$$\chi^2 = \frac{\left(S_{12}^{2,ex} - S_{12}^{2,th}\right)^2}{\cdots} + \frac{\left(S_{23}^{2,ex} - S_{23}^{2,th}\right)^2}{\cdots} + \frac{\left(S_{13}^{2,ex} - S_{13}^{2,th}\right)^2}{\cdots}$$

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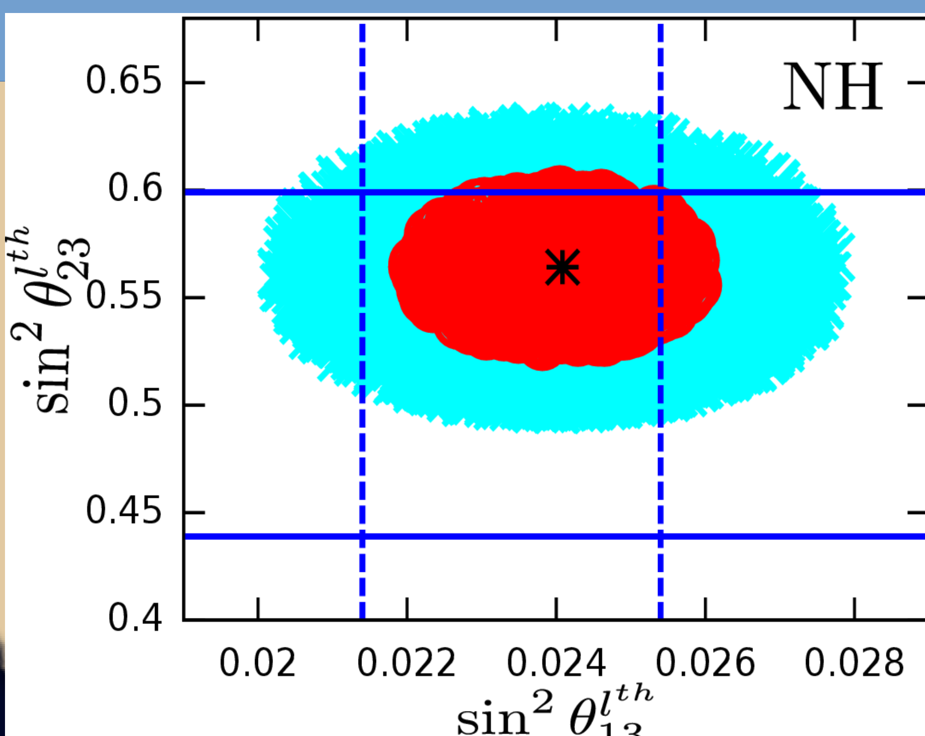
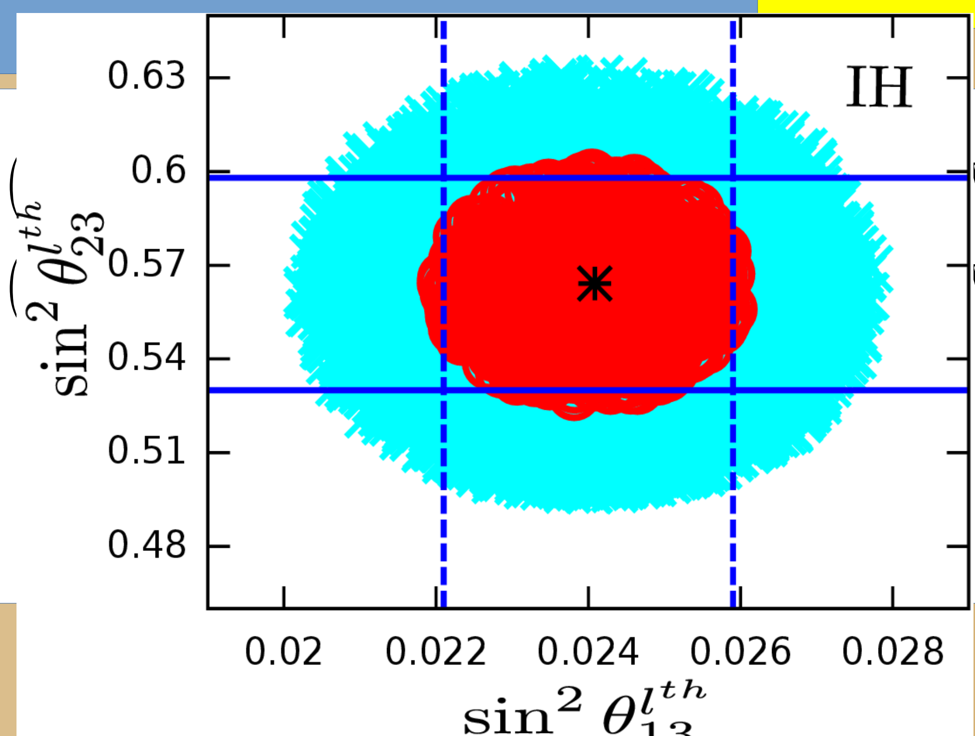
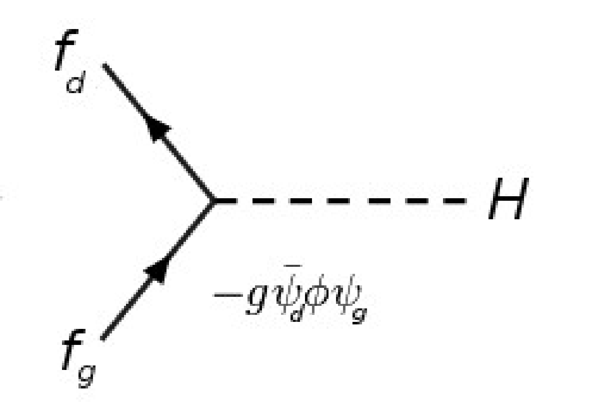




Fermions-Higgs couplings of type Cheng-Sher



The elements of $\mathbf{O}^i_{fj}$ matrices in terms of shifted mass ratios are:

$$O^f_{11} = \sqrt{\frac{\widehat{\sigma}_{f2[1]}\,\xi_{f1[3]}}{\mathcal{D}_{f1[3]}}},\quad O^f_{12} = \sqrt{\frac{\widehat{\sigma}_{f1[3]}\,\xi_{f2[1]}}{\mathcal{D}_{f2[1]}}},\quad O^f_{13} = \sqrt{\frac{\widehat{\sigma}_{f1[3]}\,\widehat{\sigma}_{f2[1]}\delta_f}{\mathcal{D}_{f3[2]}}},$$

$$O^f_{21} = \sqrt{\frac{\widehat{\sigma}_{f1[3]}\,(1-\delta_f)\,\xi_{f1[3]}}{\mathcal{D}_{f1[3]}}},\quad O^f_{22} = \sqrt{\frac{\widehat{\sigma}_{f2[1]}\,(1-\delta_f)\,\xi_{f2[1]}}{\mathcal{D}_{f2[1]}}},\quad O^f_{23} = \sqrt{\frac{(1-\delta_f)\,\delta_f}{\mathcal{D}_{f3[2]}}},$$

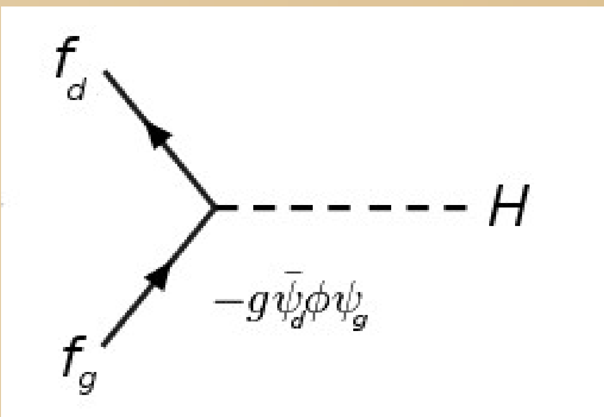
$$O^f_{31} = \sqrt{\frac{\widehat{\sigma}_{f1[3]}\,\delta_f\,\xi_{f2[1]}}{\mathcal{D}_{f1[3]}}},\quad O^f_{32} = \sqrt{\frac{\widehat{\sigma}_{f2[1]}\,\delta_f\xi_{f1[3]}}{\mathcal{D}_{f2[1]}}},\quad O^f_{33} = \sqrt{\frac{\xi_{f1[3]}\xi_{f2[1]}}{\mathcal{D}_{f3[2]}}},$$

$\mathcal{D}_{f1[3]} = (1-\delta_f)\left(\widehat{\sigma}_{f1[3]} + \widehat{\sigma}_{f2[1]}\right)\left(1-\widehat{\sigma}_{f1[3]}\right)$, $\mathcal{D}_{f2[1]} = (1-\delta_f)\left(\widehat{\sigma}_{f1[3]} + \widehat{\sigma}_{f2[1]}\right)\left(1+\widehat{\sigma}_{f2[1]}\right)$ and $\mathcal{D}_{f3[2]} = (1-\delta_f)\left(1+\widehat{\sigma}_{f2[1]}\right)\left(1-\widehat{\sigma}_{f1[3]}\right)$.

The shifted mass ratios are:

$$\widehat{\sigma}_{f1[3]} = \frac{\widehat{m}_{f1[3]} - \widehat{\mu}^f_0}{1-\widehat{\mu}^f_0} \quad \text{and} \quad \widehat{\sigma}_{f2[1]} = \frac{\left|\widehat{m}_{f2[1]} - \widehat{\mu}^f_0\right|}{1-\widehat{\mu}^f_0},$$

Fermions-Higgs couplings of type Cheng-Sher



The elements of $\mathbf{O}^{i}_{fj}$ matrices in terms of shifted mass ratios are:

$$O^f_{11} = \sqrt{\frac{\widehat{\sigma}_{f2[1]}\,\xi_{f1[3]}}{\mathcal{D}_{f1[3]}}}, \qquad O^f_{12} = \sqrt{\frac{\widehat{\sigma}_{f1[3]}\,\xi_{f2[1]}}{\mathcal{D}_{f2[1]}}}, \qquad O^f_{13} = \sqrt{\frac{\widehat{\sigma}_{f1[3]}\,\widehat{\sigma}_{f2[1]}\delta_f}{\mathcal{D}_{f3[2]}}},$$

$$O^f_{21} = \sqrt{\frac{\widehat{\sigma}_{f1[3]}\,(1-\delta_f)\,\xi_{f1[3]}}{\mathcal{D}_{f1[3]}}}, \qquad O^f_{22} = \sqrt{\frac{\widehat{\sigma}_{f2[1]}\,(1-\delta_f)\,\xi_{f2[1]}}{\mathcal{D}_{f2[1]}}}, \qquad O^f_{23} = \sqrt{\frac{(1-\delta_f)\,\delta_f}{\mathcal{D}_{f3[2]}}},$$

$$O^f_{31} = \sqrt{\frac{\widehat{\sigma}_{f1[3]}\,\delta_f\,\xi_{f2[1]}}{\mathcal{D}_{f1[3]}}}, \qquad O^f_{32} = \sqrt{\frac{\widehat{\sigma}_{f2[1]}\,\delta_f\xi_{f1[3]}}{\mathcal{D}_{f2[1]}}}, \qquad O^f_{33} = \sqrt{\frac{\xi_{f1[3]}\xi_{f2[1]}}{\mathcal{D}_{f3[2]}}},$$

$\mathcal{D}_{f1[3]} = (1 - \ldots)$

and $\mathcal{D}_{f3[2]} = (\ldots$

The shifted mass ratios are:

$$\widehat{\sigma}_{f1[3]} = \frac{\widehat{m}_{f1[3]} - \widehat{\mu}^f_0}{1-\widehat{\mu}^f_0} \quad \text{and} \quad \widehat{\sigma}_{f2[1]} = \frac{|\widehat{m} \ldots}{\ldots}$$

The elements of Yukawa matrices in the mass basis

$$\left(\widetilde{\mathbf{Y}}^k_i\right)_{\mathtt{hi}} = \frac{\sqrt{m_{k\mathtt{h}}m_{k\mathtt{i}}}}{v}\left(\widetilde{\chi}^k_i\right)_{\mathtt{hi}},$$

Flavour Changing neutral Currents

FCNC processes	Theoretical BR	Experimental upper bound BR
$\tau \to 3\mu$	8.43×10^{-14}	2×10^{-7}
$\tau \to \mu e^+ e^-$	3.15×10^{-17}	2.7×10^{-7}
$\tau \to \mu\gamma$	9.24×10^{-15}	6.8×10^{-8}
$\tau \to e\gamma$	5.22×10^{-16}	1.1×10^{-11}
$\mu \to 3e$	2.53×10^{-16}	1×10^{-12}
$\mu \to e\gamma$	2.42×10^{-20}	1.2×10^{-11}

Summary

- The permutational symmetry S_3 is the smallest non-abelian discrete symmetry suggested by data.
- Used as flavor symmetry in the quark, lepton and Higgs sectors allows for “unified” treatment of Dirac fermion masses.
- Godd phenomenology both in the quark an lepton sectors.

Thank you!!!