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Hadronic matter phases and the application to rapidly rotating neutron stars

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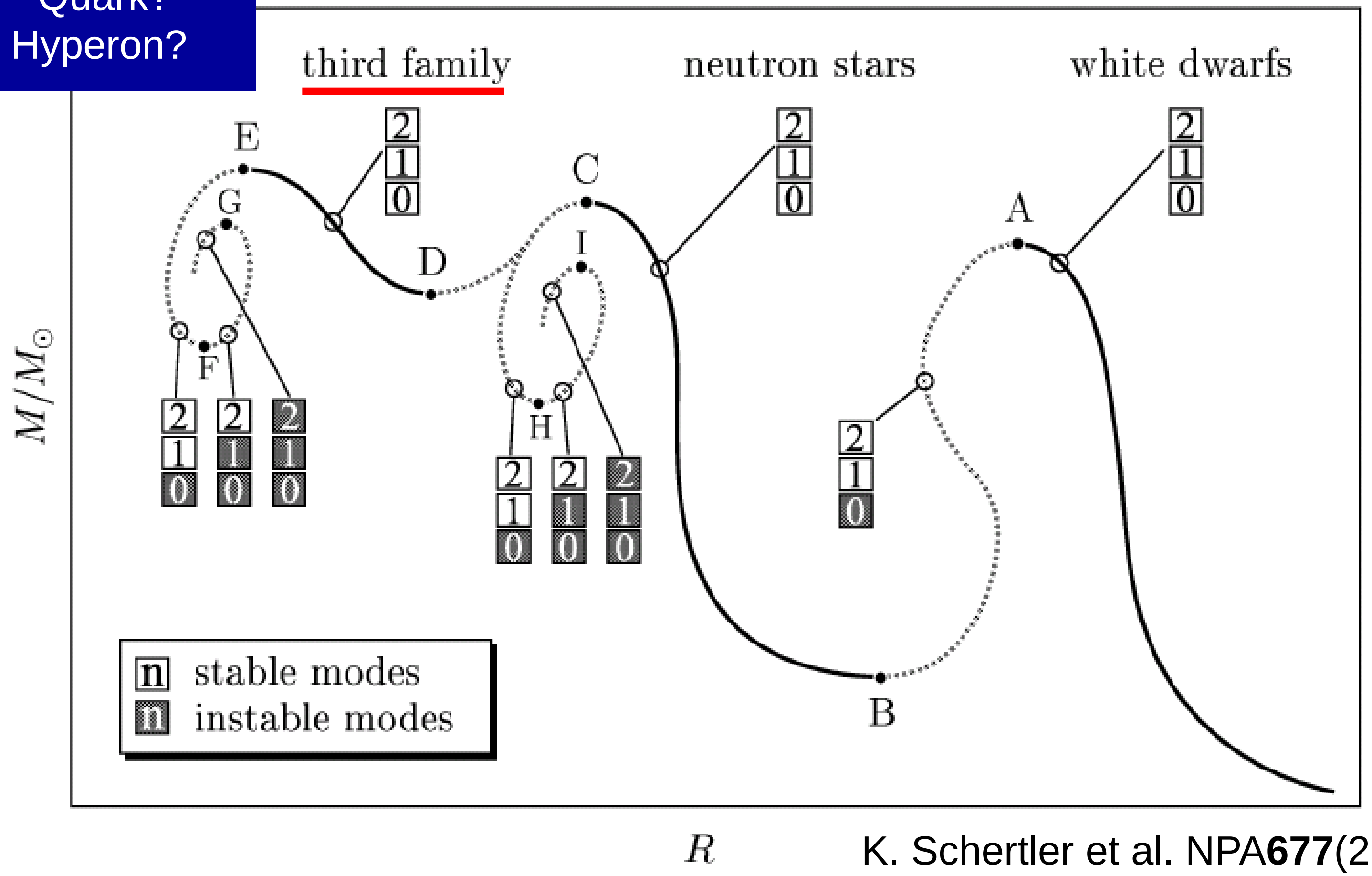
Outline

- Compact stars and equation of state(EOS) for high density matter
- Phase transition between hadronic matter and quark matter
- Application to TOV (Tolman-Oppenheimer-Volkoff) equation
- Inner structures of the hybrid star
- Application to the rotating neutron stars

Compact stars

Quark?
Hyperon?

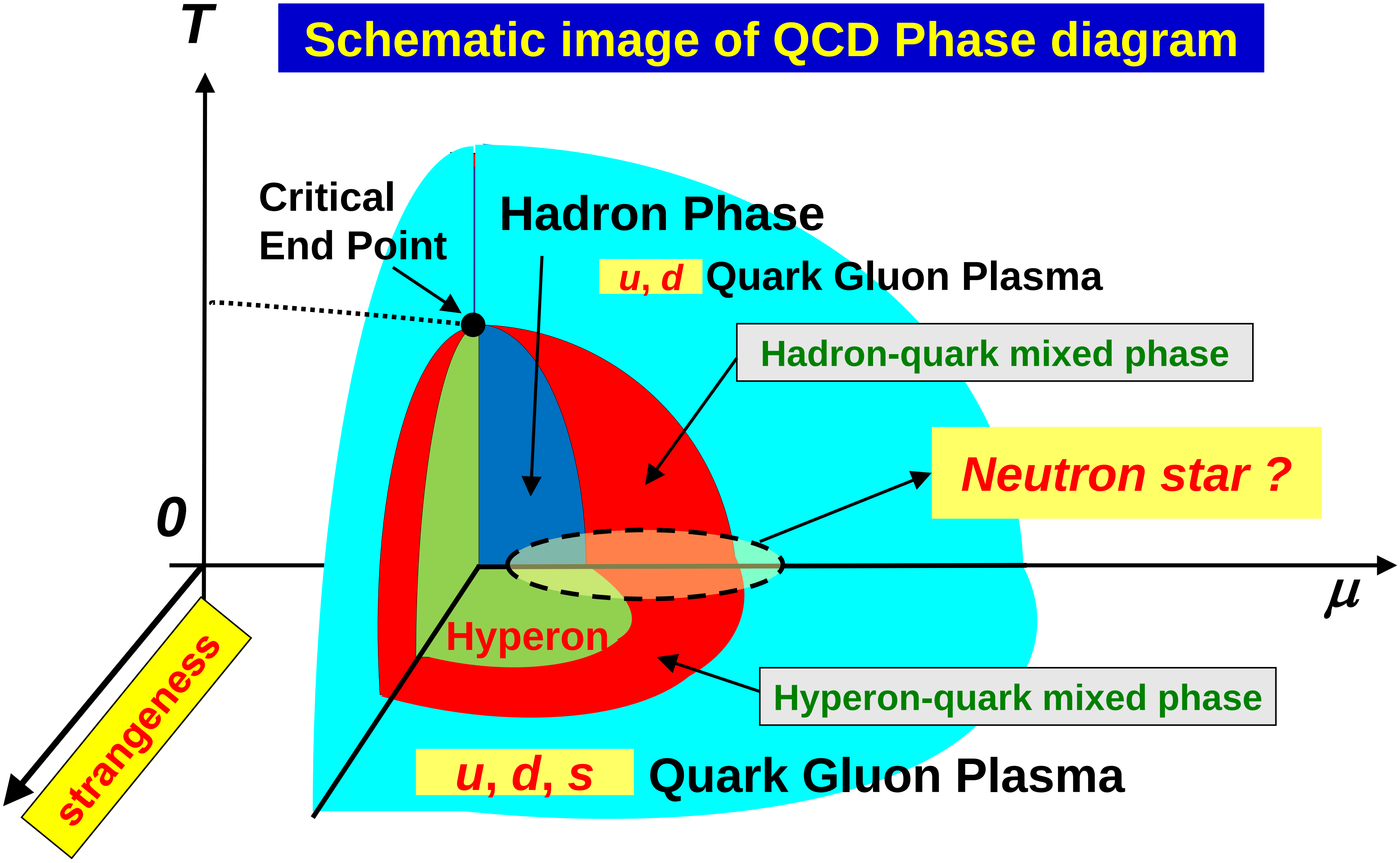


K. Schertler et al. NPA**677**(2000)463

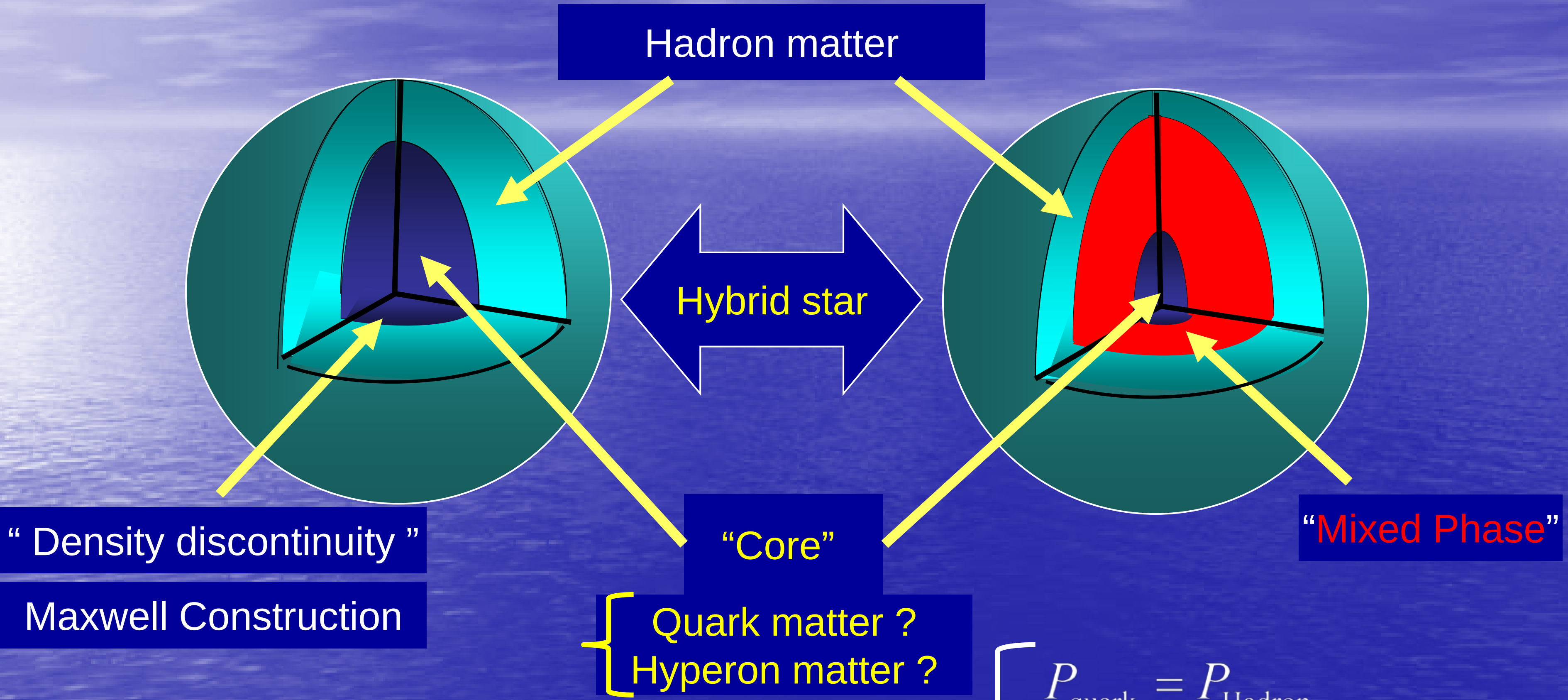
Inner structures → Equation of State (EOS)

EOS: Supernovae explosion, Relativistic Heavy-Ion collision

Schematic image of QCD Phase diagram



Inner structures of the neutron star…



“Gibbs conditions”

$$\begin{cases} P_{\text{quark}} = P_{\text{Hadron}} \\ T_{\text{quark}} = T_{\text{Hadron}} \\ \mu^{\text{B}}_{\text{quark}} = \mu^{\text{B}}_{\text{Hadron}} \\ \mu^{e}_{\text{Hadron}} = \mu^{e}_{\text{quark}} \end{cases}$$

Implication to a hybrid star

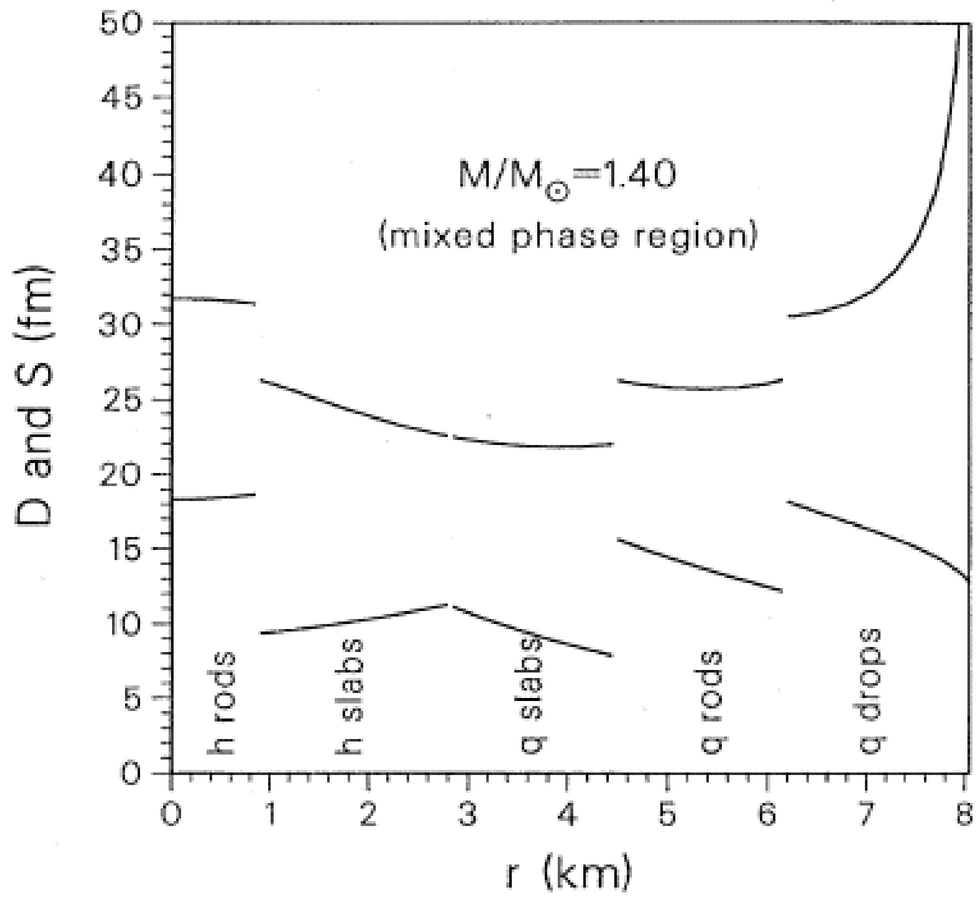
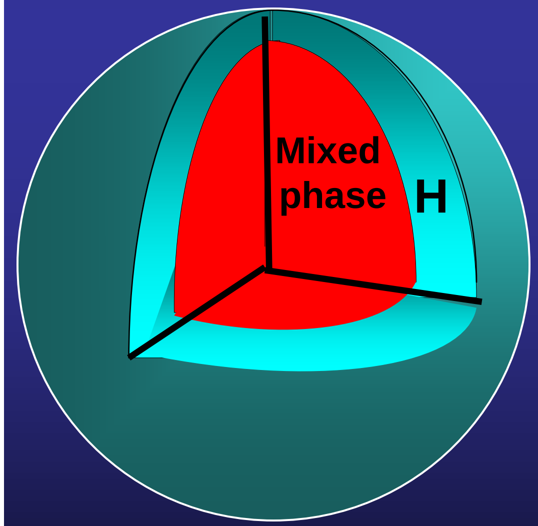


FIG. 2. Similar to Fig. 1 but for slightly less massive star. Mixed crystalline phase now extends to star's center. Radius is 12.3 km.

"Gibbs conditions"

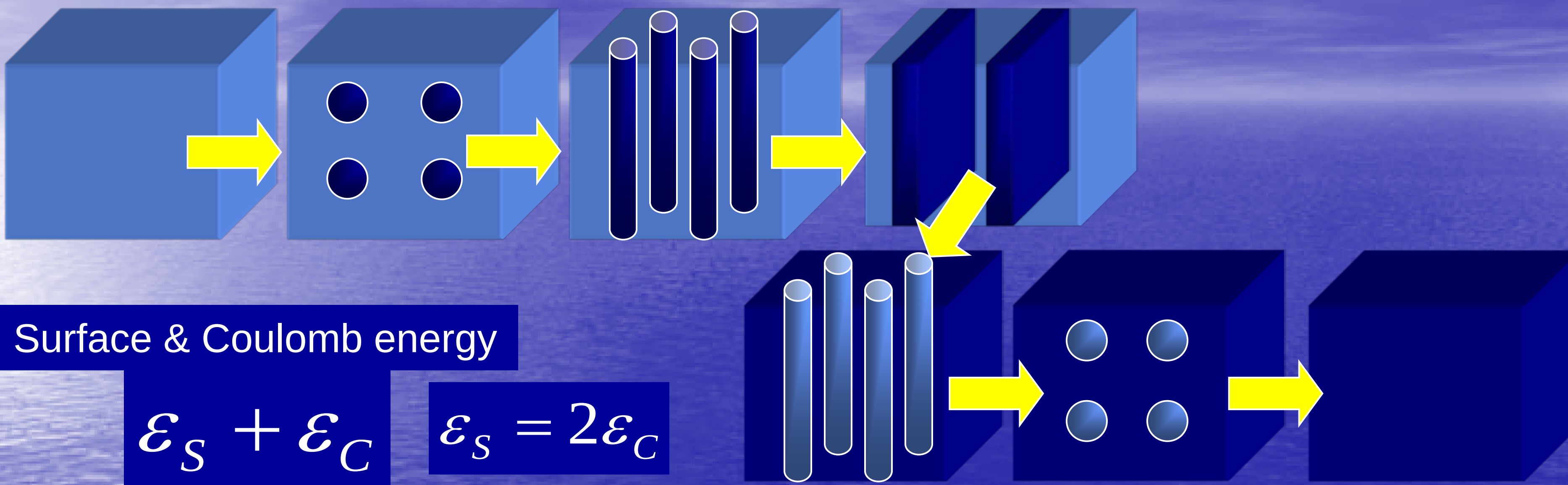
$$P_{\text{quark}} = P_{\text{Hadron}}$$
$$T_{\text{quark}} = T_{\text{Hadron}}$$
$$\mu^{\text{B}}_{\text{quark}} = \mu^{\text{B}}_{\text{Hadron}}$$
$$\mu^{e}_{\text{Hadron}} = \mu^{e}_{\text{quark}}$$



Glendenning and Pei Phys.Rev.C 52(1995) 2250

“Crystaline structure appear”

Glendenning and Pei Phys.Rev.C 52(1995) 2250



Surface & Coulomb energy

$$\varepsilon_S + \varepsilon_C$$

$$\varepsilon_S = 2\varepsilon_C$$

They didn’t solve the Poisson equation

- Voskresensky ,Yasuhira and Tatsumi, PLB**541**(2002)93；NPA**723**(2003)291

with screening effect → “Maxwell construction picture” ･･･○

solve Poisson equation with linear approximation

Density Functional Theory

T.E., T.Maruyama, S.Chiba
and T.Tatsumi PTP115(2006)337

$$\Omega_{\rm tot} = \Omega_{\rm Q} + \Omega_{\rm H} + \Omega_{\rm surface} + \Omega_{\rm e} + E_{\rm V}$$

Quark Phase

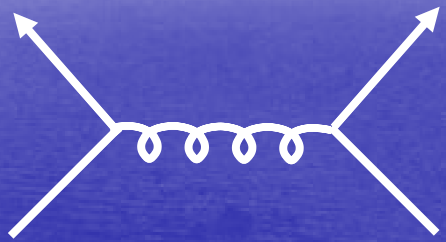
$$\Omega_{\rm u} = \int d^3r \left[\frac{3\pi^{\frac{2}{3}}}{4} \left(1 + \frac{2\alpha_{\rm c}}{3\pi}\right) \rho_{\rm u}^{\frac{4}{3}} - \mu_{\rm u}\rho_{\rm u} \right]$$

$$\Omega_{\rm d} = \int d^3r \left[\frac{3\pi^{\frac{2}{3}}}{4} \left(1 + \frac{2\alpha_{\rm c}}{3\pi}\right) \rho_{\rm d}^{\frac{4}{3}} - \mu_{\rm d}\rho_{\rm d} \right]$$

$$\Omega_{\rm s} = \int d^3r \left[\epsilon_{\rm s}(\rho_{\rm s}) - \mu_{\rm s}\rho_{\rm s} + B\right] \quad B \cdots \text{bag constant}$$

$$\Omega_{\rm I} = \Omega_{\rm u} + \Omega_{\rm d} + \Omega_{\rm s}$$

interaction : One Gluon Exchange



Hadron (Nucleon) Phase

$$\Omega_{\rm n} = \int d^3r \left[\frac{3}{10m} \left(3\pi^2\right)^{\frac{2}{3}} \rho_{\rm n}^{\frac{5}{3}} - \mu_{\rm n}\left(\rho_{\rm p}, \rho_{\rm n}\right)\rho_{\rm n} + \epsilon_{\rm pot}\left(\rho_{\rm p}, \rho_{\rm n}\right) \right]$$

$$\Omega_{\rm p} = \int d^3r \left[\frac{3}{10m} \left(3\pi^2\right)^{\frac{2}{3}} \rho_{\rm p}^{\frac{5}{3}} - \mu_{\rm p}\left(\rho_{\rm p}, \rho_{\rm n}\right) - V\rho_{\rm p} \right]$$

$$\Omega_{\rm II} = \Omega_{\rm n} + \Omega_{\rm p}$$

interaction **:**effective potential to reproduce the nuclear matter saturation property

Electron： Phase Ⅰ &Phase Ⅱ

$$\Omega_{\rm em} = \int d^3r \left[-\frac{1}{8\pi e^2} \left(\nabla V\right)^2 - \frac{\left(V - \mu_e\right)^4}{12\pi^2} \right]$$

$$V(r) = -\int d^3r' \frac{Q_i\rho_i(\boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|}$$

$$E_{\rm V} = \frac{1}{2}\int d^3r d^3r' \frac{Q_i\rho_i(\boldsymbol{r})Q_j\rho_j(\boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|}$$

We can get "equation of motion" from

$$\frac{\delta\Omega_{tot}}{\delta\rho_i} = 0 \quad \Rightarrow \quad \mu_i = \frac{\delta\varepsilon}{\delta\rho_i} - Q_i V$$

gauge invariant form

Equations of motion

T.E., T.Maruyama, S.Chiba
and T.Tatsumi PTP115(2006)337

● quark phase

$$\mu_u = \left(1 + \frac{2\alpha_c}{3\pi}\right)\pi^{\frac{2}{3}}\rho_u^{\frac{1}{3}} - \frac{2}{3}V$$

$$\mu_d = \left(1 + \frac{2\alpha_c}{3\pi}\right)\pi^{\frac{2}{3}}\rho_d^{\frac{1}{3}} + \frac{1}{3}V$$

$$\mu_s = \epsilon_s + \frac{2\alpha_c}{3\pi}\left[p_{Fs} - 3\frac{m_{Fs}^2}{\epsilon_{Fs}}\ln\left(\frac{\epsilon_{Fs} + p_{Fs}}{m_s}\right)\right] + \frac{1}{3}V$$

● nucleon phase

$$\mu_n = \frac{p_{Fn}^2}{2m} + \frac{2S_0(\rho_n - \rho_p)}{\rho_0} + \epsilon_{bind} + \frac{K_0}{6}\left(\frac{\rho_n + \rho_p}{\rho_0} - 1\right)^2 + \frac{K_0}{9}\left(\frac{\rho_n + \rho_p}{\rho_0} - 1\right) + 2C_{sat}\frac{\rho_n + \rho_p}{\rho_0} - C_{sat}$$

$$\mu_p = \mu_n - \frac{p_{Fn}^2}{2m} + \frac{p_{Fp}^2}{2m} - \frac{4S_0(\rho_B - 2\rho_p)}{\rho_0} - V$$

$$\mu_e = (3\pi^2\rho_e)^{\frac{1}{3}} + V$$

Poisson equation

$$\nabla^2 V = 4\pi e^2\left[\left(\frac{2}{3}\rho_u - \frac{1}{3}\rho_d - \frac{1}{3}\rho_s\right)\theta(R - r) + \rho_p\theta(r - R) - \rho_e\right]$$

Chemical equilibrium

Quark phase:

$$\mu_u - \mu_s + \mu_e = 0$$

$$\mu_d = \mu_s$$

Nucleon phase:

$$\mu_n (\equiv \mu_B) = \mu_p + \mu_e$$

Quark & nucleon boundary:

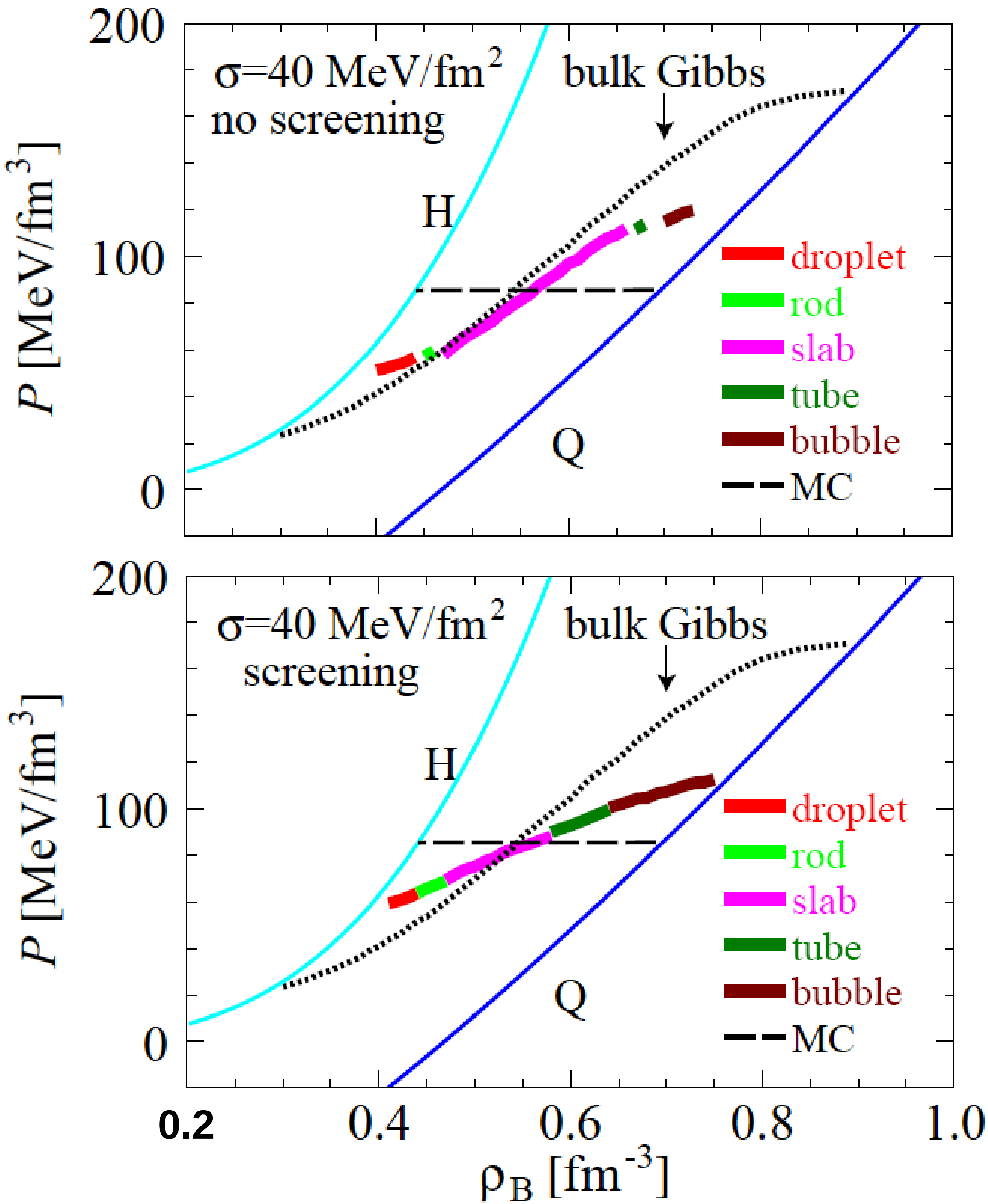
$$\mu_n = \mu_u + 2\mu_d$$

$$\mu_p = 2\mu_u + \mu_d$$

With Gibbs conditions

ρ_i is the function of V
V is the function of ρ_i

Poisson equation become highly nonlinear equation. With screening effect, it asks for rearrangement of ρ_i.



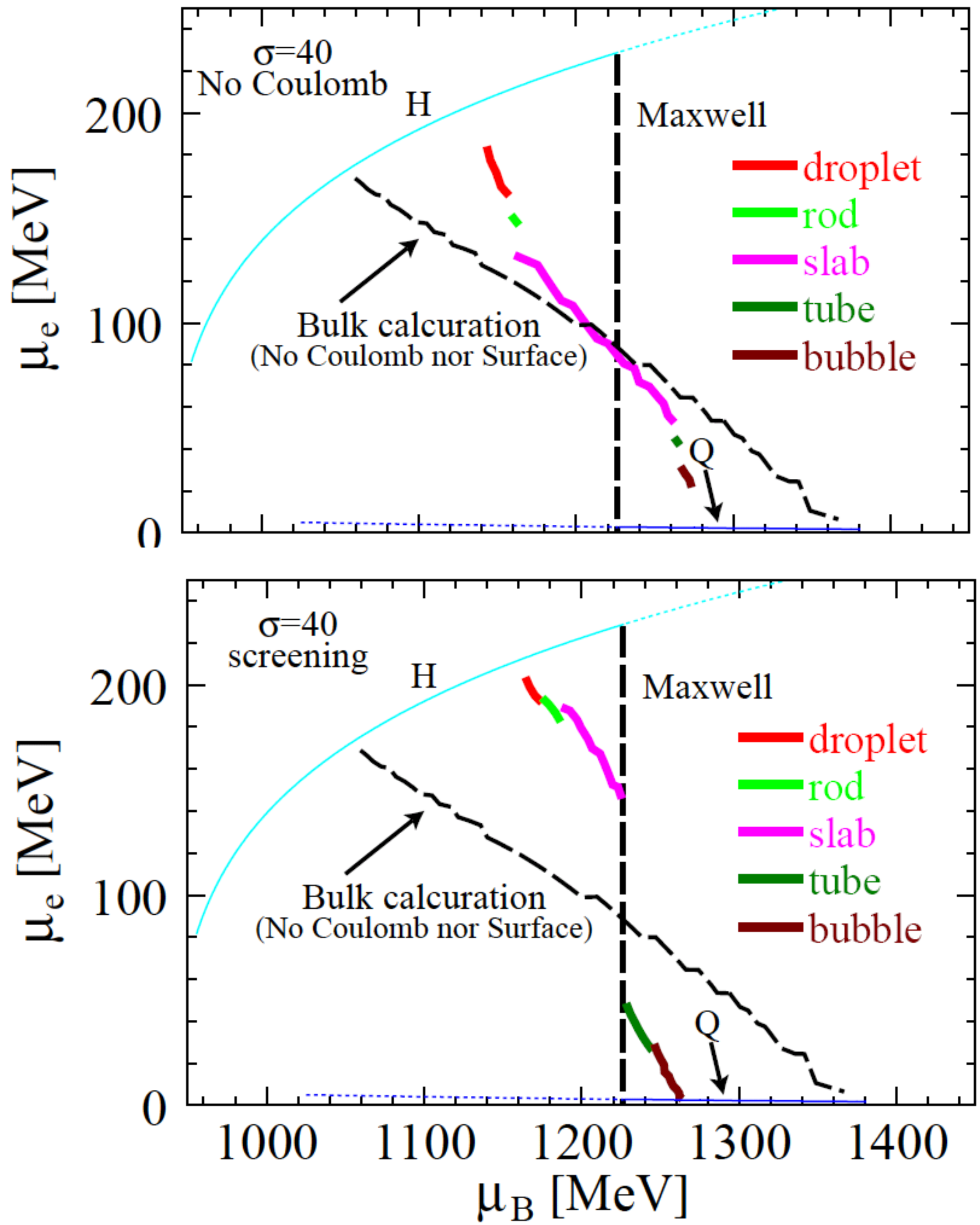
T.E., T.Maruyama, S.Chiba
and T.Tatsumi PTP**115**(2006)337

"Finite size effects"

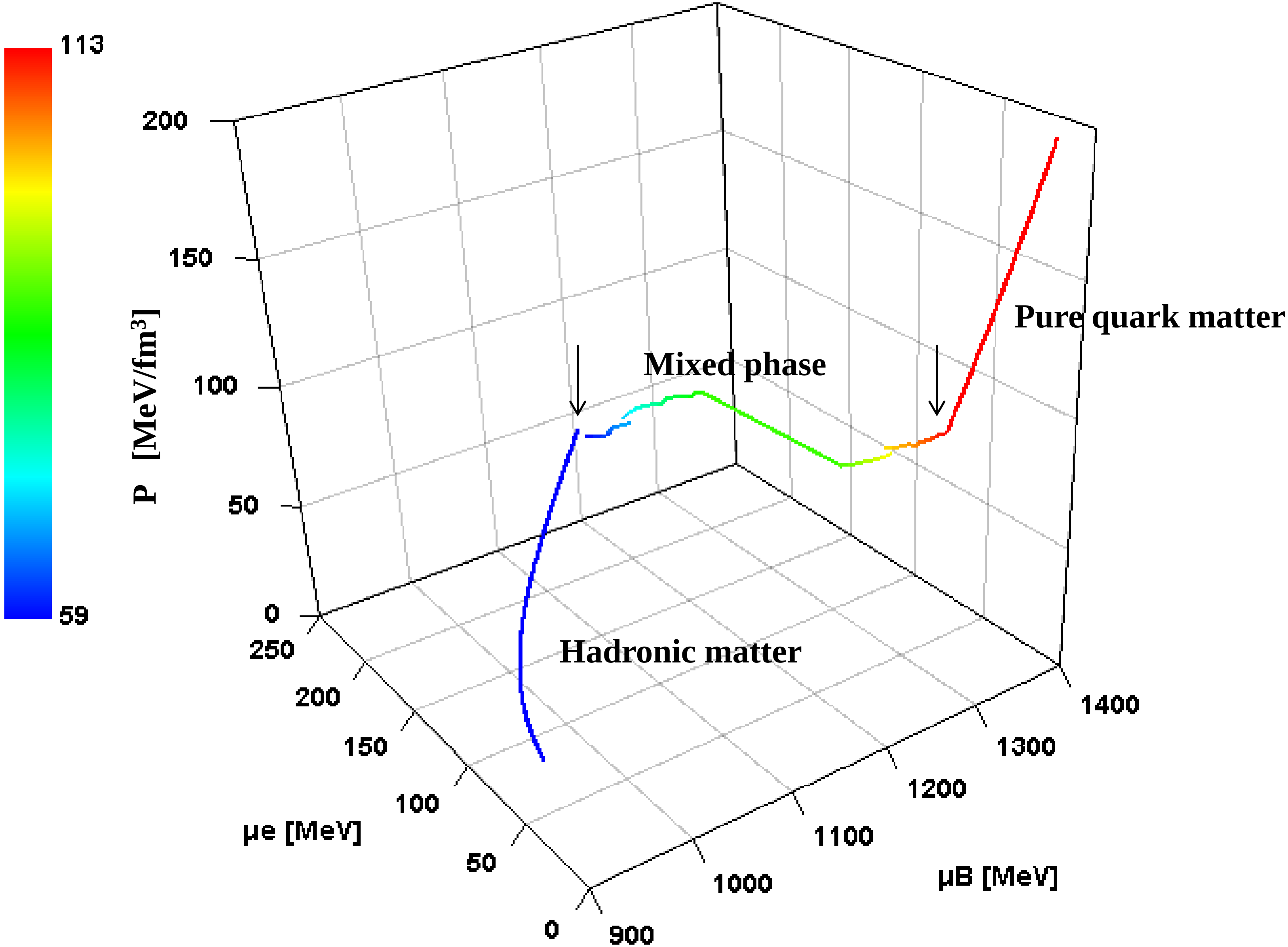
- Screening effect
- Surface tension

Lattice QCD (finite temperature)
10 ~ 100 [Mev/fm^2]
Kajantie et al NPB**357** (1991)693
Huang et al PRD**42**(1990)2864

T.E., T.Maruyama, S.Chiba and T.Tatsumi PTP**115**(2006)337



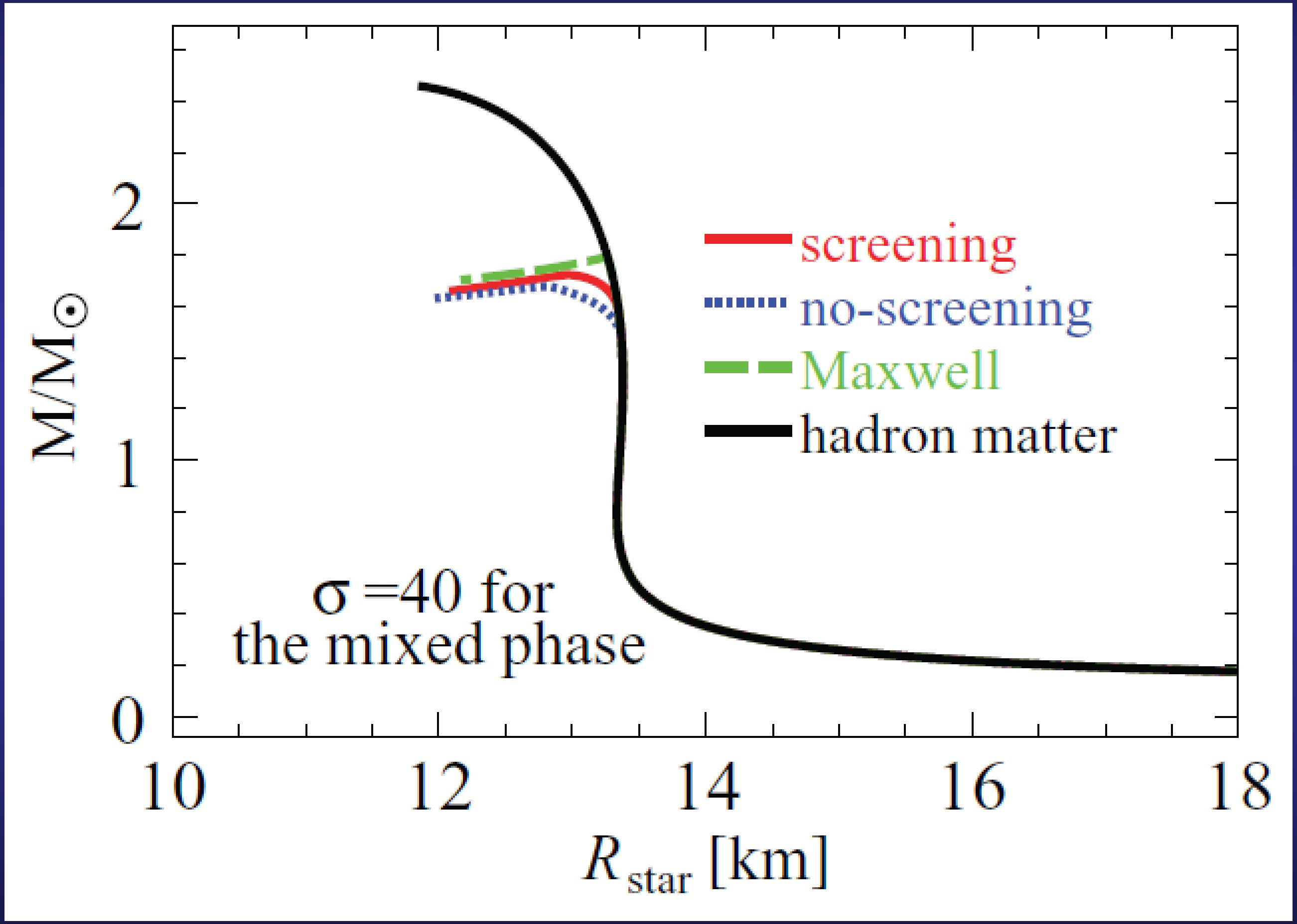
Screening effect



EOS (with screening) in μ_B − μ_e − Pressure.

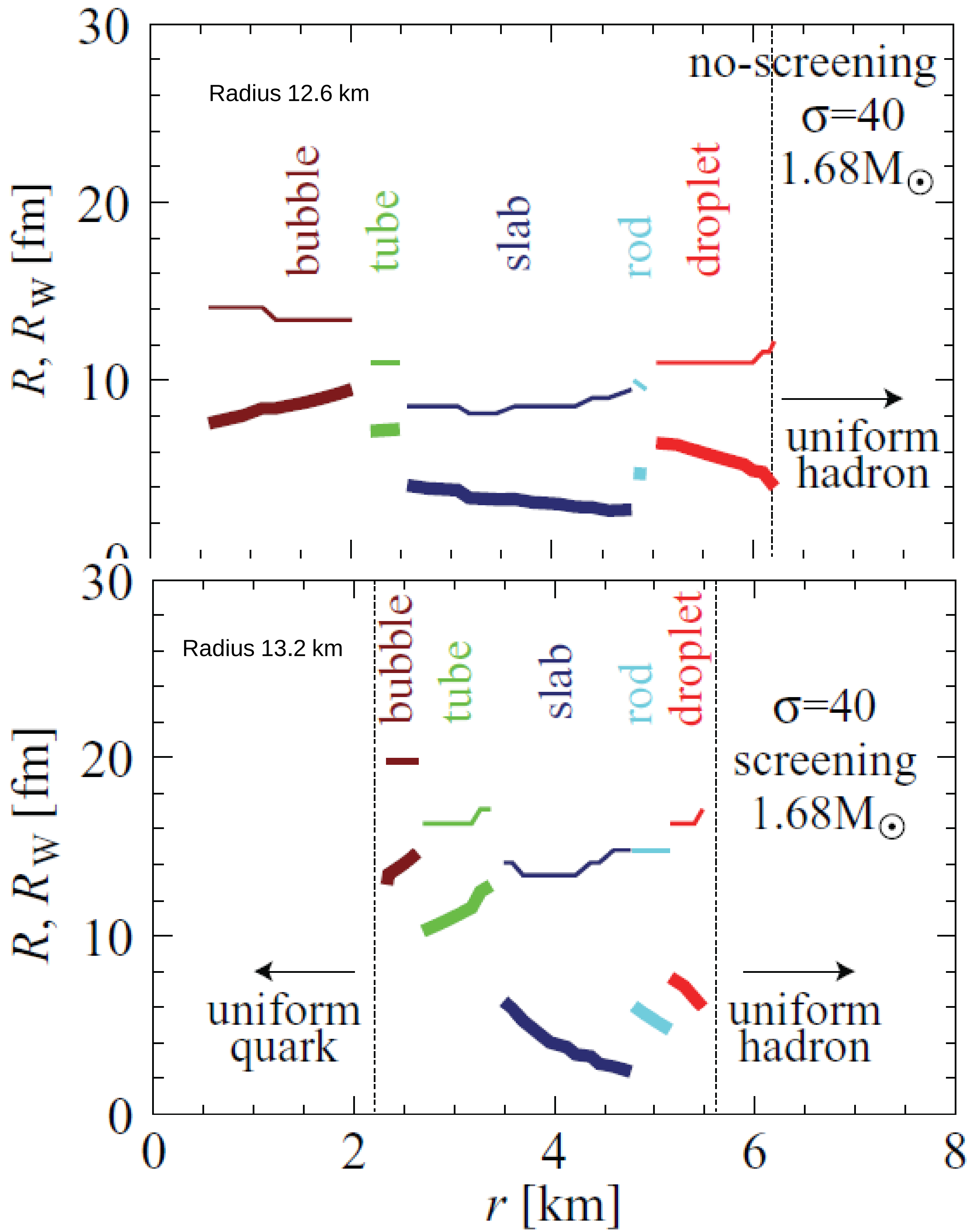
Implication to hybrid stars

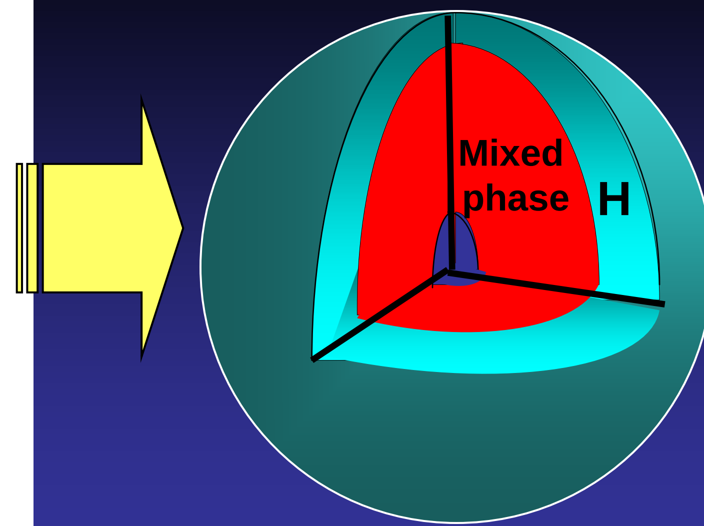
Our EOS ⇒ Tolman-Oppenheimer-Volkoff (TOV) equation

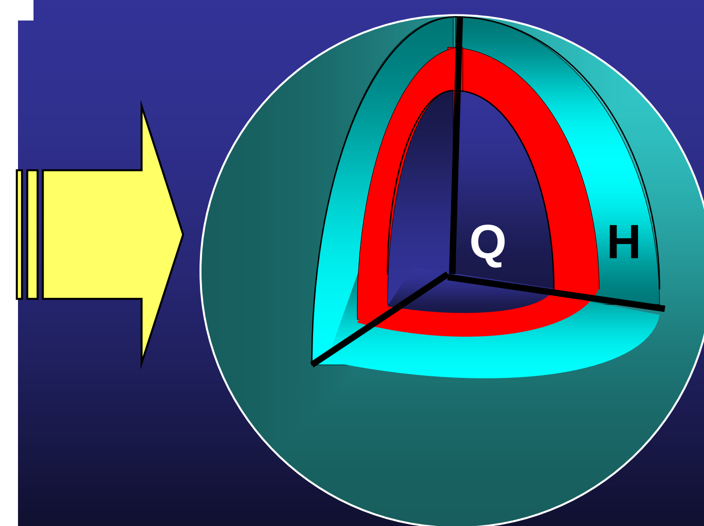


T.E., PRC83, 068801 (2011)

T.E. Phys.Rev.C83, 068801 (2011)

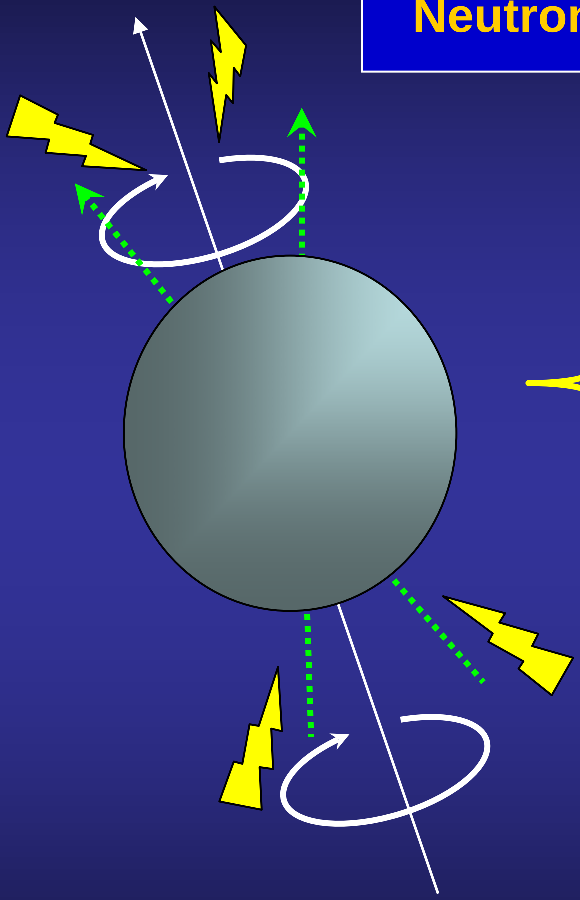






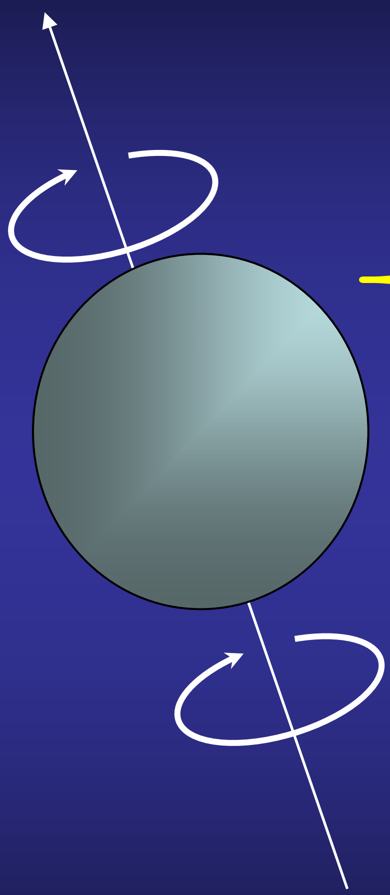
About 1000 pulsars are observed…

Neutron stars (hybrid stars) have many physical phenomena



- **Glitch phenomena**
- **Cooling problem**
- **Strong magnetic field** — 10^{12} G～10^{15} G(magneters)
- **Maximum mass** — $\sim 2.1 M_\odot$
- **Othres…**
- **Rotation**

Including the rotation effect



Approximation

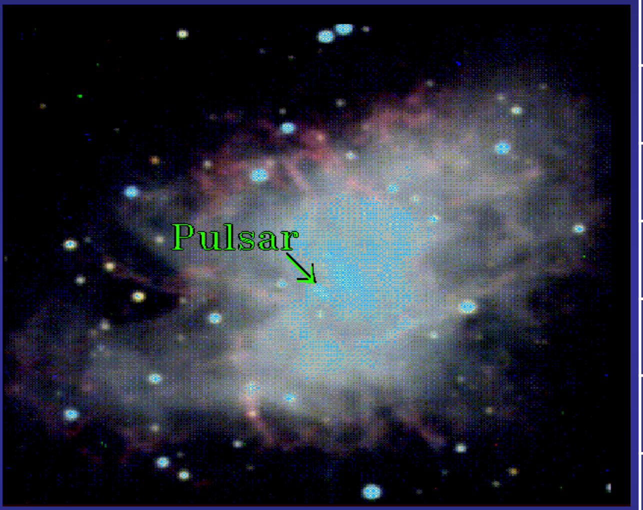
- **Stationary rigid rotation (Uniform rotation)**
- **Axially symmetric with respect to the spin axis**
- **Matter is Perfect fluid**

Review of stationary rotation in General Relativity : Stergioulas, (2003)

A. Kurkela et al. arXiv:1006.4062[astro-ph.HE]

Our EOS ⇒ **Rotating Neutron Star (RNS)**

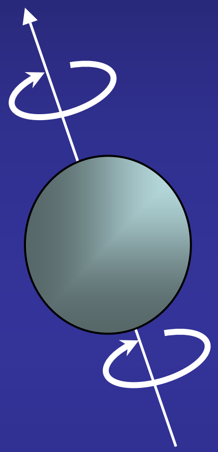
Observations

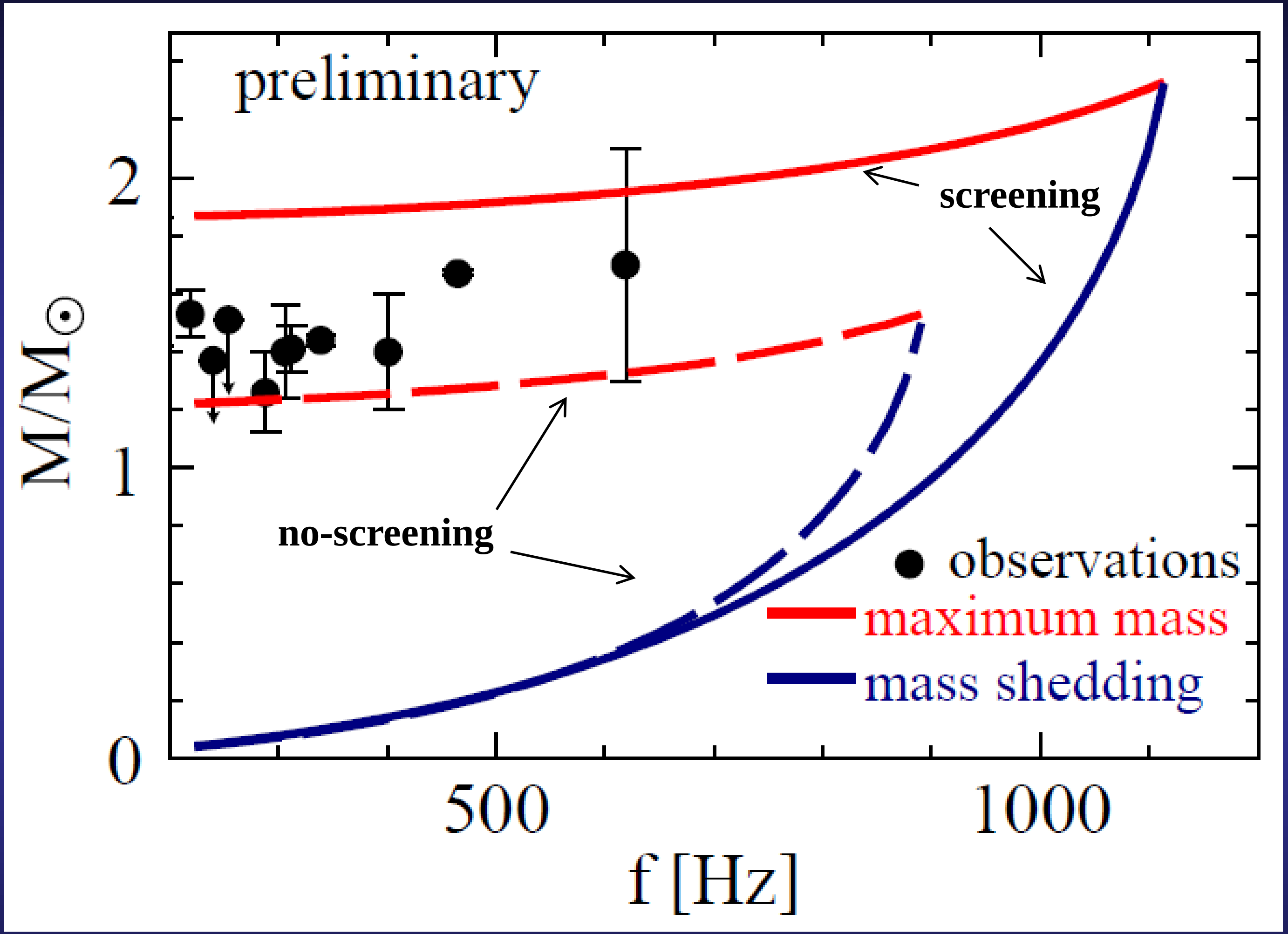


Name	Spin [Hz]	Mass/M ⊙
J0024-7204H	312	1.41±0.08
J0437-4715	174	1.76±0.20
J0514-4002A	126	< 1.52
J0751+1807	288	1.26±0.14
J1012+5307	190	1.64±0.22
J1713+0747	219	1.53±0.08
4U1608-52	619	1.70±0.40
J1748-2446I	105	1.85±0.05
SAXJ1808.4-3658	401	1.40±0.20
J1824-2452C	240	< 1.37
B1855+09	187	1.58±0.13
J1903+0327	465	1.67±0.01
J1909-3744	339	1.44±0.02
J1911-5958A	306	1.40±0.16
J2019+2425	254	< 1.51

Masses of neutron stars with millisecond periods

Including rotation: EOS(screening, no-screening

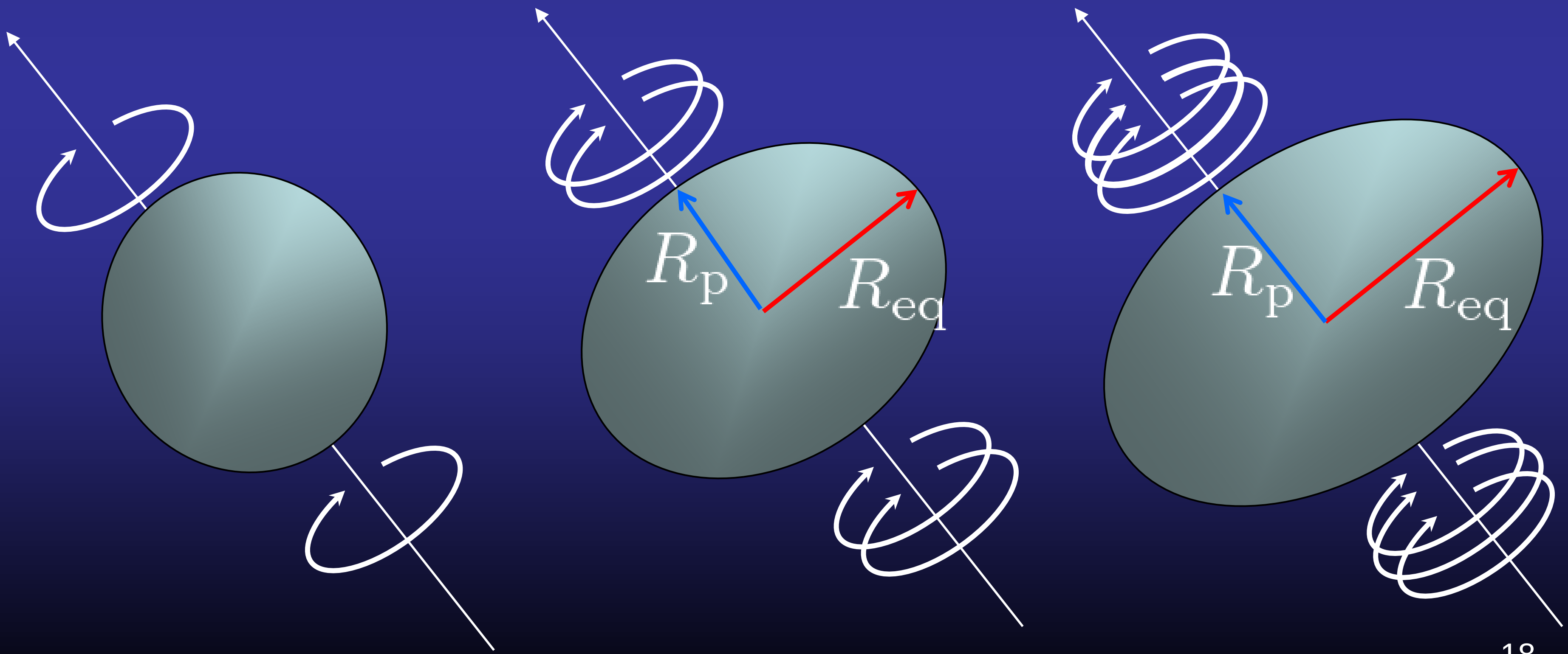


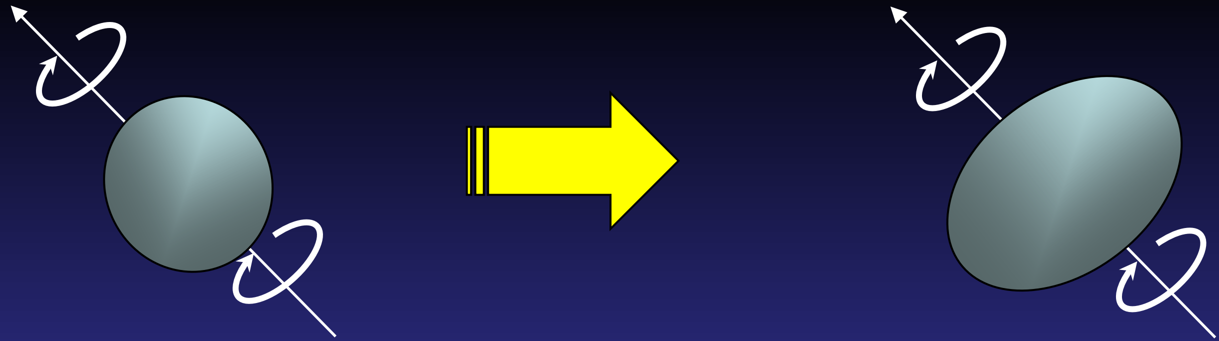


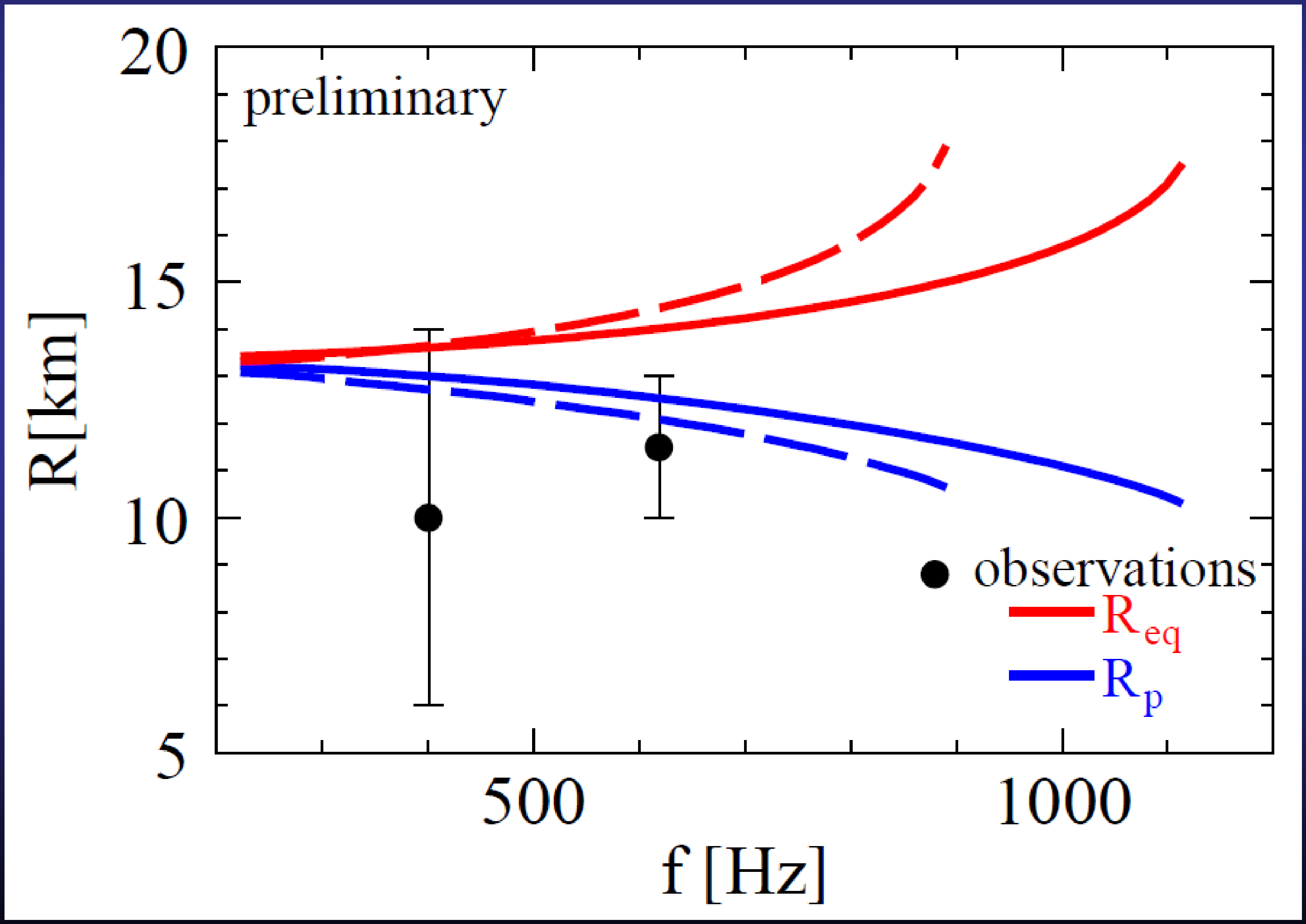
no-screening EOS is not appropriate

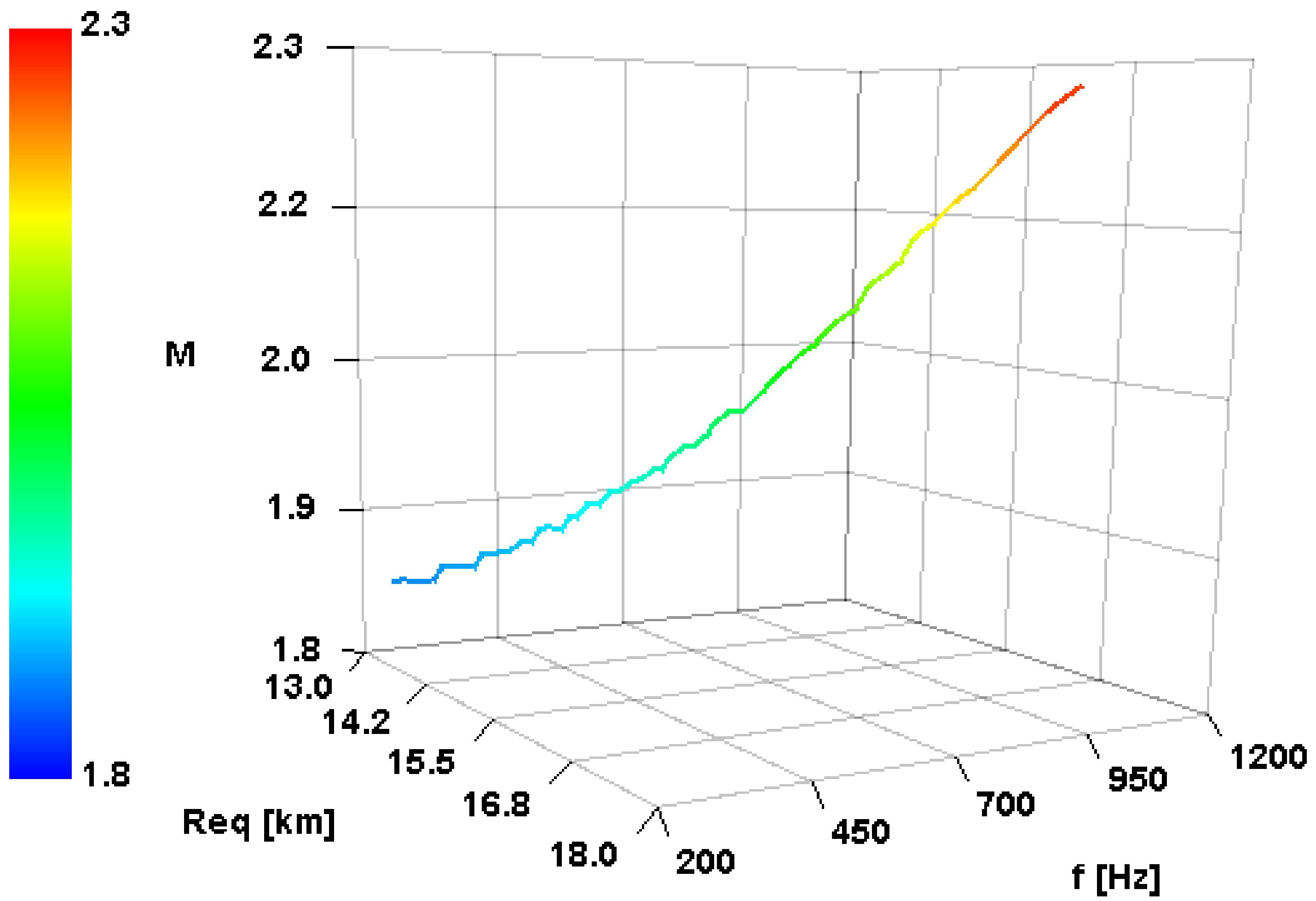
Name	Spin [Hz]	Radius
4U1608-52	619	11.5±2.0
SAXJ1808.4-3658	401	10.0±4.0

Neutron star periods and radii

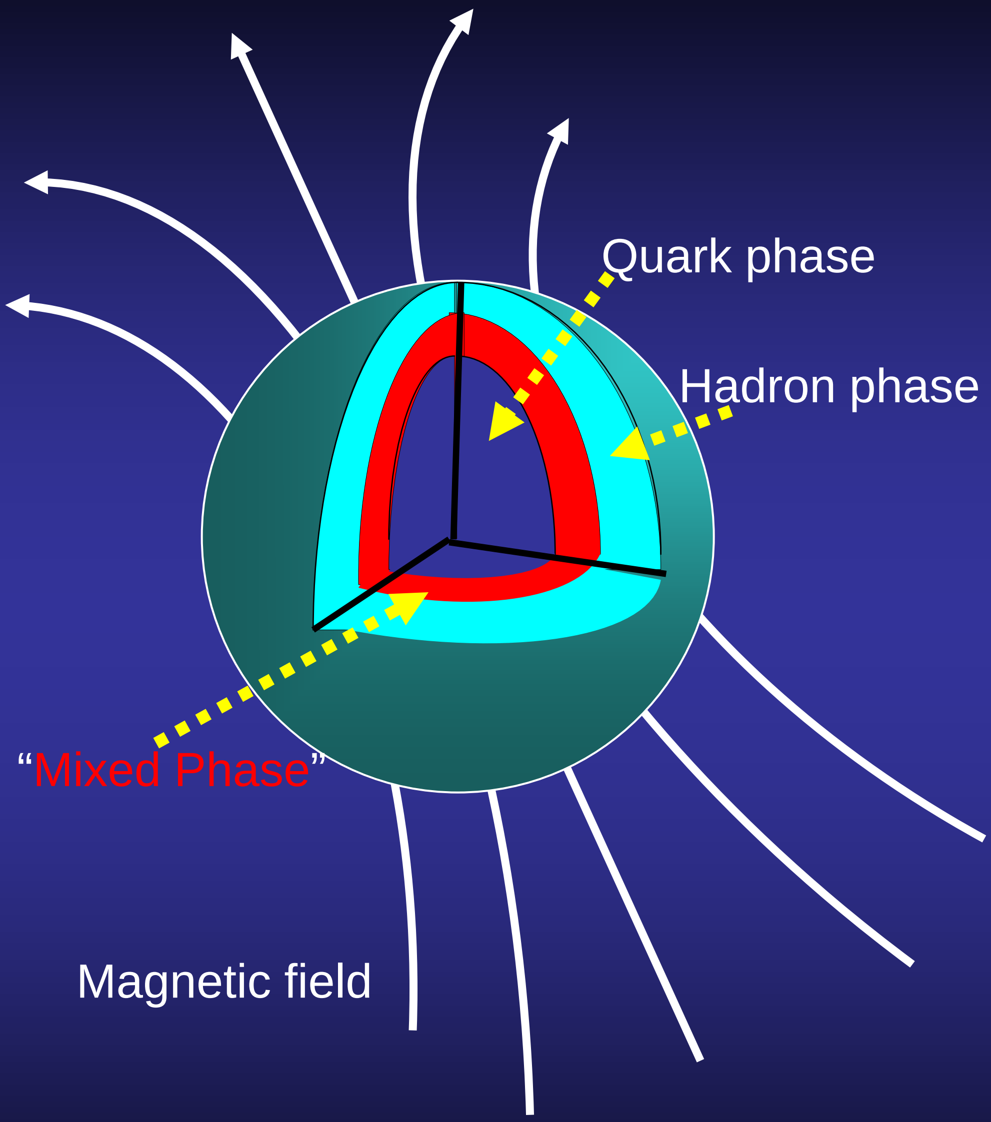





preliminary
20
15
10
5
500
1000
R[km]
f [Hz]
observations
R_{eq}
R_p



Mass－Radius(equatorial)－Spin relation.



Strong magnetic field～10^{12} G

Magnetars～10^{15} G

The origin of magnetic field unknown…

spin-polarization of nuclear matter

⇒many calculations in 1970s,

But negative results…

cf. J.M. Pearson et al. PRL**24**(1970)325

J. Dabrowski et al. PRC**17**(1978)1516

⇒**spin-polarization** of liquid ^{3}He "favorable"

M. Takano, **T. E.**, R. Kimura and M. Yamada, PTP**109**(2003)213

How about quark matter?

spin-polarization⇒may be possible

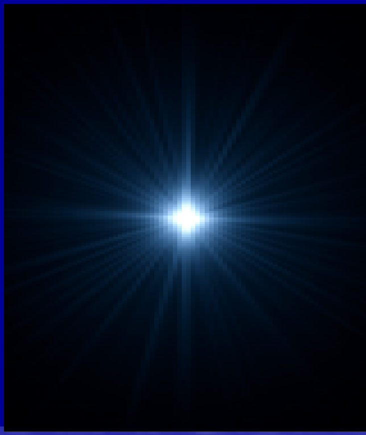
T. Tatsumi PLB**489**(2000)280

Quark matter would exist or not?

cf. Dynamo effect

Summary

- Inner structures of the star strongly depend on EOSs
- EOSs confront observations.
- “Rotation” restricts EOSs of the matter. no-screening case is not appropriate



Future plans ;

- Rotation effects on inner structures of the star
- Magnetic fields and rotation relation
- Strong magnetic fields – what is the origin ?

Thank you for your attention.

Back UP Slides

Maxwell construction and Gibbs condition

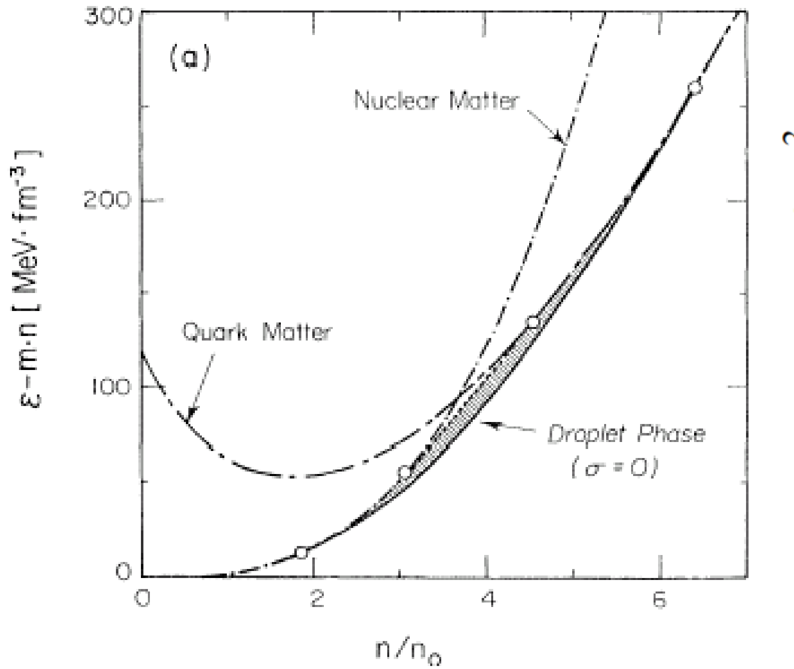
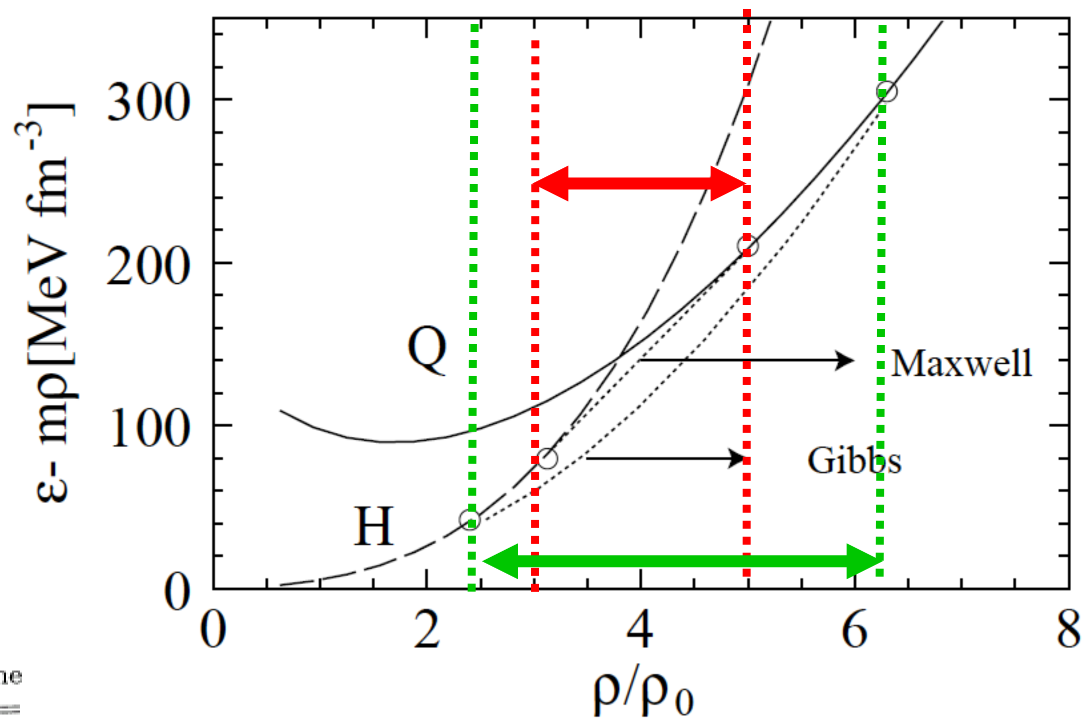
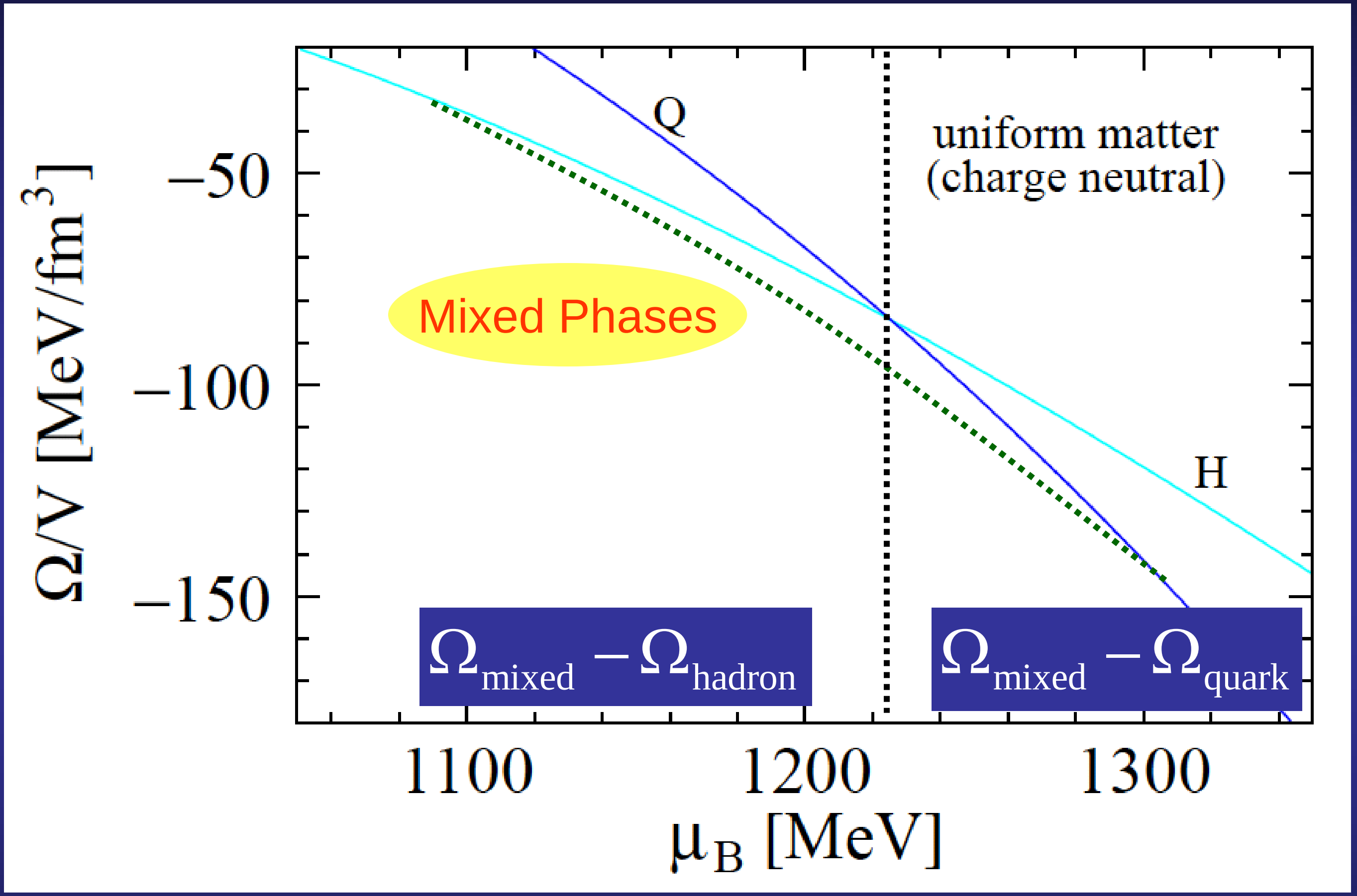


FIG. 1. (a) The full line gives the energy density of the droplet phase without surface and Coulomb energies ($\sigma = 0$). Also shown are the energy densities of electrically neutral bulk nuclear matter, quark matter in β equilibrium, and the double-tangent construction (dashed line) corresponding to the coexistence of bulk electrically neutral phases. (b) Energy densities of the droplet phase relative to its value for $\sigma = 0$ for $\sigma = 10$, 50, and 90 MeV$\cdot$fm^{-2}. When the energy density of the droplet phase falls within the hatched area it is energetically favored.



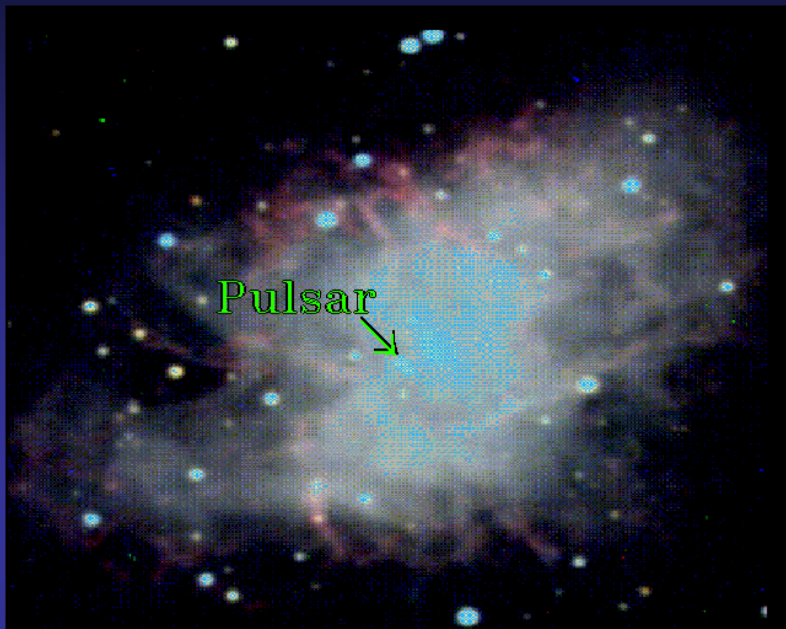
EOS for Hadronic matter , quark matter and Mixed phase (Gibbs and Maxwell)

Heiselberg et al PRL70(1993)1355



Q
uniform matter
(charge neutral)
Mixed Phases
H
$\Omega_{mixed} - \Omega_{hadron}$
$\Omega_{mixed} - \Omega_{quark}$
−50
−100
−150
1100
1200
1300
Ω/V [MeV/fm^3]
μ_B [MeV]

digression

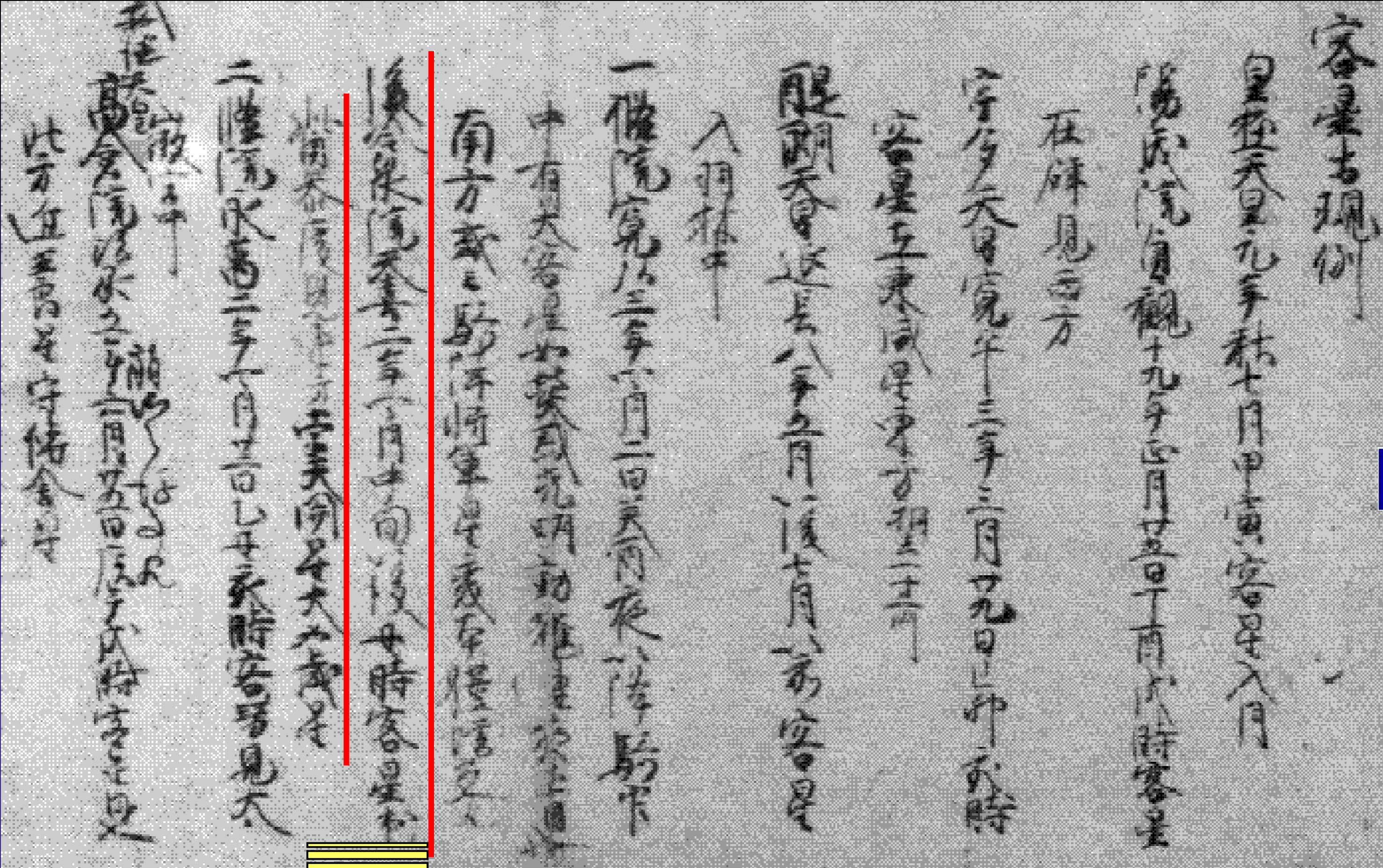


The pulsar in the Crab nebula

This explosion was observed more than 900 years ago…

Teika Fujiwara（藤原定家）

Nobleman in old Japan
He wrote in his diary
“Meigetsu-ki”



Teika Fujiwara（藤原定家）

“Meigetsu-ki(明月記)”

“May 1054, a star appeared in the east sky.
The size was as same as Jupiter”.

The historical record in China “宋史”
Chinese people also saw this explosion