The Leptonic Dirac CP-Violating Phase from Sum Rules

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Outline

- 3-neutrino mixing
- General setup
- Sum rules
- Predictions
- Conclusions

Based on I. Girardi, S.T. Petcov, A.T., NPB **894** (2015) 733 [arXiv:1410.8056]
I. Girardi, S.T. Petcov, A.T., EPJC **75** (2015) 7, 345 [arXiv:1504.00658]

3-neutrino mixing

$$\nu_{lL} = \sum_{j=1}^{3} U_{lj} \nu_{jL}\,, \quad l = e, \mu, \tau$$

U is the Pontecorvo-Maki-Nakagawa-Sakata neutrino mixing matrix

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix}$$

	Best fit	**3σ range**
$\sin^2\theta_{12}$	0.308	0.259 ÷ 0.359
$\sin^2\theta_{23}$ (NO)	0.437	0.374 ÷ 0.626
$\sin^2\theta_{23}$ (IO)	0.455	0.380 ÷ 0.641
$\sin^2\theta_{13}$ (NO)	0.0234	0.0176 ÷ 0.0295
$\sin^2\theta_{13}$ (IO)	0.0240	0.0178 ÷ 0.0298
δ/π (NO)	1.39	0 ÷ 2
δ/π (IO)	1.31	0 ÷ 2

Capozzi *et. al.*, PRD **89** (2014) 093018

$\theta_{12} \approx \pi/4 - 0.20$

$\theta_{23} \approx \pi/4 - 0.06$

$\theta_{13} \approx 0 + 0.15$

Symmetry?

General setup

$$U = U_e^\dagger U_\nu$$

$$U_e^\dagger M_e M_e^\dagger U_e = \mathrm{diag}\left(m_e^2, m_\mu^2, m_\tau^2\right) \qquad U_\nu^T M_\nu U_\nu = \mathrm{diag}\left(m_1, m_2, m_3\right)$$

M_e is the charged lepton mass matrix

M_ν is the neutrino Majorana mass matrix

$$U = \tilde{U}_e^\dagger \, \Psi \tilde{U}_\nu \, Q_0$$

Frampton, Petcov, Rodejohann,
NPB **687** (2004) 31

$\tilde{U}_e$ and $\tilde{U}_\nu$ are CKM-like 3×3 unitary matrices

$$\Psi = \mathrm{diag}\left(1, e^{-i\psi}, e^{-i\omega}\right) \qquad Q_0 = \mathrm{diag}\left(1, e^{i\frac{\xi_{21}}{2}}, e^{i\frac{\xi_{31}}{2}}\right)$$

$\tilde{U}_\nu$ is assumed to have a symmetry form which is dictated by, or associated with, a flavour (discrete) symmetry, e.g., A_4, S_4, A_5, T'

General setup

Symmetry forms of $\tilde{U}_\nu$: bimaximal, tri-bimaximal, golden ratio, hexagonal…

$$U_{\mathrm{BM}} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} \qquad U_{\mathrm{TBM}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}$$

$$\tilde{U}_\nu = R_{23}(\theta^\nu_{23})\, R_{12}(\theta^\nu_{12})$$

where

$$R_{12}\,(\theta^\nu_{12}) = \begin{pmatrix} \cos\theta^\nu_{12} & \sin\theta^\nu_{12} & 0 \\ -\sin\theta^\nu_{12} & \cos\theta^\nu_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- Tri-bimaximal (TBM) — A_4/T' — $\theta^\nu_{12} = \arcsin(1/\sqrt{3}) \approx 35°$
- Bimaximal (BM) — S_4 — $\theta^\nu_{12} = \pi/4 = 45°$
- Golden ratio A (GRA) — A_5 — $\theta^\nu_{12} = \arcsin(1/\sqrt{(2+r)}) \approx 31°$
- Golden ratio B (GRB) — D_{10} — $\theta^\nu_{12} = \arcsin(\sqrt{(3-r)/2}) = 36°$
- Hexagonal (HG) — D_{12} — $\theta^\nu_{12} = \pi/6 = 30°$

$\theta^\nu_{23} = -\pi/4$ for all these symmetry forms

r is the golden ratio: $r = (1 + \sqrt{5})/2$

General setup

Charged lepton (CL) corrections:

- 1 rotation from the CL sector and 2 rotations from the neutrino sector

$$U = R_{ij}(\theta^e_{ij})\, \Psi\, R_{23}(\theta^\nu_{23})\, R_{12}(\theta^\nu_{12})\, Q_0$$

- 2 rotations from the CL sector and 2 rotations from the neutrino sector

$$U = R_{ij}(\theta^e_{ij})\, R_{kl}(\theta^e_{kl})\, \Psi\, R_{23}(\theta^\nu_{23})\, R_{12}(\theta^\nu_{12})\, Q_0$$

- 1 rotation from the CL sector and 3 rotations from the neutrino sector

$$U = R_{ij}(\theta^e_{ij})\, \Psi\, R_{23}(\theta^\nu_{23})\, R_{13}(\theta^\nu_{13})\, R_{12}(\theta^\nu_{12})\, Q_0$$

=> sum rules for cos δ, i.e., cos δ as a function of the observable mixing angles θ_{12}, θ_{23}, θ_{13} and the angles θ^ν_{ij}, whose values are fixed

Sum rules

1 rotation from the CL sector and 2 rotations from the neutrino sector

$R_{12}(\theta^e_{12})$

$$U = R_{12}(\theta^e_{12})\, \Psi\, R_{23}(\theta^\nu_{23})\, R_{12}(\theta^\nu_{12})\, Q_0$$

$$\sin^2\theta_{23} = \frac{\sin^2\theta^\nu_{23} - \sin^2\theta_{13}}{1 - \sin^2\theta_{13}}$$

$$\cos\delta = \frac{(\cos 2\theta_{13} - \cos 2\theta^\nu_{23})^{\frac{1}{2}}}{\sqrt{2}\sin 2\theta_{12}\sin\theta_{13}|\cos\theta^\nu_{23}|}\left[\cos 2\theta^\nu_{12} + (\sin^2\theta_{12} - \cos^2\theta^\nu_{12})\,\frac{2\sin^2\theta^\nu_{23} - (3 + \cos 2\theta^\nu_{23})\sin^2\theta_{13}}{\cos 2\theta_{13} - \cos 2\theta^\nu_{23}}\right]$$

$R_{13}(\theta^e_{13})$

$$U = R_{13}(\theta^e_{13})\, \Psi\, R_{23}(\theta^\nu_{23})\, R_{12}(\theta^\nu_{12})\, Q_0$$

$$\sin^2\theta_{23} = \frac{\sin^2\theta^\nu_{23}}{1 - \sin^2\theta_{13}}$$

$$\cos\delta = -\frac{(\cos 2\theta_{13} + \cos 2\theta^\nu_{23})^{\frac{1}{2}}}{\sqrt{2}\sin 2\theta_{12}\sin\theta_{13}|\sin\theta^\nu_{23}|}\left[\cos 2\theta^\nu_{12} + (\sin^2\theta_{12} - \cos^2\theta^\nu_{12})\,\frac{2\cos^2\theta^\nu_{23} - (3 - \cos 2\theta^\nu_{23})\sin^2\theta_{13}}{\cos 2\theta_{13} + \cos 2\theta^\nu_{23}}\right]$$

Sum rules

2 rotations from the CL sector and 2 rotations from the neutrino sector

$R_{12}(\theta^e_{12})\, R_{23}(\theta^e_{23})$

$$U = R_{12}(\theta^e_{12})\, R_{23}(\theta^e_{23})\, \Psi\, R_{23}(\theta^\nu_{23})\, R_{12}(\theta^\nu_{12})\, Q_0$$

$$\cos\delta = \frac{\tan\theta_{23}}{\sin 2\theta_{12} \sin\theta_{13}} \left[\cos 2\theta^\nu_{12} + \left(\sin^2\theta_{12} - \cos^2\theta^\nu_{12}\right)\left(1 - \cot^2\theta_{23}\, \sin^2\theta_{13}\right)\right]$$

Petcov, NPB **892** (2015) 400

$R_{13}(\theta^e_{13})\, R_{23}(\theta^e_{23})$

$$U = R_{13}(\theta^e_{13})\, R_{23}(\theta^e_{23})\, \Psi\, R_{23}(\theta^\nu_{23})\, R_{12}(\theta^\nu_{12})\, Q_0$$

$$\cos\delta = -\frac{\cot\theta_{23}}{\sin 2\theta_{12} \sin\theta_{13}} \left[\cos 2\theta^\nu_{12} + \left(\sin^2\theta_{12} - \cos^2\theta^\nu_{12}\right)\left(1 - \tan^2\theta_{23}\, \sin^2\theta_{13}\right)\right]$$

Sum rules

1 rotation from the CL sector and 3 rotations from the neutrino sector

$R_{12}(\theta^e_{12})$

$$U = R_{12}(\theta^e_{12})\,\Psi\,R_{23}(\theta^\nu_{23})\,R_{13}(\theta^\nu_{13})\,R_{12}(\theta^\nu_{12})\,Q_0$$

$$\sin^2\theta_{23} = 1 - \frac{\cos^2\theta^\nu_{23}\cos^2\theta^\nu_{13}}{1-\sin^2\theta_{13}}$$

$$\cos\delta = \frac{1}{\sin 2\theta_{12}\sin\theta_{13}|\cos\theta^\nu_{13}\cos\theta^\nu_{23}|(\cos^2\theta_{13}-\cos^2\theta^\nu_{13}\cos^2\theta^\nu_{23})^{\frac{1}{2}}}\Big[(\cos^2\theta_{13}-\cos^2\theta^\nu_{13}\cos^2\theta^\nu_{23})\sin^2\theta_{12} + \cos^2\theta_{12}\sin^2\theta_{13}\cos^2\theta^\nu_{13}\cos^2\theta^\nu_{23} - \cos^2\theta_{13}(\cos\theta^\nu_{12}\sin\theta^\nu_{13}\cos\theta^\nu_{23} - \sin\theta^\nu_{12}\sin\theta^\nu_{23})^2\Big]$$

$R_{13}(\theta^e_{13})$

$$U = R_{13}(\theta^e_{13})\,\Psi\,R_{23}(\theta^\nu_{23})\,R_{13}(\theta^\nu_{13})\,R_{12}(\theta^\nu_{12})\,Q_0$$

$$\sin^2\theta_{23} = \frac{\sin^2\theta^\nu_{23}\cos^2\theta^\nu_{13}}{1-\sin^2\theta_{13}}$$

$$\cos\delta = -\frac{1}{\sin 2\theta_{12}\sin\theta_{13}|\cos\theta^\nu_{13}\sin\theta^\nu_{23}|(\cos^2\theta_{13}-\cos^2\theta^\nu_{13}\sin^2\theta^\nu_{23})^{\frac{1}{2}}}\Big[(\cos^2\theta_{13}-\cos^2\theta^\nu_{13}\sin^2\theta^\nu_{23})\sin^2\theta_{12} + \cos^2\theta_{12}\sin^2\theta_{13}\cos^2\theta^\nu_{13}\sin^2\theta^\nu_{23} - \cos^2\theta_{13}(\cos\theta^\nu_{12}\sin\theta^\nu_{13}\sin\theta^\nu_{23} + \sin\theta^\nu_{12}\cos\theta^\nu_{23})^2\Big]$$

Predictions for cos δ

Using best fit values of mixing angles for NO neutrino mass spectrum

Scheme	TBM	GRA	GRB	HG	BM
$\theta^e_{12} - (\theta^\nu_{23}, \theta^\nu_{12})$	-0.114	0.289	-0.200	0.476	—
$\theta^e_{13} - (\theta^\nu_{23}, \theta^\nu_{12})$	0.114	-0.289	0.200	-0.476	—
$(\theta^e_{12}, \theta^e_{23}) - (\theta^\nu_{23}, \theta^\nu_{12})$	-0.091	0.275	-0.169	0.445	—
$(\theta^e_{13}, \theta^e_{23}) - (\theta^\nu_{23}, \theta^\nu_{12})$	0.151	-0.315	0.251	-0.531	—

Scheme	$[\pi/20, -\pi/4]$	$[\pi/10, -\pi/4]$	$[a, -\pi/4]$	$[\pi/20, b]$	$[\pi/20, \pi/6]$
$\theta^e_{12} - (\theta^\nu_{23}, \theta^\nu_{13}, \theta^\nu_{12})$	-0.222	0.760	0.911	-0.775	-0.562

Scheme	$[\pi/20, c]$	$[\pi/20, \pi/4]$	$[\pi/10, \pi/4]$	$[a, \pi/4]$	$[\pi/20, d]$
$\theta^e_{13} - (\theta^\nu_{23}, \theta^\nu_{13}, \theta^\nu_{12})$	-0.866	0.222	-0.760	-0.911	-0.791

$\theta^\nu_{23} = -\pi/4$ $[\theta^\nu_{13}, \theta^\nu_{12}]$

$a = \arcsin(1/3)$ $b = \arcsin(1/\sqrt{(2+r)})$ $c = \arcsin(1/\sqrt{3})$ $d = \arcsin(\sqrt{(3-r)/2})$

Non-zero values of θ^ν_{13}: Bazzocchi, arXiv:1108.2497, Toorop *et. al.*, PLB **703** (2011) 447, Rodejohann and Zhang, PLB **732** (2014) 174

Predictions for J_{CP}: statistical analysis

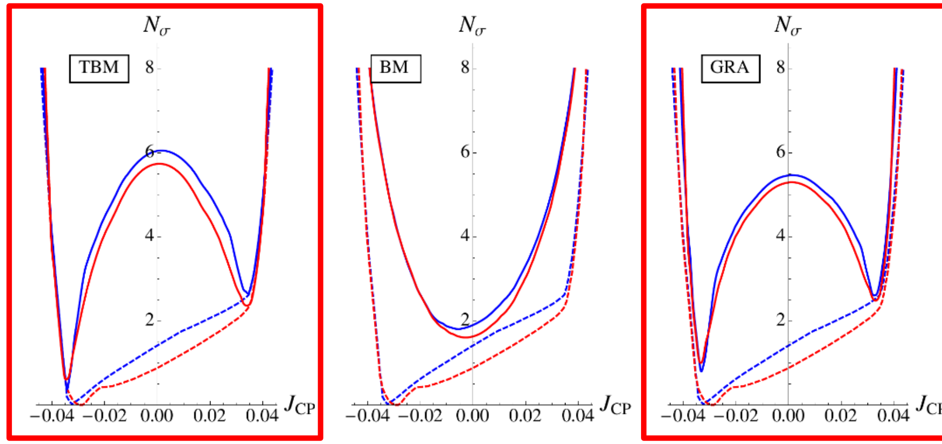
$R_{12}(\theta^e_{12})\, R_{23}(\theta^e_{23})$

$$U = R_{12}(\theta^e_{12})\, R_{23}(\theta^e_{23})\, \Psi\, R_{23}(\theta^\nu_{23})\, R_{12}(\theta^\nu_{12})\, Q_0$$

$$J_{\rm CP} = {\rm Im}\left\{U^*_{e1} U^*_{\mu 3} U_{e3} U_{\mu 1}\right\} = \frac{1}{8} \sin\delta \sin 2\theta_{13} \sin 2\theta_{23} \sin 2\theta_{12} \cos\theta_{13}$$

J_{CP} determines the magnitude of CP-violating effects in neutrino oscillations

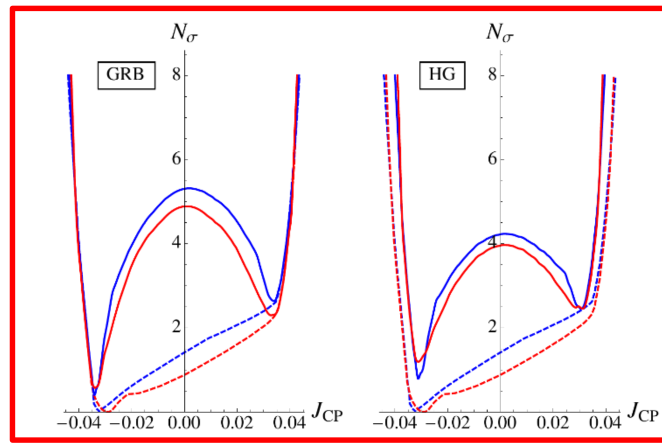
Krastev and Petcov, PLB **205** (1988) 84



$$N_\sigma \equiv \sqrt{\chi^2}$$

NO scheme
IO scheme

NO global fit
IO global fit



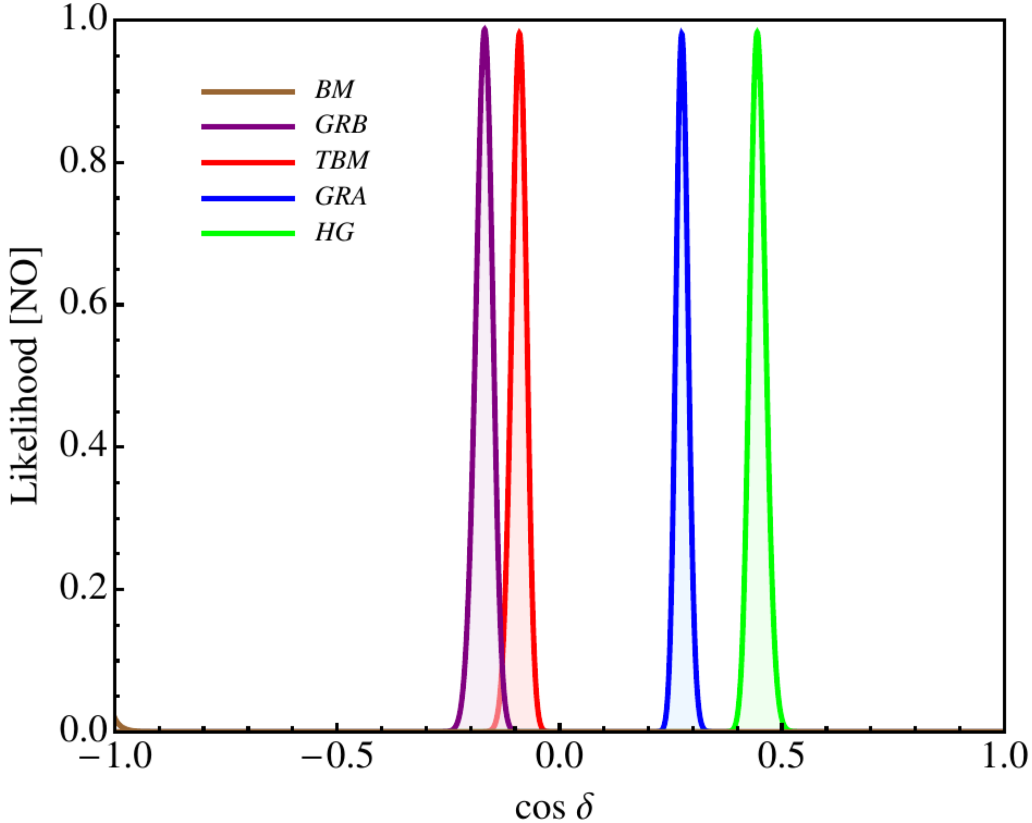
Relatively large CP-violating effects in neutrino oscillations in the cases of TBM, GRA, GRB, HG:
$J_{CP} \approx -0.03$, $|J_{CP}| \gtrsim 0.02$ @ 3σ
and suppressed ones in the case of BM:
$J_{CP} \approx 0$

Using latest results on $\sin^2\theta_{ij}$ and δ, obtained in global analysis of neutrino oscillation data in Capozzi *et. al.*, PRD **89** (2014) 093018

Predictions for cos δ: statistical analysis

Likelihood function $$L(\cos\delta) \propto \exp\left(-\frac{\chi^2(\cos\delta)}{2}\right)$$

$R_{12}(\theta^e_{12})\, R_{23}(\theta^e_{23})$ $$U = R_{12}(\theta^e_{12})\, R_{23}(\theta^e_{23})\, \Psi\, R_{23}(\theta^\nu_{23})\, R_{12}(\theta^\nu_{12})\, Q_0$$

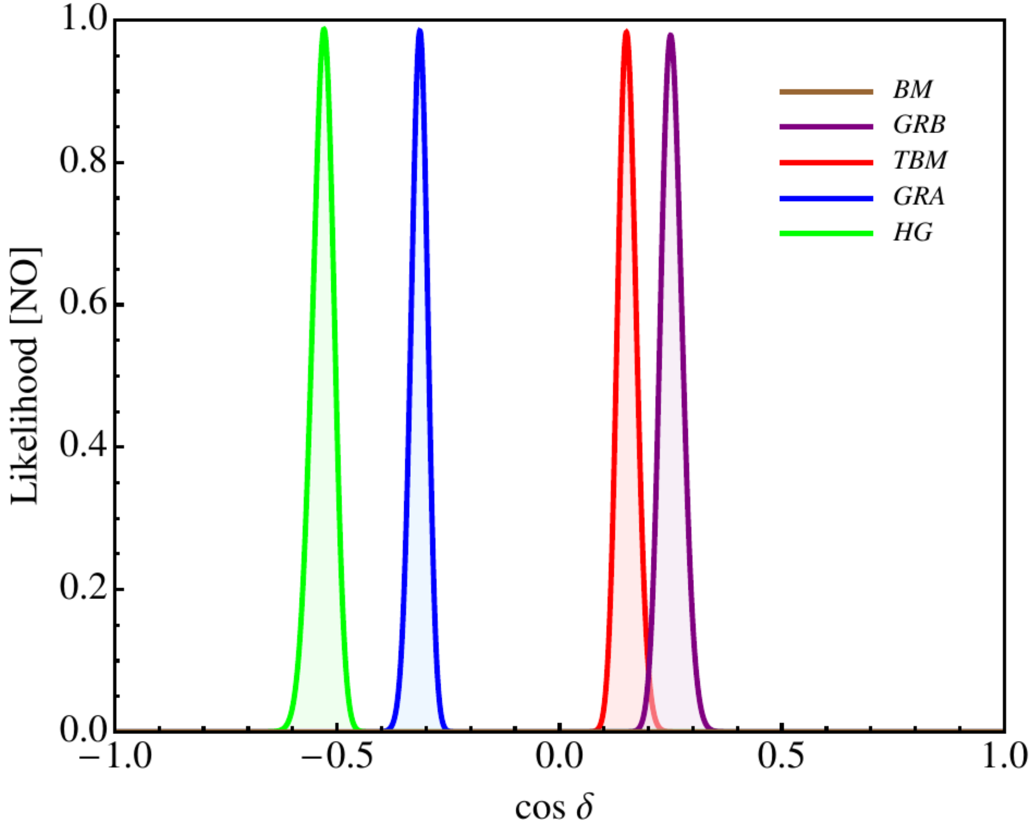


Using current best fit values and prospective 1σ uncertainties for $\sin^2\theta_{ij}$: 0.7% for $\sin^2\theta_{12}$ (JUNO), 3% for $\sin^2\theta_{13}$ (Daya Bay), 5% for $\sin^2\theta_{23}$ (NOvA and T2K) + Gaussian approximation

Predictions for cos δ: statistical analysis

Likelihood function $$L(\cos\delta) \propto \exp\left(-\frac{\chi^2(\cos\delta)}{2}\right)$$

$R_{13}(\theta^e_{13})\, R_{23}(\theta^e_{23})$ $$U = R_{13}(\theta^e_{13})\, R_{23}(\theta^e_{23})\, \Psi\, R_{23}(\theta^\nu_{23})\, R_{12}(\theta^\nu_{12})\, Q_0$$

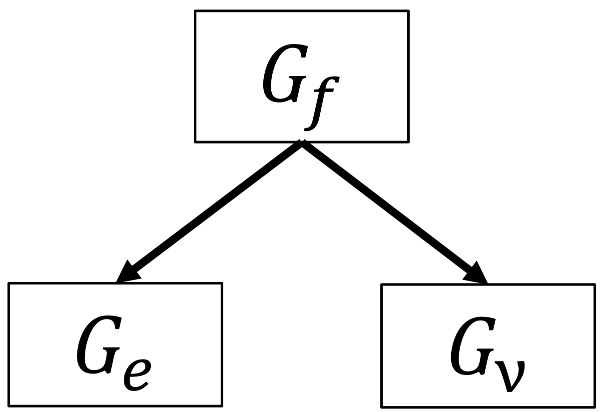


Using current best fit values and prospective 1σ uncertainties for $\sin^2\theta_{ij}$: 0.7% for $\sin^2\theta_{12}$ (JUNO), 3% for $\sin^2\theta_{13}$ (Daya Bay), 5% for $\sin^2\theta_{23}$ (NOvA and T2K) + Gaussian approximation

Extension of the study

Today on arXiv:

I. Girardi, S.T. Petcov, A.J. Stuart, A.T., arXiv:1509.02502



Flavour symmetry group (non-Abelian discrete)

Residual symmetries of the CL and neutrino mass matrices

$\Downarrow$

Sum rules for $\cos\delta$

Conclusions

- ❖ Exact (within the schemes considered) sum rules for $\cos\delta$
- ❖ Relatively large CP-violating effects in neutrino oscillations in the cases of TBM, GRA, GRB, HG and suppressed ones in the case of BM
- ❖ Sufficiently precise measurement of δ combined with prospective precision on the neutrino mixing angles can provide information about the existence of a new type of fundamental symmetry in the lepton sector

Backup

Statistical details

$$\chi^2(\sin^2\theta_{12}, \sin^2\theta_{13}, \sin^2\theta_{23}, \delta) = \chi_1^2(\sin^2\theta_{12}) + \chi_2^2(\sin^2\theta_{13}) + \chi_3^2(\sin^2\theta_{23}) + \chi_4^2(\delta)$$

1) χ_i^2 being extracted from Capozzi *et. al.*, PRD **89** (2014) 093018

2) Gaussian approximation:

$$\chi_i^2 = \frac{(x_i - \overline{x}_i)^2}{\sigma_{x_i}^2}$$

$x_i = \{\sin^2\theta_{12}, \sin^2\theta_{13}, \sin^2\theta_{23}, \delta\}$

Future: $x_i = \{\sin^2\theta_{12}, \sin^2\theta_{13}, \sin^2\theta_{23}\}$

$\overline{x}_i$ and σ_{x_i} are b.f.v. and 1σ uncertainties

$$\chi^2\left(x_i^m(y_j^m)\right) = \sum_{i=1} \chi_i^2\left(x_i^m(y_j^m)\right)$$

y_j^m are parameters of the scheme

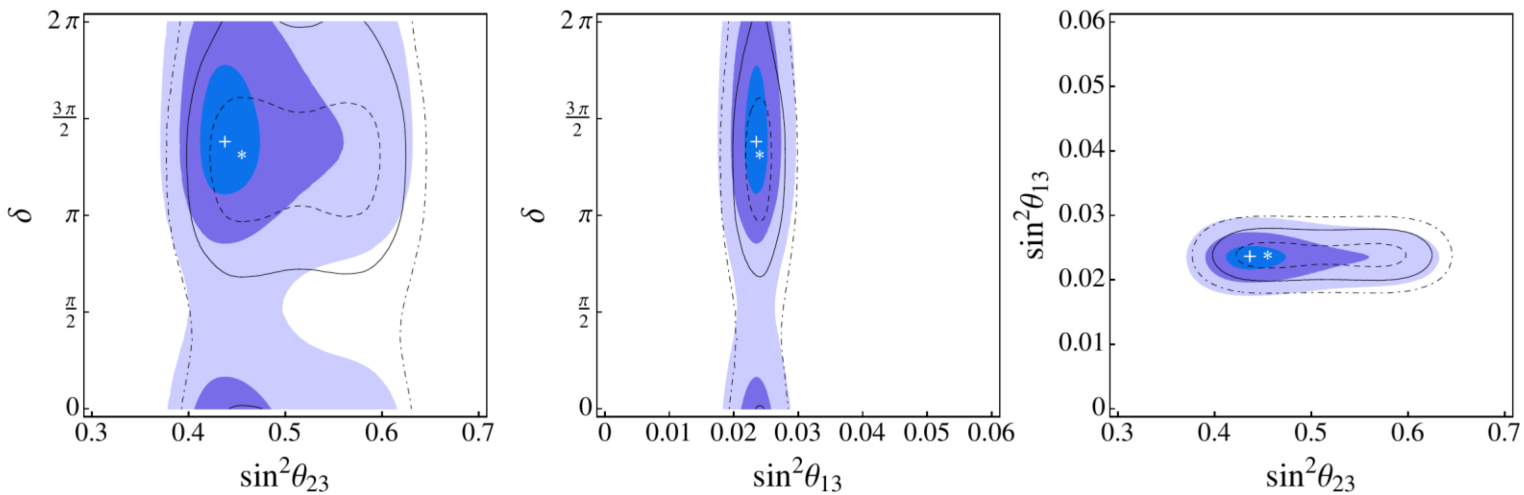
$$\chi^2(\alpha) = \min\left[\chi^2\left(x_i^m(y_j^m)\right)\Big|_{\alpha=\text{const}}\right]$$

$\alpha = \{\delta, J_{\text{CP}}, \sin^2\theta_{23}\}$

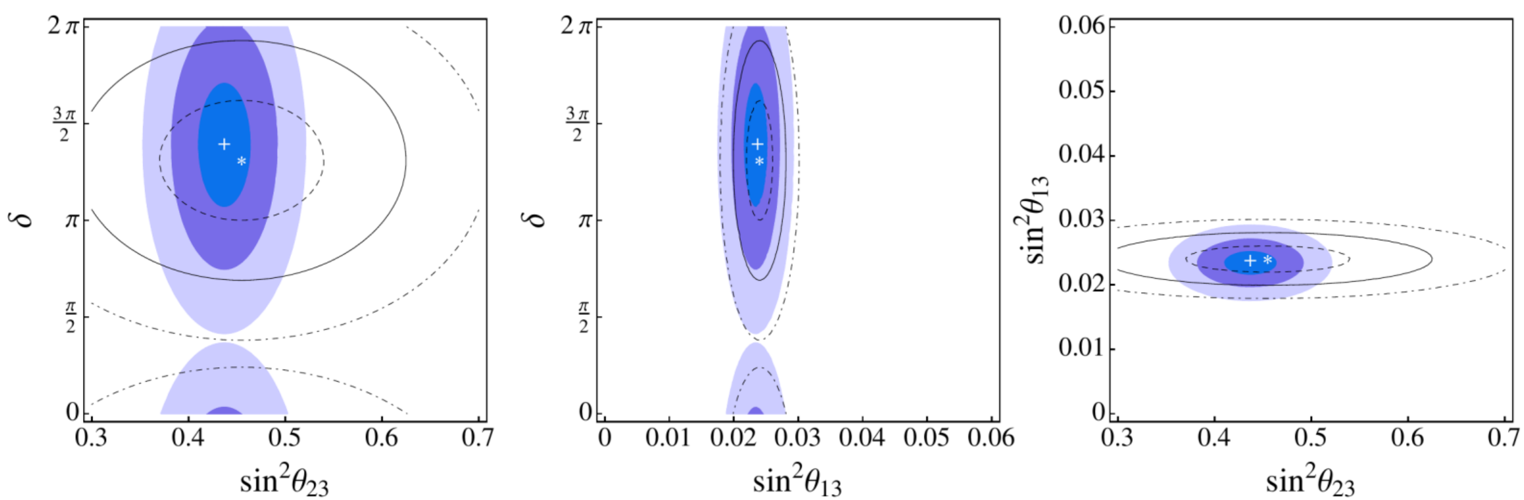
$$L(\cos\delta) \propto \exp\left(-\frac{\chi^2(\cos\delta)}{2}\right)$$

Statistical details: comparison

1)



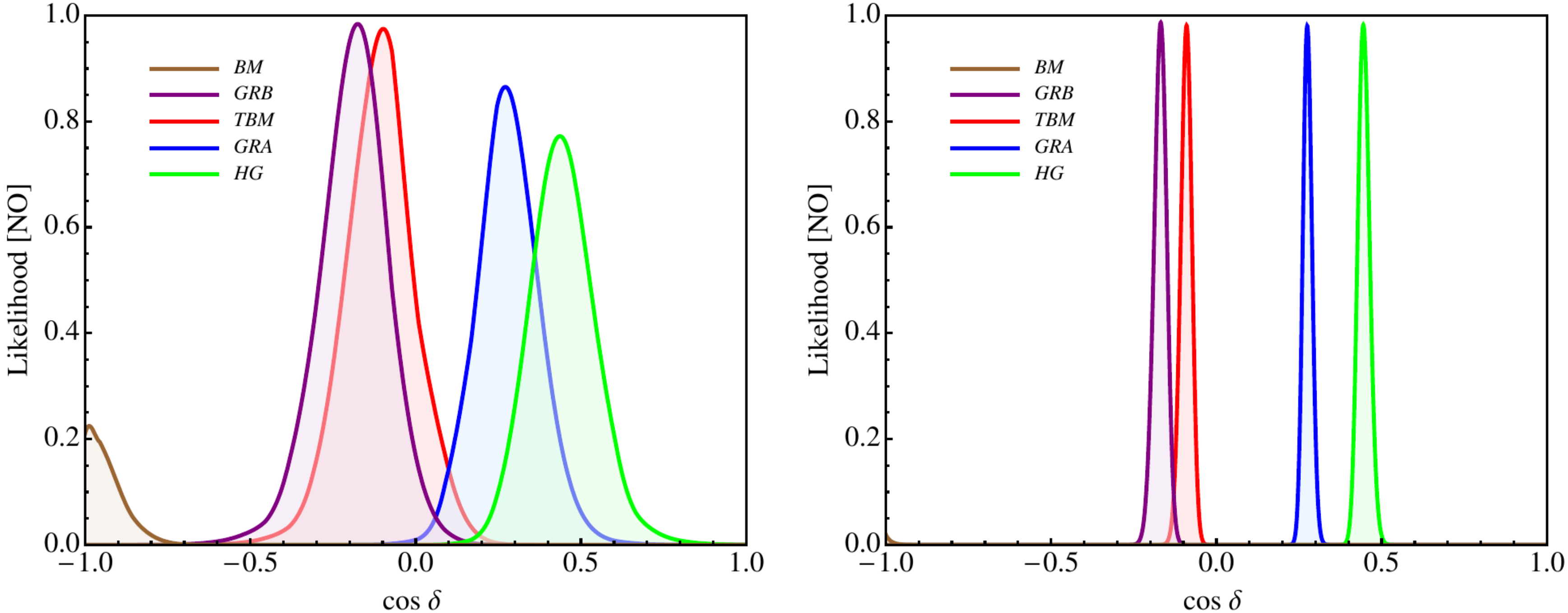
2)



Predictions for cos δ: statistical analysis

Likelihood function $L(\cos\delta) \propto \exp\left(-\frac{\chi^2(\cos\delta)}{2}\right)$

$R_{12}(\theta^e_{12})\, R_{23}(\theta^e_{23})$ $U = R_{12}(\theta^e_{12})\, R_{23}(\theta^e_{23})\, \Psi\, R_{23}(\theta^\nu_{23})\, R_{12}(\theta^\nu_{12})\, Q_0$



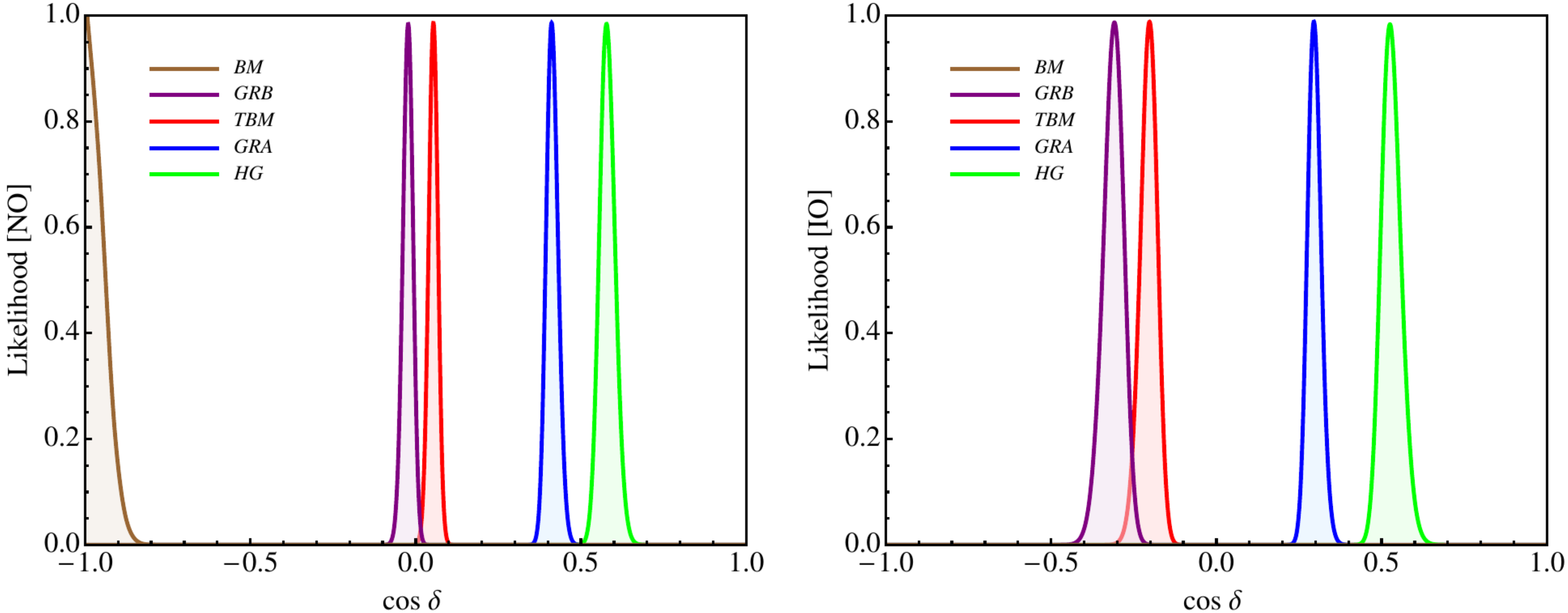
Using latest results on $\sin^2\theta_{ij}$ and δ, obtained in global analysis of neutrino oscillation data in Capozzi *et. al.*, PRD **89** (2014) 093018

Using prospective 1σ uncertainties on $\sin^2\theta_{ij}$:
0.7% for $\sin^2\theta_{12}$ (JUNO)
3% for $\sin^2\theta_{13}$ (Daya Bay)
5% for $\sin^2\theta_{23}$ (NOvA and T2K)
+ Gaussian approximation

Dependence on best fit values

Likelihood function $L(\cos\delta) \propto \exp\left(-\frac{\chi^2(\cos\delta)}{2}\right)$

$R_{12}(\theta^e_{12})\, R_{23}(\theta^e_{23})$ $U = R_{12}(\theta^e_{12})\, R_{23}(\theta^e_{23})\, \Psi\, R_{23}(\theta^\nu_{23})\, R_{12}(\theta^\nu_{12})\, Q_0$



$(\sin^2\theta_{12})_{bf} = 0.332$

$(\sin^2\theta_{23})_{bf} = 0.437$

$(\sin^2\theta_{13})_{bf} = 0.0234$

$(\sin^2\theta_{12})_{bf} = 0.304$

$(\sin^2\theta_{23})_{bf} = 0.579$

$(\sin^2\theta_{13})_{bf} = 0.0219$

IO neutrino mass spectrum

Gonzalez-Garcia *et. al.*, JHEP **1411** (2014) 052

Predictions for δ: statistical analysis

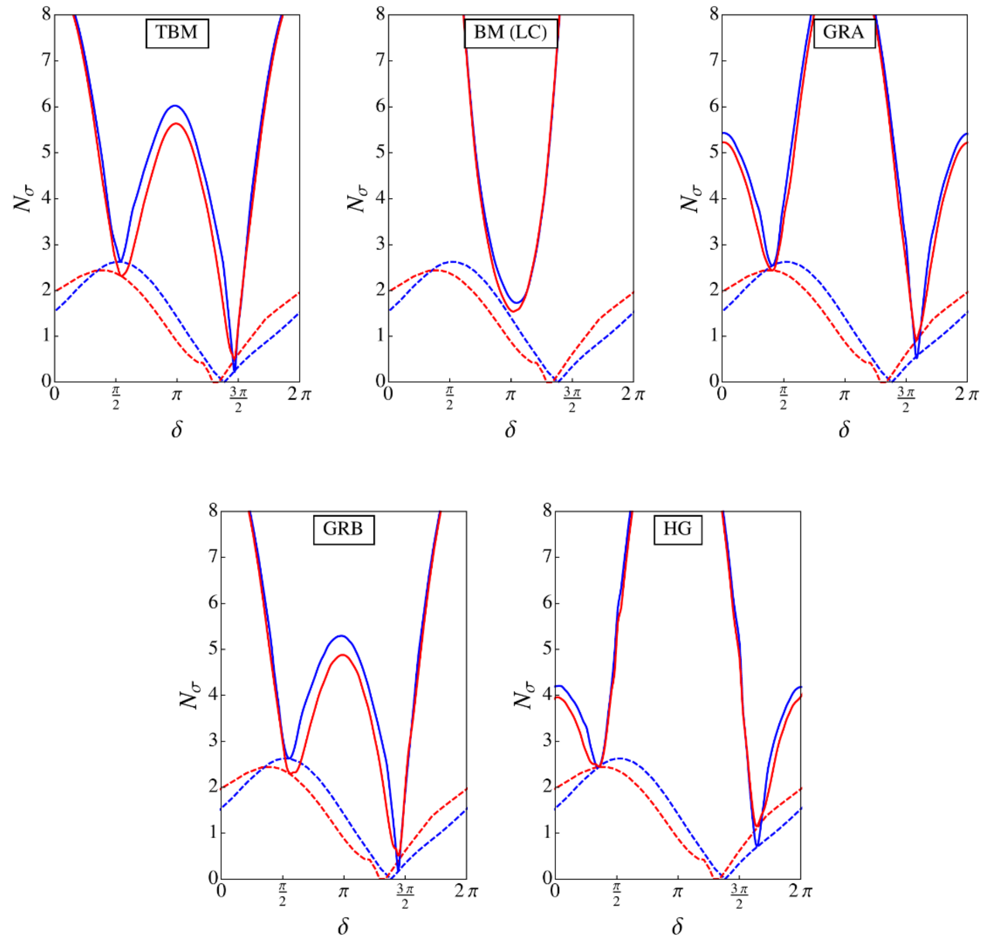
$R_{12}(\theta^e_{12})\, R_{23}(\theta^e_{23})$

$$U = R_{12}(\theta^e_{12})\, R_{23}(\theta^e_{23})\, \Psi\, R_{23}(\theta^\nu_{23})\, R_{12}(\theta^\nu_{12})\, Q_0$$

$$N_\sigma \equiv \sqrt{\chi^2}$$

NO scheme
IO scheme

NO global fit
IO global fit



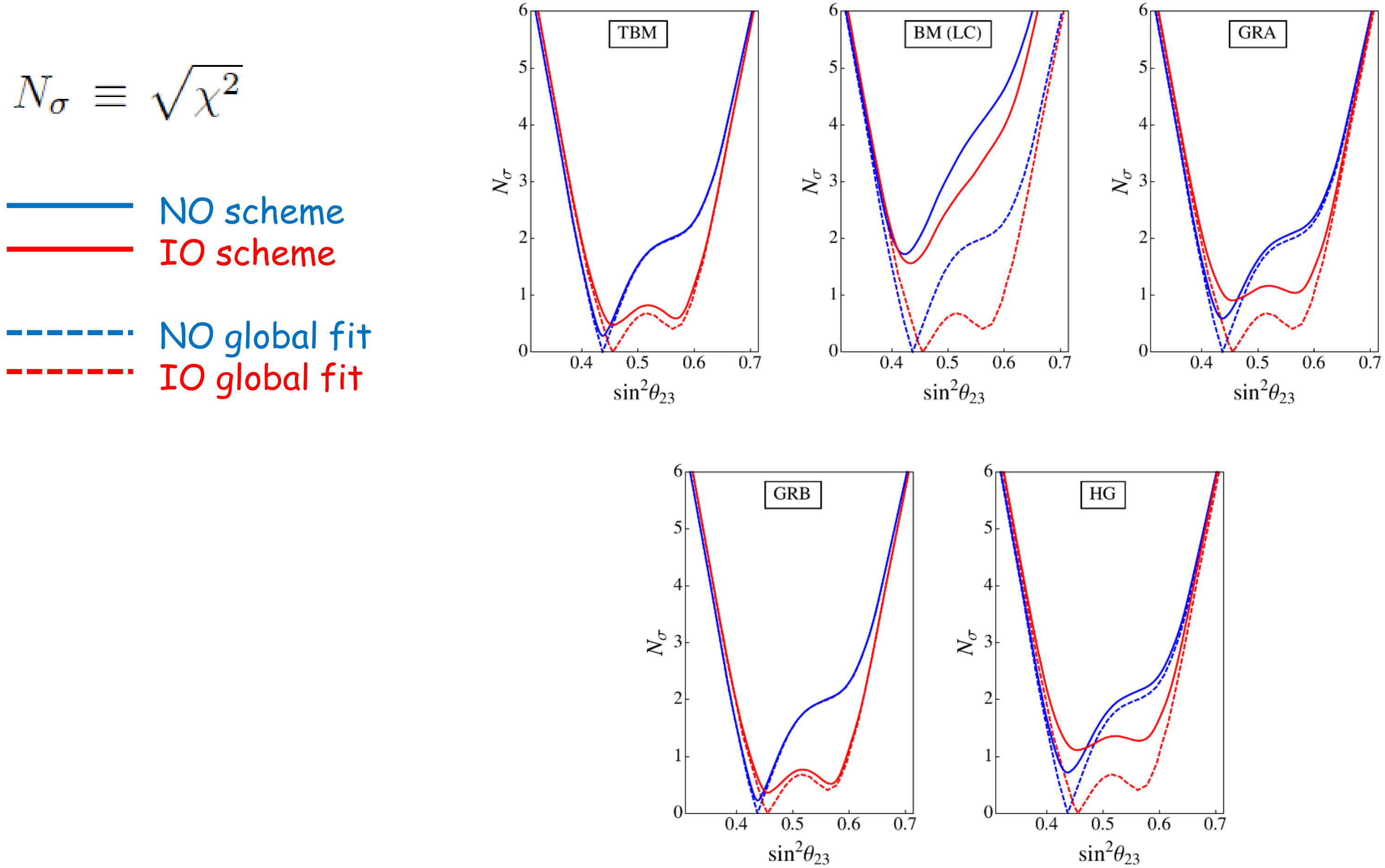
Using latest results on $\sin^2\theta_{ij}$ and δ, obtained in global analysis of neutrino oscillation data in Capozzi *et. al.*, PRD **89** (2014) 093018

Results for $\sin^2\theta_{23}$: statistical analysis

$R_{12}(\theta^e_{12})\, R_{23}(\theta^e_{23})$

$$U = R_{12}(\theta^e_{12})\, R_{23}(\theta^e_{23})\, \Psi\, R_{23}(\theta^\nu_{23})\, R_{12}(\theta^\nu_{12})\, Q_0$$

$$N_\sigma \equiv \sqrt{\chi^2}$$

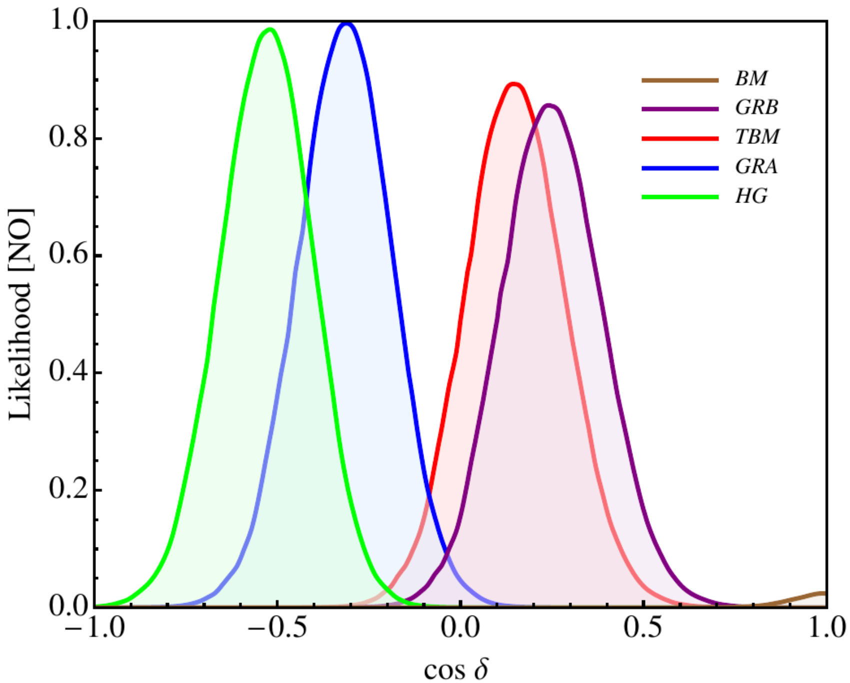


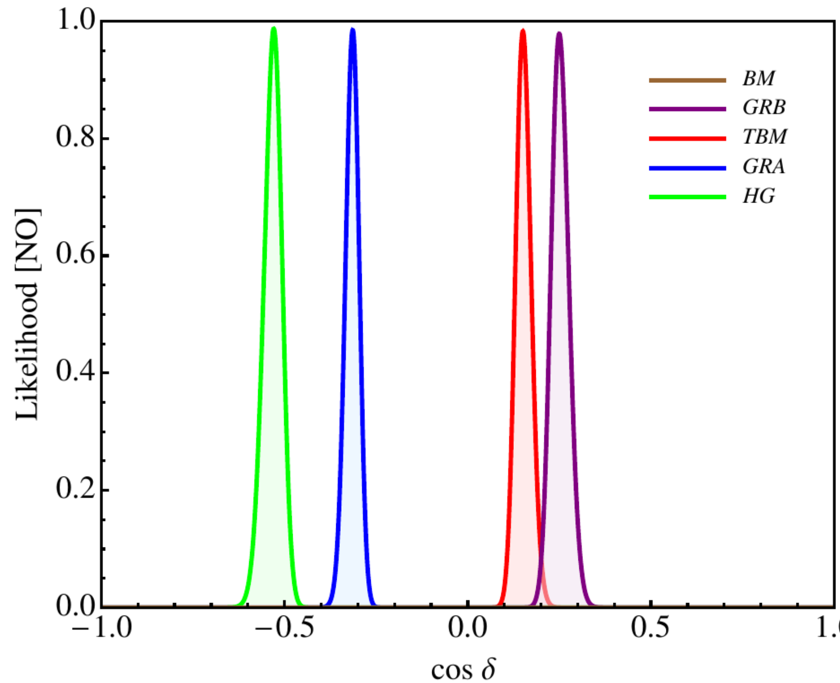
Using latest results on $\sin^2\theta_{ij}$ and δ, obtained in global analysis of neutrino oscillation data in Capozzi *et. al.*, PRD **89** (2014) 093018

Predictions for cos δ: statistical analysis

Likelihood function $L(\cos\delta) \propto \exp\left(-\frac{\chi^2(\cos\delta)}{2}\right)$

$R_{13}(\theta^e_{13})\, R_{23}(\theta^e_{23})$ $U = R_{13}(\theta^e_{13})\, R_{23}(\theta^e_{23})\, \Psi\, R_{23}(\theta^\nu_{23})\, R_{12}(\theta^\nu_{12})\, Q_0$





Using latest results on $\sin^2\theta_{ij}$ and δ, obtained in global analysis of neutrino oscillation data in Capozzi *et. al.*, PRD **89** (2014) 093018

Using prospective 1σ uncertainties on $\sin^2\theta_{ij}$:
0.7% for $\sin^2\theta_{12}$ (JUNO)
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5% for $\sin^2\theta_{23}$ (NOvA and T2K)
+ Gaussian approximation

Results for $\sin^2\theta_{23}$: statistical analysis

$R_{13}(\theta^e_{13})\, R_{23}(\theta^e_{23})$ $$U = R_{13}(\theta^e_{13})\, R_{23}(\theta^e_{23})\, \Psi\, R_{23}(\theta^\nu_{23})\, R_{12}(\theta^\nu_{12})\, Q_0$$

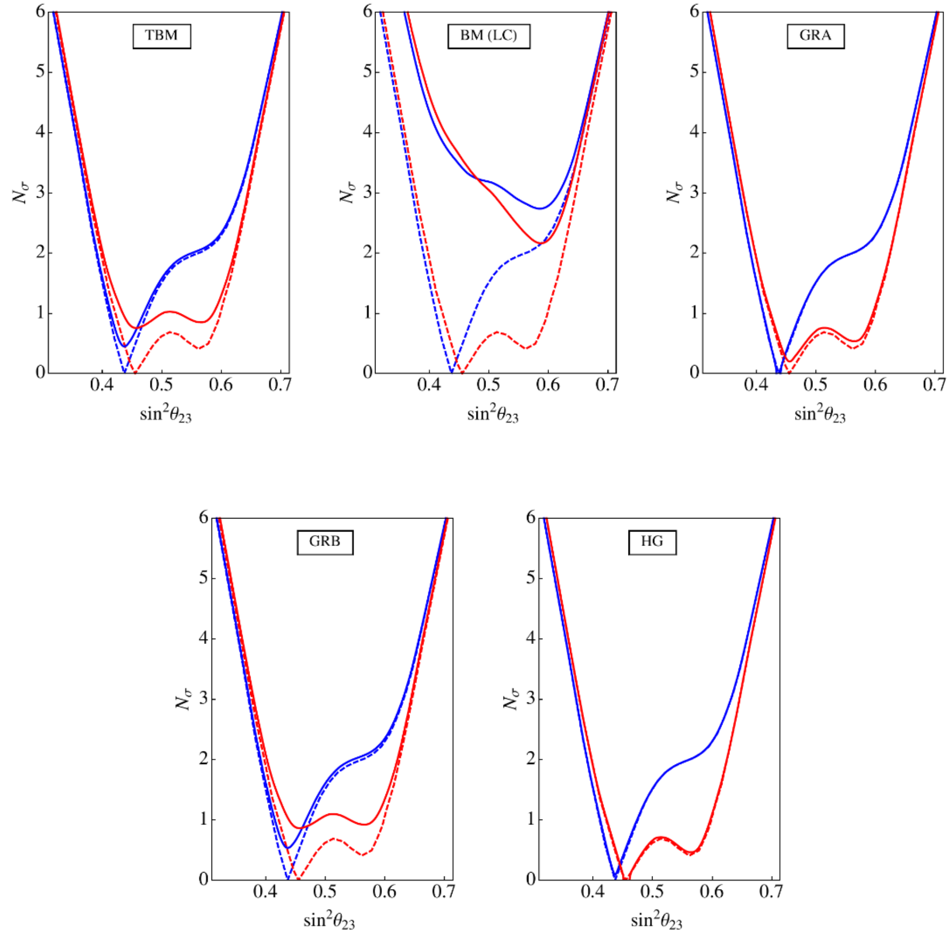
$$N_\sigma \equiv \sqrt{\chi^2}$$

NO scheme

IO scheme

NO global fit

IO global fit

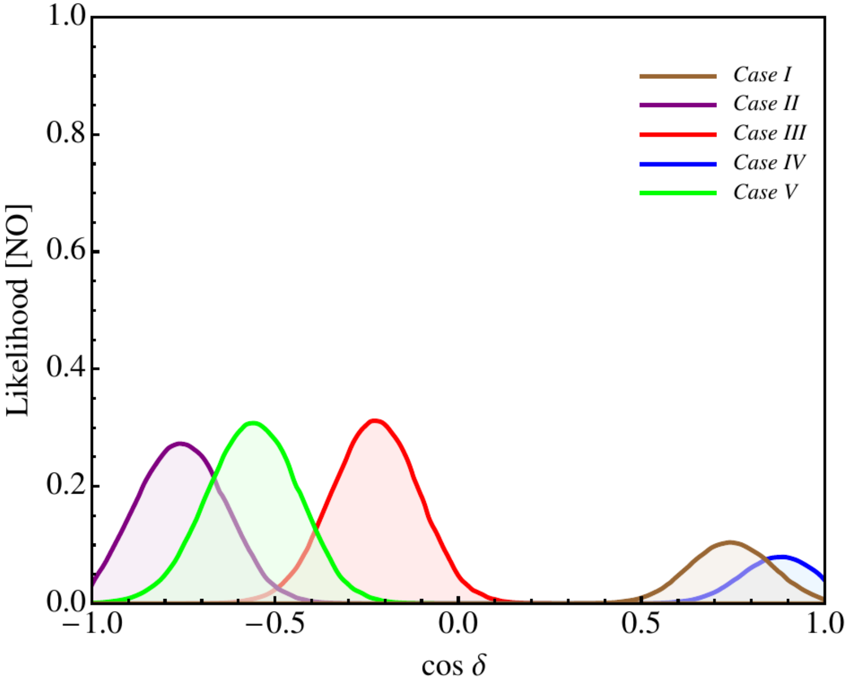


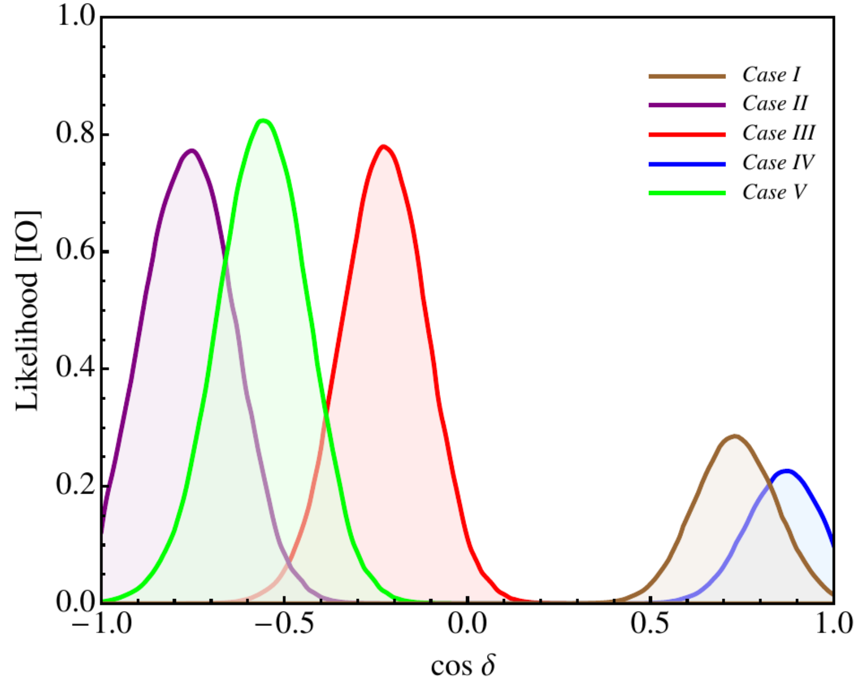
Using latest results on $\sin^2\theta_{ij}$ and δ, obtained in global analysis of neutrino oscillation data in Capozzi *et. al.*, PRD **89** (2014) 093018

Predictions for cos δ: statistical analysis

Likelihood function $$L(\cos\delta) \propto \exp\left(-\frac{\chi^2(\cos\delta)}{2}\right)$$

$R_{12}(\theta^e_{12})$ $$U = R_{12}(\theta^e_{12})\,\Psi\,R_{23}(\theta^\nu_{23})\,R_{13}(\theta^\nu_{13})\,R_{12}(\theta^\nu_{12})\,Q_0$$





Using latest results on $\sin^2\theta_{ij}$ and δ, obtained in global analysis of neutrino oscillation data in Capozzi *et. al.*, PRD **89** (2014) 093018

$[\theta^\nu_{13}, \theta^\nu_{12}]$: Case I = [π/10, − π/4] Case II = [π/20, arcsin (1/√(2+r))]
Case III = [π/20, − π/4] Case IV = [arcsin (1/3), − π/4] Case V = [π/20, π/6]

Predictions for cos δ: statistical analysis

$R_{12}(\theta^e_{12})$

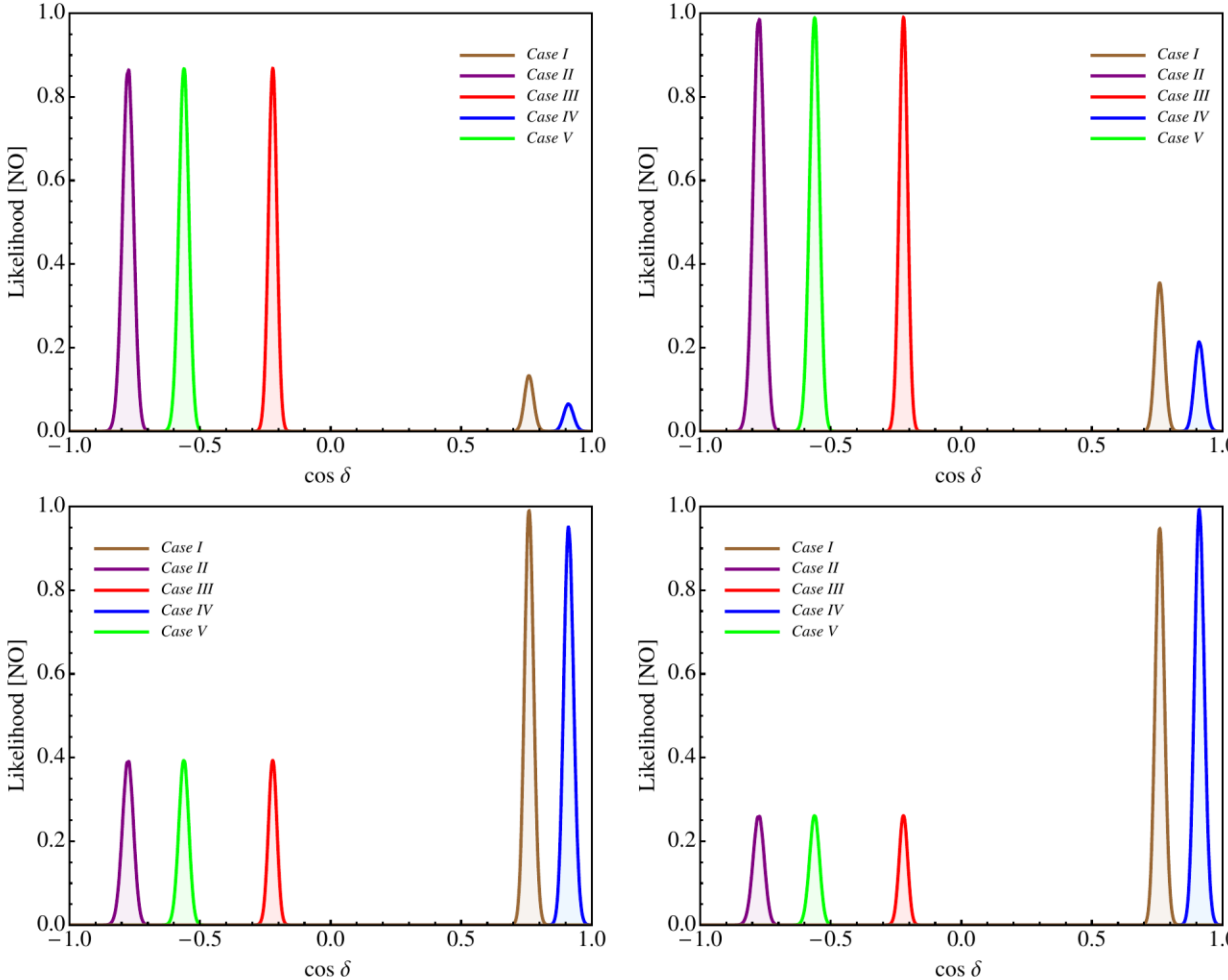
$$U = R_{12}(\theta^e_{12})\,\Psi\,R_{23}(\theta^\nu_{23})\,R_{13}(\theta^\nu_{13})\,R_{12}(\theta^\nu_{12})\,Q_0$$

$(\sin^2\theta_{23})_{pbf} = 0.488$, $\theta^\nu_{13} = 0$

$(\sin^2\theta_{23})_{pbf} = 0.501$, $\theta^\nu_{13} = \pi/20$

$(\sin^2\theta_{23})_{pbf} = 0.537$, $\theta^\nu_{13} = \pi/10$

$(\sin^2\theta_{23})_{pbf} = 0.545$, $\theta^\nu_{13} = \arcsin(1/3)$

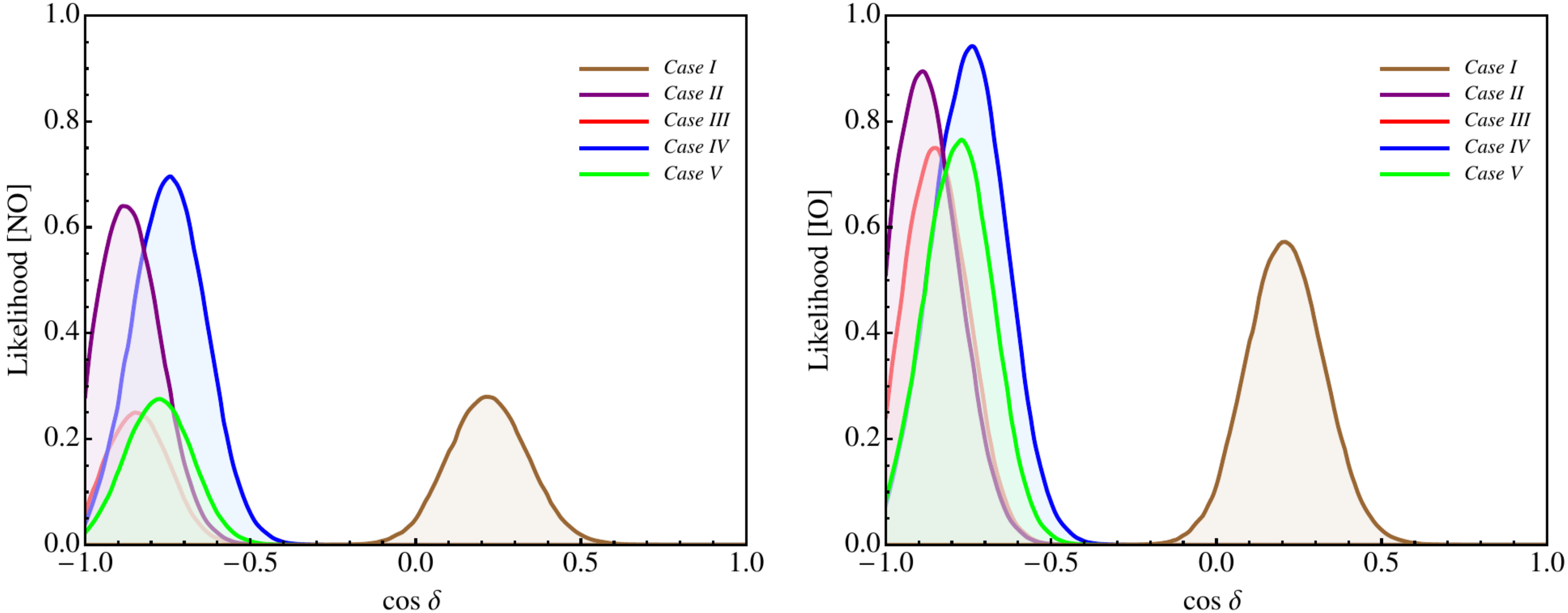


$[\theta^\nu_{13}, \theta^\nu_{12}]$: Case I = $[\pi/10, -\pi/4]$ Case II = $[\pi/20, \arcsin(1/\sqrt{(2+r)})]$
Case III = $[\pi/20, -\pi/4]$ Case IV = $[\arcsin(1/3), -\pi/4]$ Case V = $[\pi/20, \pi/6]$

Predictions for cos δ: statistical analysis

Likelihood function $L(\cos\delta) \propto \exp\left(-\frac{\chi^2(\cos\delta)}{2}\right)$

$R_{13}(\theta^e_{13})$: $U = R_{13}(\theta^e_{13})\,\Psi\,R_{23}(\theta^\nu_{23})\,R_{13}(\theta^\nu_{13})\,R_{12}(\theta^\nu_{12})\,Q_0$



Using latest results on $\sin^2\theta_{ij}$ and δ, obtained in global analysis of neutrino oscillation data in F. Capozzi *et. al.*, PRD **89** (2014) 093018

$[\theta^\nu_{13}, \theta^\nu_{12}]$: Case I = [π/20, π/4] Case II = [arcsin (1/3), π/4]
Case III = [π/20, arcsin (1/√3)] Case IV = [π/10, π/4] Case V = [π/20, arcsin (√(3−r)/2)]

Predictions for cos δ: statistical analysis

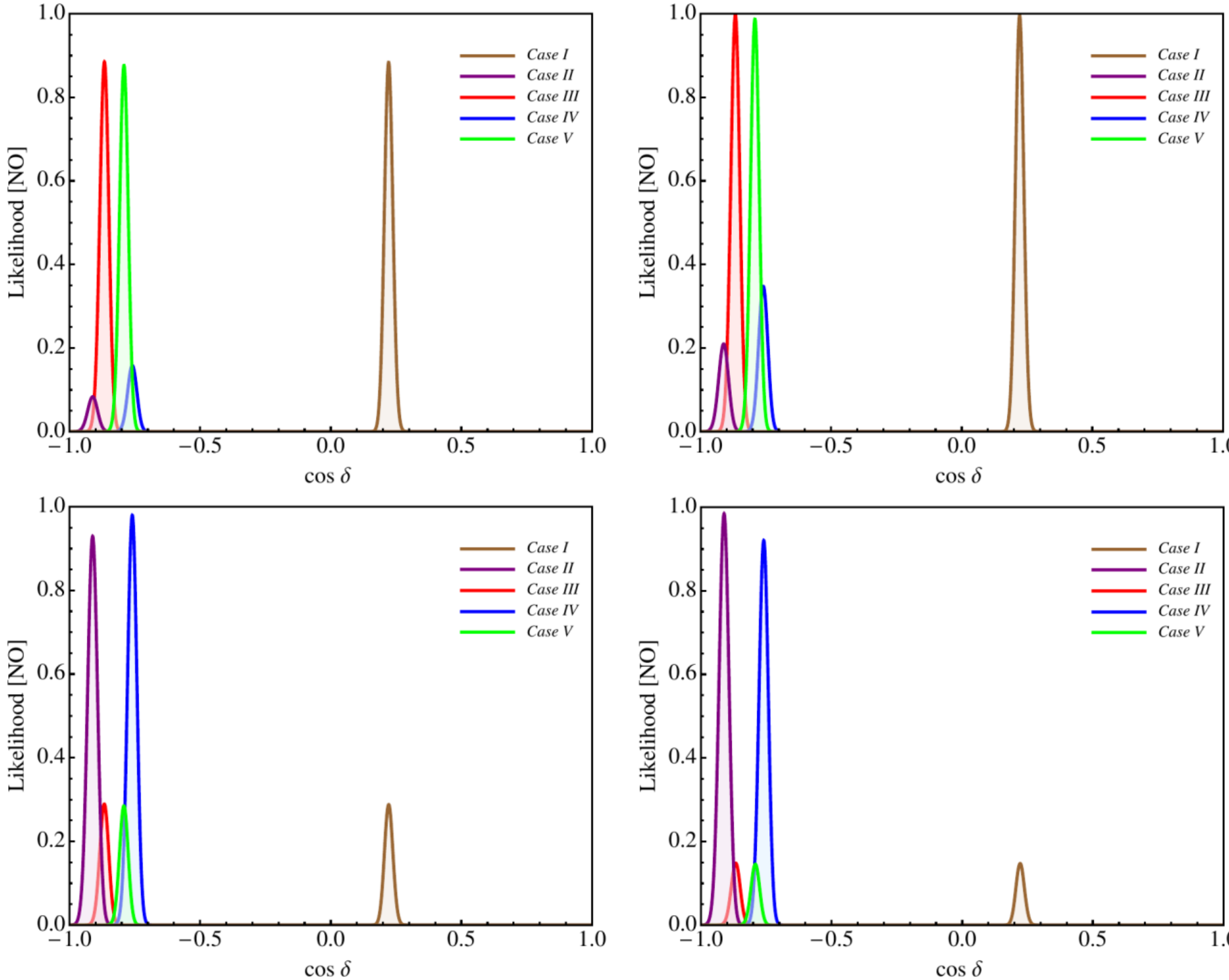
$R_{13}(\theta^e_{13})$ $$U = R_{13}(\theta^e_{13})\,\Psi\,R_{23}(\theta^\nu_{23})\,R_{13}(\theta^\nu_{13})\,R_{12}(\theta^\nu_{12})\,Q_0$$

$(\sin^2\theta_{23})_{\rm pbf} = 0.512$, $\theta^\nu_{13} = 0$

$(\sin^2\theta_{23})_{\rm pbf} = 0.499$, $\theta^\nu_{13} = \pi/20$

$(\sin^2\theta_{23})_{\rm pbf} = 0.463$, $\theta^\nu_{13} = \pi/10$

$(\sin^2\theta_{23})_{\rm pbf} = 0.455$, $\theta^\nu_{13} = \arcsin(1/3)$



$[\theta^\nu_{13}, \theta^\nu_{12}]$: Case I = $[\pi/20, \pi/4]$ Case II = $[\arcsin(1/3), \pi/4]$
Case III = $[\pi/20, \arcsin(1/\sqrt{3})]$ Case IV = $[\pi/10, \pi/4]$ Case V = $[\pi/20, \arcsin(\sqrt{(3-r)/2})]$