# Recent developments in supernova neutrinos

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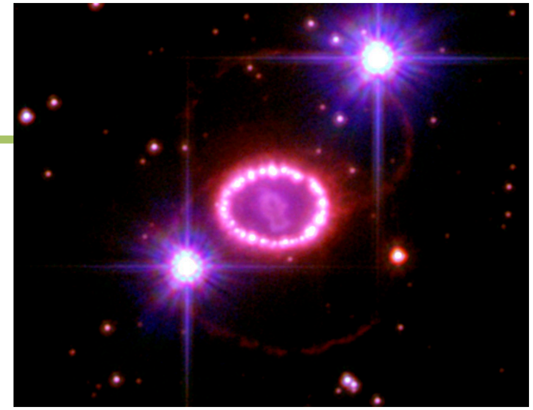
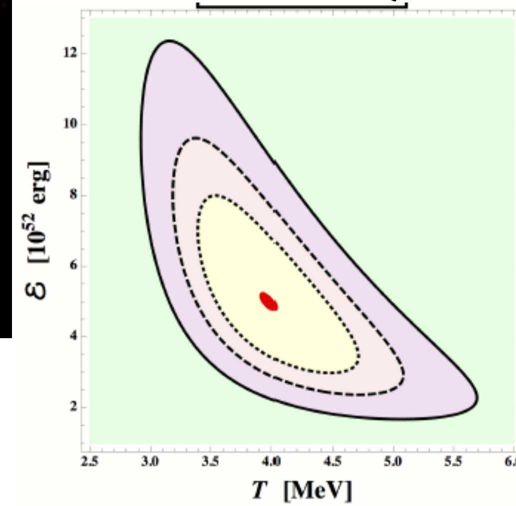
# Supernova ν

*These comprise lighter massive stars (O-Ne-Mg), iron-core stars to collapsars (AD-BH).*

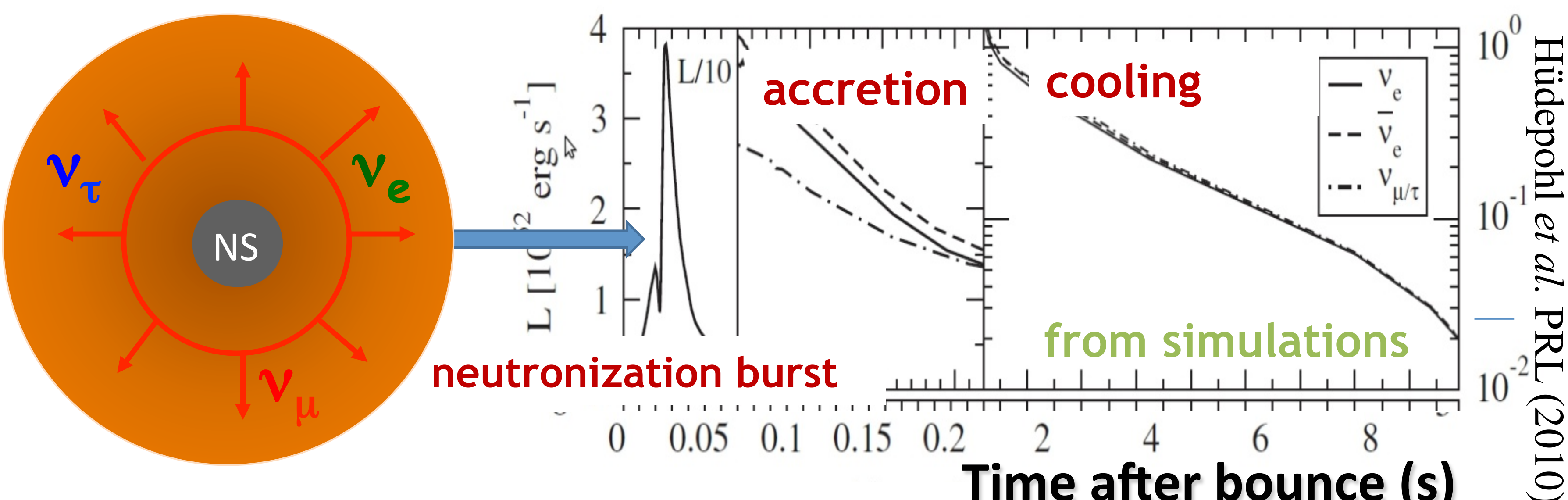


SN1987A (LMC)

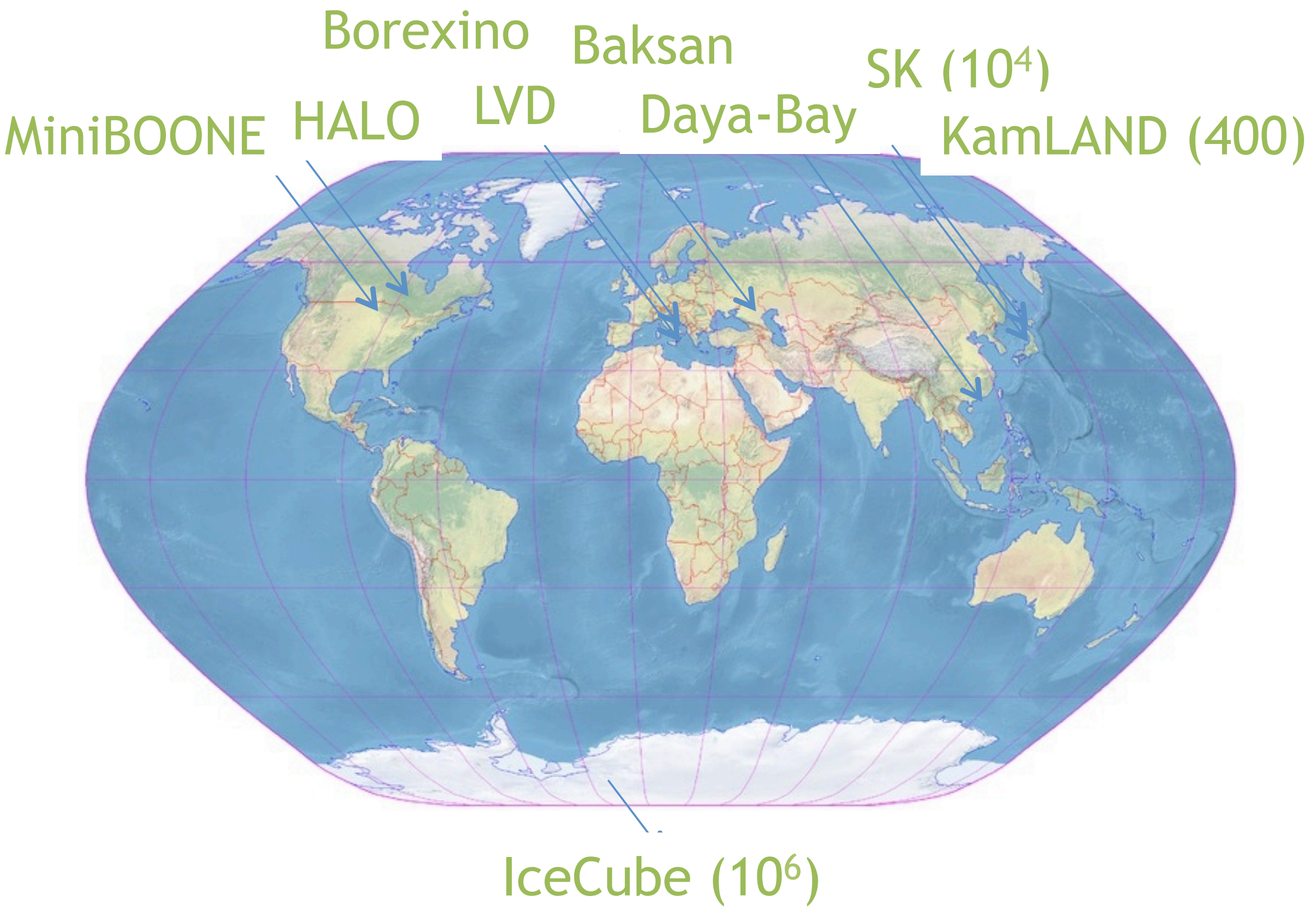
Vissani, JPG (2014)




$10^{58}$ **ν** of about 10 MeV in 10 s from the gravitational collapse of massive stars.



# Current SN observatories

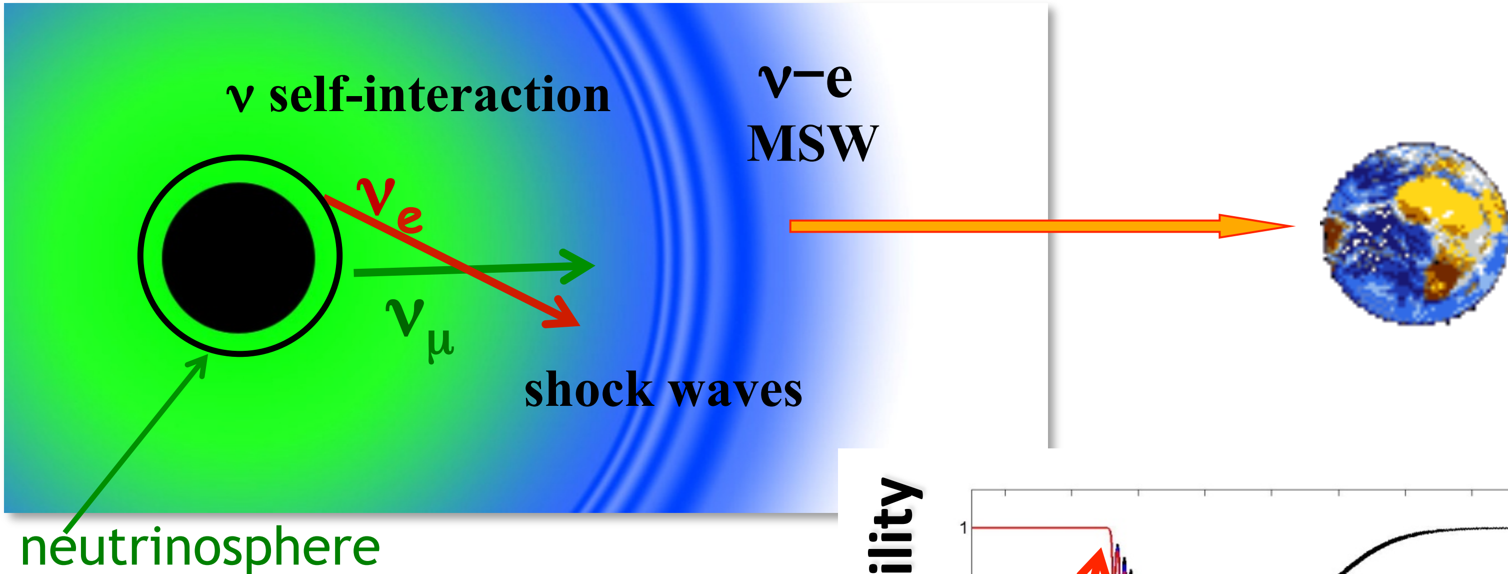



Detection channels available : scattering on p, nuclei and electrons.

Large scale detectors coming or under study -- JUNO, Hyper-K…

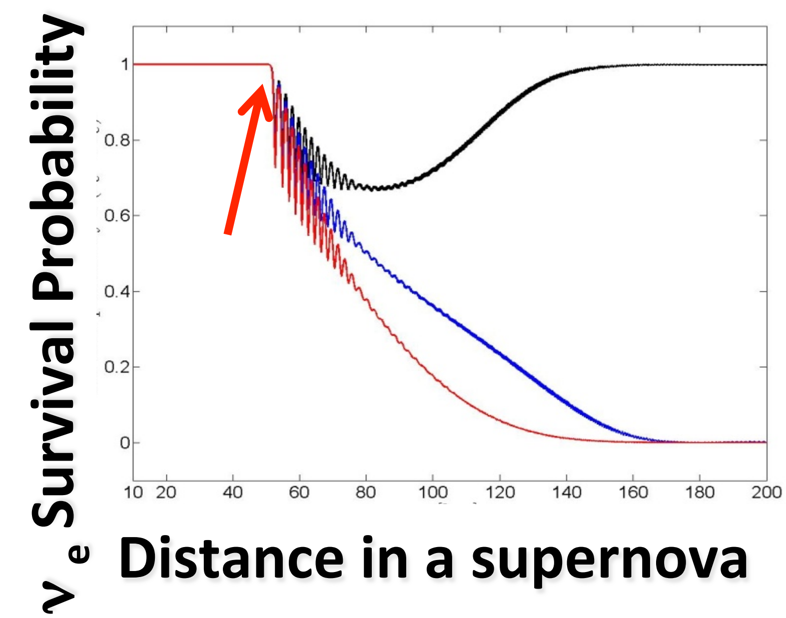
*SNEWS – Supernova Early Warning System, New J. Phys. 6 (2004)*

# Neutrinos flavour conversion in supernovae




Self-interaction effects in the "bulb model" :

collective stable and unstable modes emerge

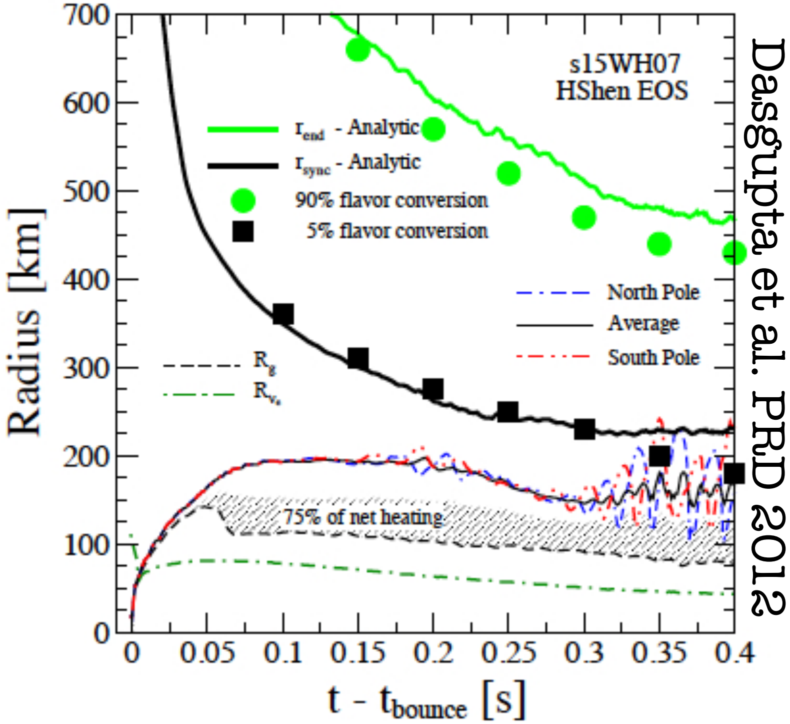



see e.g. Duan, Fuller, Qian, Ann. Rev.. 60 (2010)

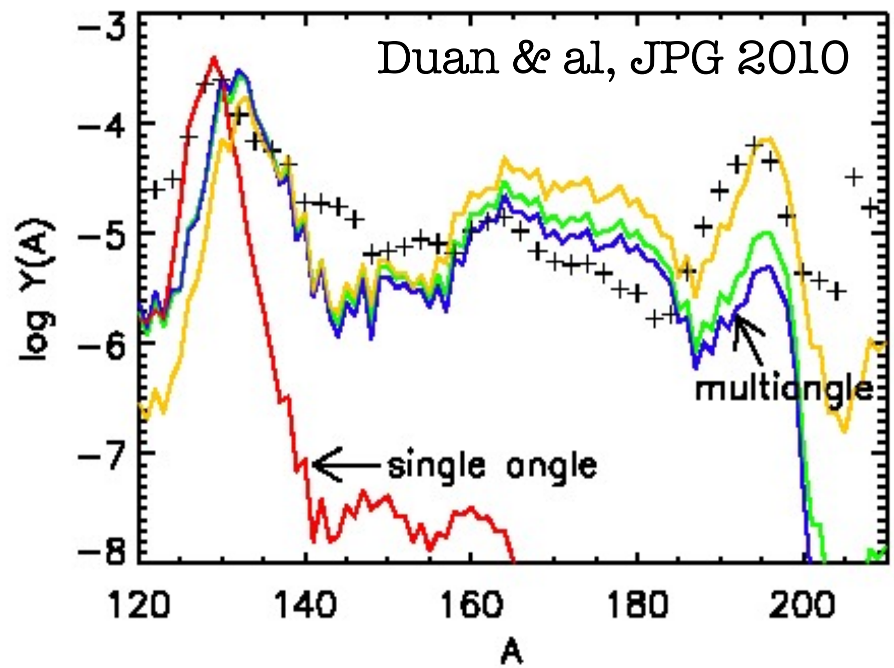
# key open questions

Impact of flavour conversion :

- on the shock wave ?




- on nucleosynthesis ?
(r-process, νp process)



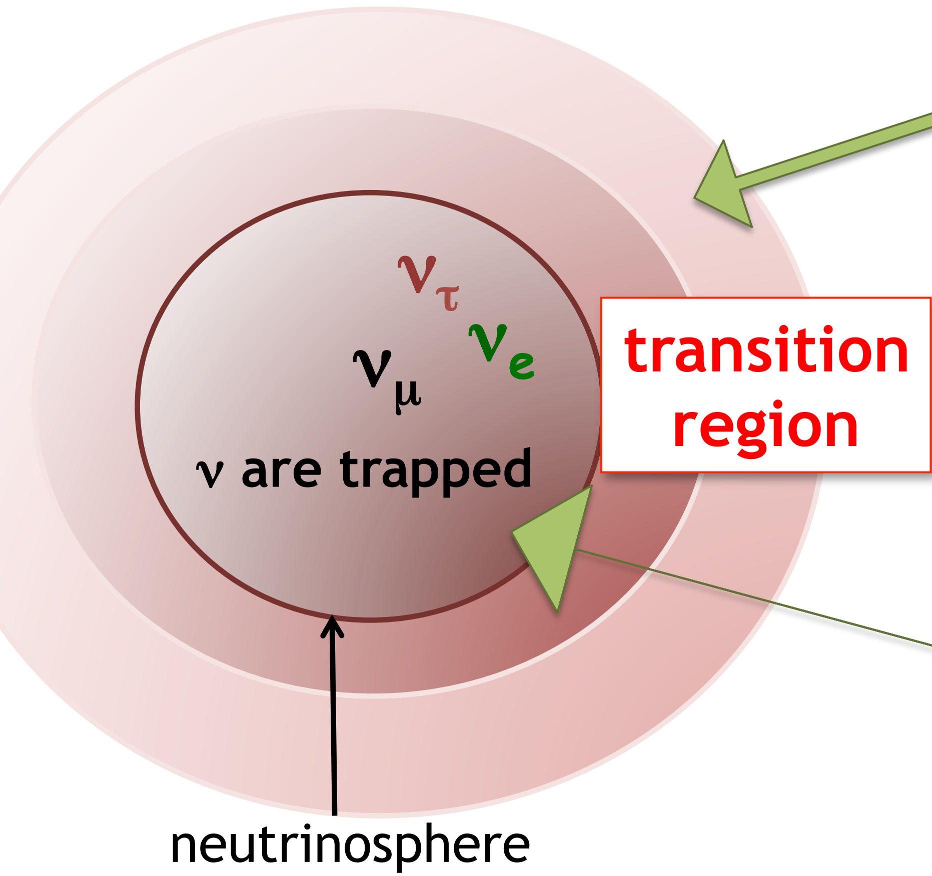

- role of decoherence by matter, of symmetry breaking ?

see e.g. Chakraborty, et al. PRL 107 (2011) and arXiv:1507.07569

# Description of ν evolution in environments

Volpe, JIMPE 24 (2015) , 1506.06222

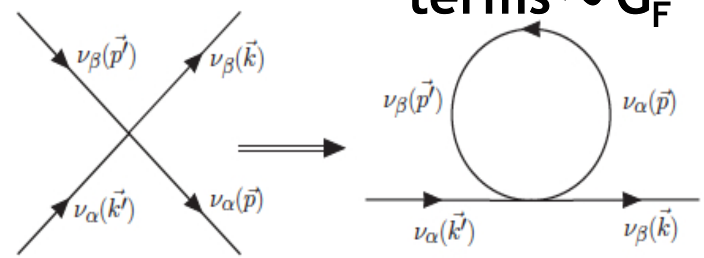
Several approaches used to derive evolution equations -- *density matrix, effective spins, Green's functions and CTP, path-integral, BBGKY hierarchy*.




Equation based on :

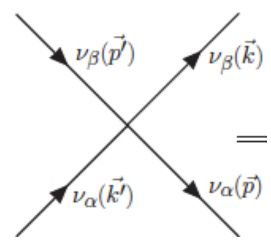
- ❑ mean-field approximation

terms $\sim G_F$

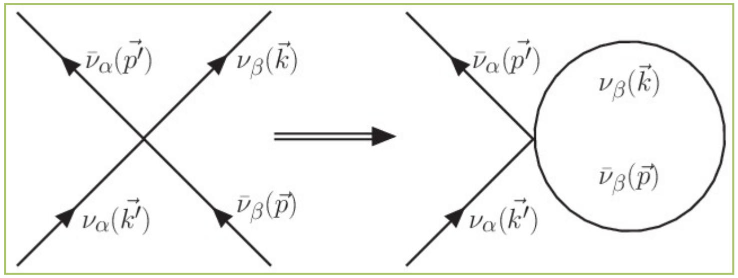



- ❑ extended mean-field
- ❑ Boltzmann approximation

terms $\sim G_F^2$



# Recent theoretical developments

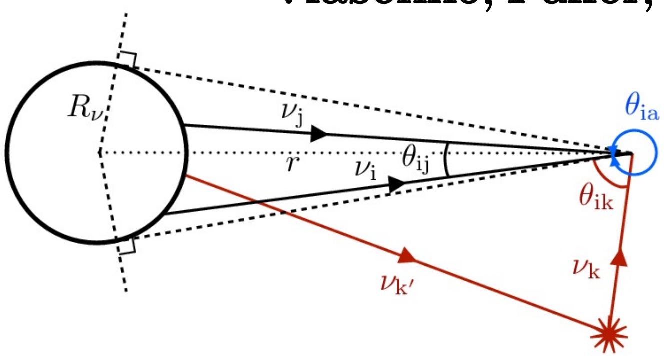



## ν-antiν pairing correlations

Volpe, Väänänen, Espinoza, PRD87 (2013), 1302.2374
With Born-Bogoliubov-Green-Kirkwood-Yvon (BBGKY) hierarchy

## mass corrections

Volpe, Väänänen, Espinoza, PRD87 (2013), 1302.2374
Vlasenko, Fuller, Cirigliano, PRD89 (2014), 1309.2628




## collisions

Cherry et al, PRL108 (2012),1203.1607

Such corrections can be amplified by the non-linearity of equations.

**Role of contributions beyond the mean-field ?**

# Extended mean-field for astrophysical media

Serreau, Volpe, PRD 90 (2014), arXiv:1409.3591

General mean-field equations for massive Dirac or Majorana ν in an inhomogeneous medium.

Possible two-point correlators :

number or mixing operators

NORMAL DENSITIES

$$\rho_{ij}(t,\vec{q},h,\vec{q}\,',h') = \langle a_j^\dagger(t,\vec{q}\,',h')a_i(t,\vec{q},h)\rangle,$$

$$\bar{\rho}_{ij}(t,\vec{q},h,\vec{q}\,',h') = \langle b_i^\dagger(t,\vec{q},h)b_j(t,\vec{q}\,',h')\rangle,$$

DENSITIES with helicity change $\zeta_{ij}(t,\vec{q}) = \langle a_j^\dagger(t,\vec{q},+)a_i(t,\vec{q},-)\rangle$

PAIRING DENSITIES

$$\kappa_{ij}(t,\vec{q},h,\vec{q}\,',h') = \langle b_j(t,\vec{q}\,',h')a_i(t,\vec{q},h)\rangle,$$

pair correlators

The Ehrenfest theorem for each two-point correlators.

# Extended mean-field for astrophysical media

Serreau, Volpe, PRD 90 (2014), arXiv:1409.3591

The most general mean-field Hamiltonian is bilinear :

$$H_{\rm eff}(t) = \int d^3x\, \bar{\psi}_i(t,\vec{x})\Gamma_{ij}(t,\vec{x})\psi_j(t,\vec{x}),$$

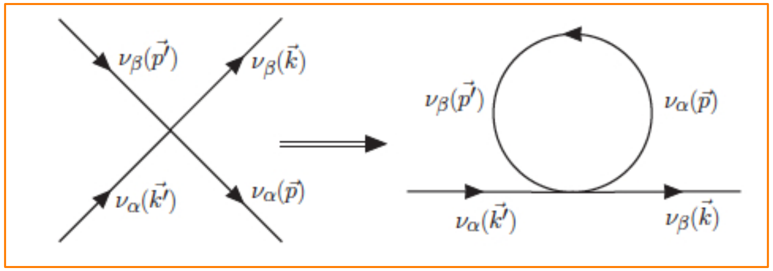
kernel

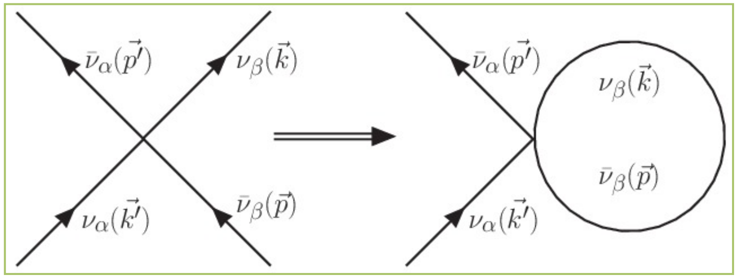
$$\psi_j(\vec{x}) = \sum_h \int \frac{d^3\vec{p}}{(2\pi)^3 2E_p}[a(\vec{p},h)u_{\vec{p},h}e^{i\vec{p}\cdot\vec{x}} + b^\dagger(\vec{p},h)v_{\vec{p},h}e^{-i\vec{p}\cdot\vec{x}}]$$

and gets quadratic form :

$$H_{\rm eff}(t) = a^\dagger(t)\cdot\Gamma^{\nu\nu}(t)\cdot a(t) + b(t)\cdot\Gamma^{\bar{\nu}\bar{\nu}}(t)\cdot b^\dagger(t) + a^\dagger(t)\cdot\Gamma^{\nu\bar{\nu}}(t)\cdot b^\dagger(t) + b(t)\cdot\Gamma^{\bar{\nu}\nu}(t)\cdot a(t),$$

For example, for the neutrino self-interaction :



# Extended mean-field for astrophysical media

Serreau, Volpe, PRD 90 (2014), arXiv:1409.3591

General mean-field equations for inhomogeneous media

$$i\dot{\mathcal{R}}(t) = [\mathcal{H}(t), \mathcal{R}(t)] . \qquad \mathcal{H}(t) = \begin{pmatrix} \Gamma^{\nu\nu}(t) & \Gamma^{\nu\bar{\nu}}(t) \\ \Gamma^{\bar{\nu}\nu}(t) & \Gamma^{\bar{\nu}\bar{\nu}}(t) \end{pmatrix}$$

New contributions from mass or pairing correlations :

- introduce new mixing between neutrino-antineutrinos.
- require anisotropy of the medium.

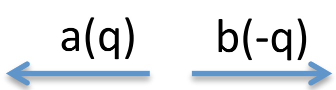
ν-antiν pairing correlations

$$\mathcal{R}(t) = \begin{pmatrix} \rho(t) & \kappa(t) \\ \kappa^{\dagger}(t) & \mathbb{1} - \bar{\rho}(t) \end{pmatrix} \qquad \kappa_{ij}(t, \vec{q}, h, \vec{q}', h') = \langle b_j(t, \vec{q}', h') a_i(t, \vec{q}, h) \rangle ,$$

$$\Gamma^{\nu\bar{\nu}}(t) \to -\hat{\epsilon}_q^{*} \cdot \vec{V}(t)$$

This term is sourced by $\rho$ and $\kappa$.

Note that for homogeneous background :




Volpe, Väänänen, Espinoza PRD87 (2013)

An order-of-magnitude estimate in

Karatsev, Raffelt, Vogel, PRD91 (2015)

# Extended mean-field for astrophysical media

Serreau, Volpe, PRD 90 (2014), arXiv:1409.3591

General mean-field equations for inhomogeneous media

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New contributions from mass or pairing correlations :

- introduce new mixing between neutrino-antineutrinos.
- require anisotropy of the medium.

mass corrections

$$\mathcal{R}(t) = \begin{pmatrix} \rho_M(t,\vec{q}) & \zeta_M(t,\vec{q}) \\ \zeta_M^{\dagger}(t,\vec{q}) & \bar{\rho}_M^T(t,-\vec{q}) \end{pmatrix} \quad \zeta_{ij}(t,\vec{q}) = \langle a_j^{\dagger}(t,\vec{q},+) a_i(t,\vec{q},-)\rangle$$

$$\Gamma^{\nu\bar{\nu}}(t) \rightarrow e^{i\phi_q}\hat{\epsilon}_q^{*} \cdot \left[\vec{V}(t)\frac{m}{2q} + \frac{m}{2q}\vec{V}^T(t)\right] .$$

helicity coherence

spin coherence - Vlasenko, Fuller, Cirigliano, PRD89 (2014)

First calculation shows it might have an impact under appropriate conditions. Vlasenko, Fuller, Cirigliano, arXiv:1406.6724

# Conclusions and perspectives

- Important questions still needs to be investigated of ν flavour conversion in dense media. Instabilities behind the shock searched.

- Extended equations beyond usual mean-field derived to investigate the transition region.

- Effects of corrections beyond mean-field :
  - pairing correlations : small, but inhomogeneities ? With collisions ?
  - mass corrections : might influence flavour evolution
  - collisions : a study needed of the competition among three timescales - collisions, flavour change, macroscopic evolution.


Thank you for your attention

# A novel perspective to ν conversion

Volpe, Väänänen, Espinoza, PRD87 (2013), arXiv: 1302.2374

BBGKY for neutrinos :

- ❖ a system of particles and anti-particles
- ❖ particles with mixings

**ν**

$$\rho_\nu = \begin{pmatrix} \langle a^\dagger_{\nu_\alpha,i} a_{\nu_\alpha,i}\rangle & \langle a^\dagger_{\nu_\beta,j} a_{\nu_\alpha,i}\rangle \\ \langle a^\dagger_{\nu_\alpha,i} a_{\nu_\beta,j}\rangle & \langle a^\dagger_{\nu_\beta,j} a_{\nu_\beta,j}\rangle \end{pmatrix}$$

**anti-ν**

$$\bar\rho_\nu = \begin{pmatrix} \langle b^\dagger_{\nu_\alpha,i} b_{\nu_\alpha,i}\rangle & \langle b^\dagger_{\nu_\beta,j} b_{\nu_\alpha,i}\rangle \\ \langle b^\dagger_{\nu_\alpha,i} b_{\nu_\beta,j}\rangle & \langle b^\dagger_{\nu_\beta,j} b_{\nu_\beta,j}\rangle \end{pmatrix}$$

**occupation number op.**

**decoherence or mixing terms**

The BBGKY is a rigorous theoretical framework :

- ✓ to go from the N-body to the 1-body description
- ✓ that is very general, equivalent the Green's function formalism (equal-time limit)

**UNIFIED APPROACH for ASTROPHYSICAL and COSMOLOGICAL APPLICATIONS**

*that allows to go beyond current approximations*

# Born-Bogoliubov-Green-Kirkwood-Yvon -BBGKY- hierarchy

Kirkwood, J. Chem. Phys. 3 (1935), Yvon, Actual. Sci. Ind. 203 (1935),
Born and Green Proc. R. Soc. A 188 (1946), Bogoliubov, J. Phys. 10 (1946)

The s-reduced density matrix : $\rho_{1\ldots s} = \langle \psi(t) | a_s^\dagger \ldots a_1^\dagger a_1 \ldots a_s | \psi(t) \rangle$

one-body density

$$\rho_1 = \langle \psi(t) | a_1^\dagger a_1 | \psi(t) \rangle$$

two-body density

$$\rho_{12} = \langle \psi(t) | a_2^\dagger a_1^\dagger a_1 a_2 | \psi(t) \rangle$$

Solving exactly the many-body problem is equivalent to

$$i\dot{\rho}_1 = [t_1, \rho_1] + \mathrm{Tr}_{(2)} \{[v_{12}, \rho_{12}]\}$$

$$i\dot{\rho}_{12} = [t_1 + t_2 + v_{12}, \rho_{12}] + \mathrm{Tr}_{(3)} \{[v_{13} + v_{23}, \rho_{123}]\}$$

$$\vdots$$

$$i\dot{\rho}_{1\cdots n} = \left[ \sum_{i=1}^{n} t_i + \sum_{j>i=1}^{n} v_{ij}, \rho_{1\cdots n} \right] + \sum_{i=1}^{n} \mathrm{Tr}_{(n+1)} \left\{ \left[ v_{i(n+1)}, \rho_{1\cdots(n+1)} \right] \right\}$$

a hierarchy of equations for s-reduced density matrices

Volpe, Väänänen, Espinoza, PRD87 (2013)
– BBGKY hierarchy, ν-antiν pairing correlations, mass cont.

# The first BBGKY equation

$$\rho_1 = \langle \psi(t) | a_1^\dagger a_1 | \psi(t) \rangle \qquad \rho_{12} = \langle \psi(t) | a_2^\dagger a_1^\dagger a_1 a_2 | \psi(t) \rangle$$

one-body density — two-body density

The two-body density matrix can be written as :

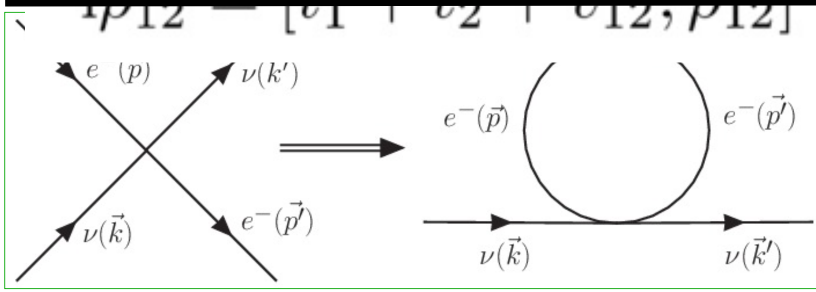
$$\rho_{12} = \rho_1 \rho_2 + c_{12}$$

two-body correlation function

The first BBGKY equations gives for the mean-field evolution equations

$$i\dot{\rho}_1 = [t_1, \rho_1] + \mathrm{Tr}_{(2)} \{[v_{12}, \rho_{12}]\}$$

$$i\dot{\rho}_{12} = [t_1 + t_2 + v_{12}, \rho_{12}] + \mathrm{Tr}_{(3)} \{[v_{13} + v_{23}, \rho_{123}]\}$$




$$\Gamma_{1,ij}(\rho) = \sum_{mn} v_{(im,jn)} \rho_{2,nm}$$

MEAN-FIELD

the mean-field approximation

# CONTRIBUTIONS FROM PAIRING CORRELATIONS

The extended Hamiltonian with mixings, ν interactions with matter and ν :

$$\mathcal{H}(t,\vec{q}) = \begin{pmatrix} S(t,q) - \hat{q}\cdot\vec{V}(t) & -\hat{\epsilon}_q^*\cdot\vec{V}(t) \\ -\hat{\epsilon}_q\cdot\vec{V}(t) & \bar{S}(t,q) + \hat{q}\cdot\vec{V}(t) \end{pmatrix}.$$

The off-diagonal term introduces neutrino-antineutrino mixing,
if medium anisotropic and pairing correlations present.

The quantities in H are *(trace terms taken out to simplify expressions)* :

$$S(t,q) = h^0(q) + h^{\rm mat}(t) + \sqrt{2}G_F\int_{\vec{p}} \ell(t,\vec{p}) \qquad \ell(t,\vec{q}) = \rho(t,\vec{q}) - \bar{\rho}(t,-\vec{q}).$$

$$\vec{V}(t) = -\sqrt{2}G_F\int_{\vec{p}}\left\{\hat{p}\,\ell(t,\vec{p}) + \hat{\epsilon}_p\kappa(t,\vec{p}) + \hat{\epsilon}_p^*\kappa^\dagger(t,\vec{p})\right\}.$$

If medium is isotropic and ν-ν pairing correlations are negligible, one recovers the « usual » mean-field equations :

$$i\dot{\rho} = [h(\rho),\rho] \qquad h = h^0 + h^{mat} \qquad h_{\nu\nu} = \sqrt{2}G_F\int_{\vec{p}}(1-\hat{q}\cdot\hat{p})\ell(t,\vec{p}).$$

# NEUTRINO MASS CONTRIBUTIONS

Serreau and Volpe, PRD 90 (2014), arXiv:1409.3591

With the same procedure extended equations with neutrino mass terms can be derived. For Majorana neutrinos one has :

$$\rho_M(t,\vec{q}) \rightarrow \begin{pmatrix} \rho_M(t,\vec{q}) & \zeta_M(t,\vec{q}) \\ \zeta_M^\dagger(t,\vec{q}) & \bar{\rho}_M^T(t,-\vec{q}) \end{pmatrix} \qquad \zeta_{ij}(t,\vec{q}) = \langle a_j^\dagger(t,\vec{q},+)a_i(t,\vec{q},-)\rangle$$

$$\Gamma_M^{\nu\nu}(t,\vec{q}) \rightarrow \begin{pmatrix} H_M(t,\vec{q}) & \Phi_M(t,\vec{q}) \\ \Phi_M^\dagger(t,\vec{q}) & -\bar{H}_M^T(t,-\vec{q}) \end{pmatrix}$$

$$H_M(t,\vec{q}) = S(t,q) - \hat{q}\cdot\vec{V}(t) - \hat{q}\cdot\vec{V}_m(t),$$

$$\bar{H}_M(t,\vec{q}) = \bar{S}(t,q) + \hat{q}\cdot\vec{V}(t) + \hat{q}\cdot\vec{V}_m(t),$$

$$\Phi_M(t,\vec{q}) = e^{i\phi_q}\hat{\epsilon}_q^* \cdot \left[\vec{V}(t)\frac{m}{2q} + \frac{m}{2q}\vec{V}^T(t)\right].$$

The off-diagonal term introduces neutrino-antineutrino mixing, if medium anisotropic. This is referred to as helicity coherence.

Vlasenko, Fuller, Cirigliano, PRD89 (2014) – spin coherence

First calculation shows it might have an impact under appropriate conditions.

Vlasenko, Fuller, Cirigliano, arXiv:1406.6724