# Light as a probe of the structure of space-time

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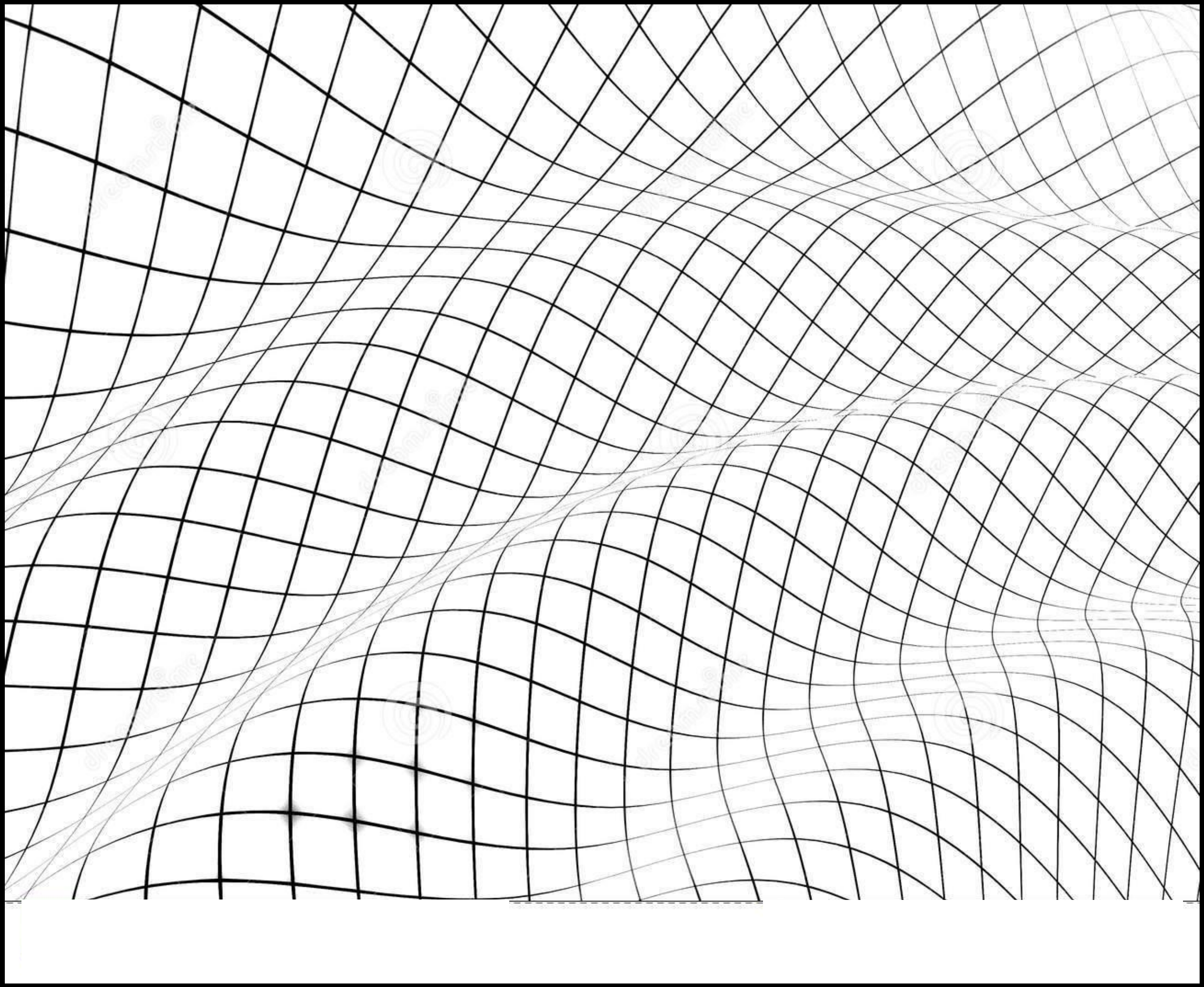
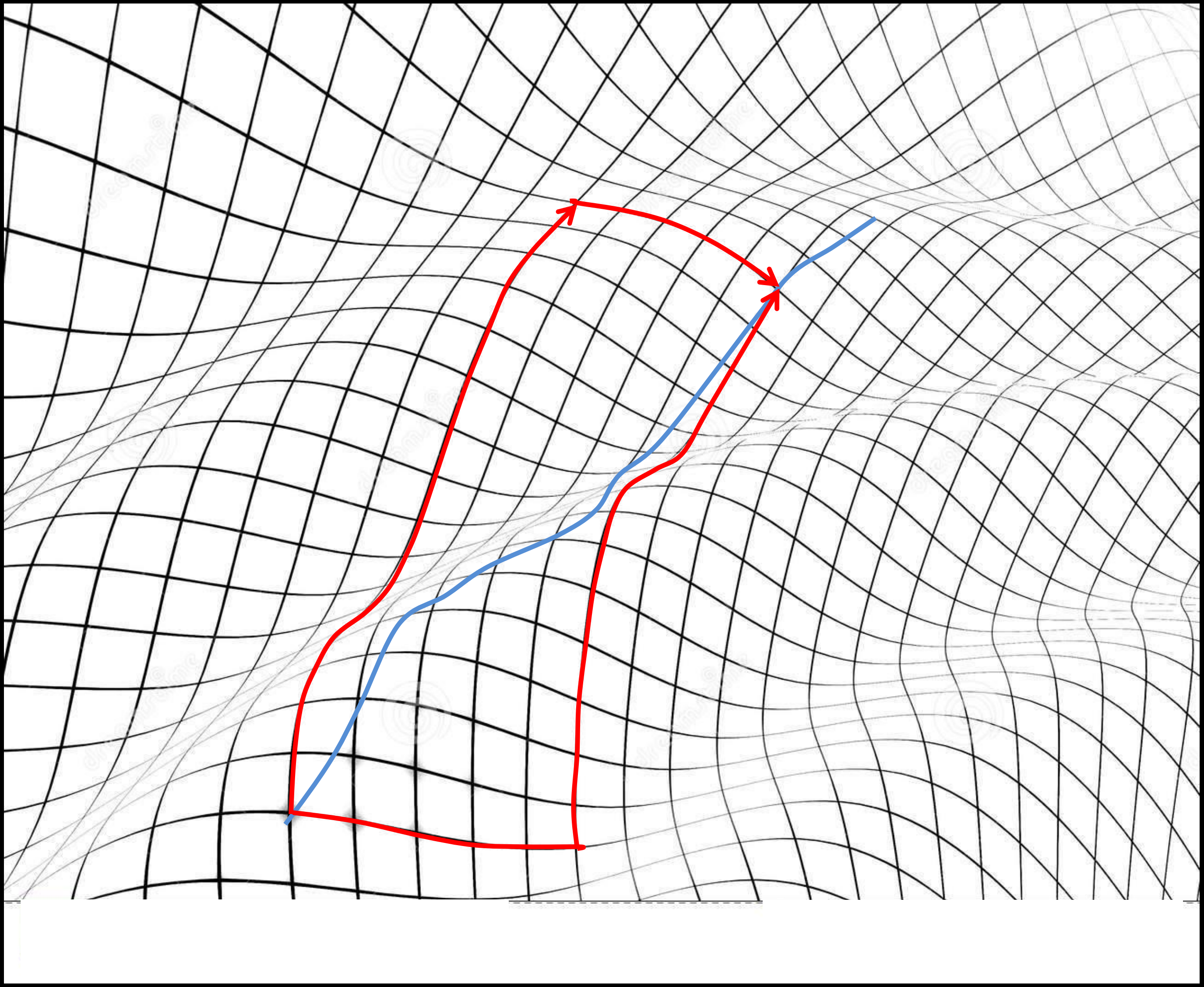
# Best covering of space-time by means of null geodesics



# Best covering of space-time by means of null geodesics



# Chiral symmetry with respect to time

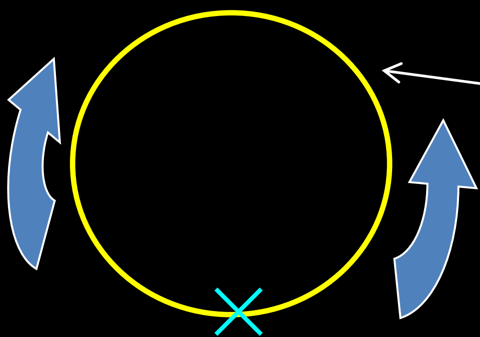
$$ds^2 = g_{00}c^2dt^2 + 2g_{0i}cdtdx^i + g_{ij}dx^idx^j$$

Light → ds = 0

$$dt = \frac{-g_{0i}dx^i \pm \sqrt{\left(g_{0i}dx^i\right)^2 - g_{00}g_{ij}dx^idx^j}}{cg_{00}}$$

+

# Closed space trajectory



Shape is irrelevant

$x^i = x^i(\ell)$

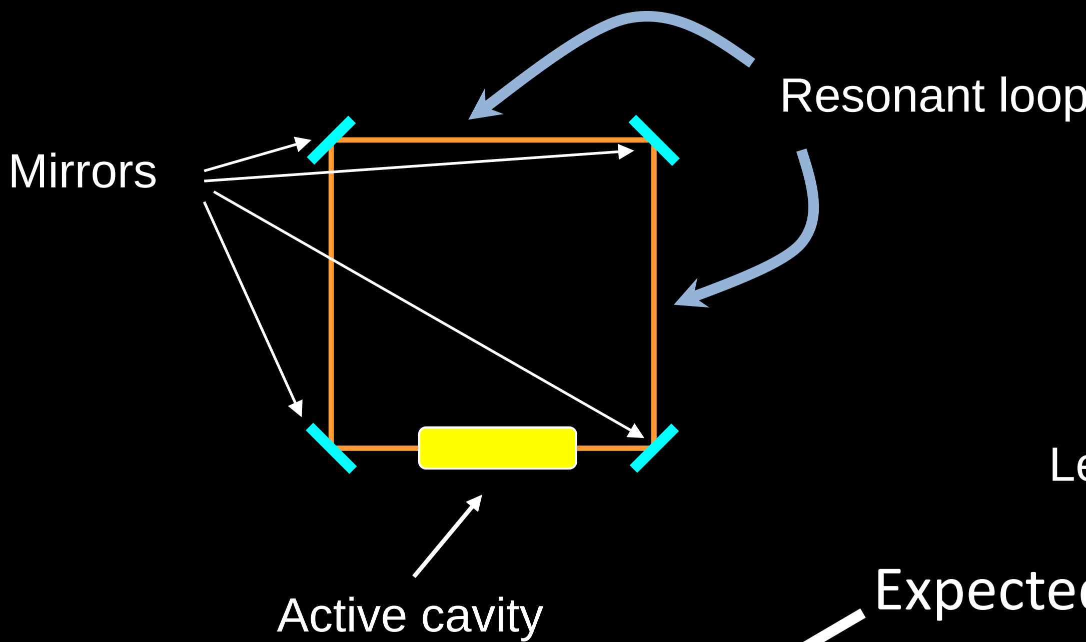
Counterrotating light beams

$$dt = \frac{-g_{0i}\dfrac{dx^i}{d\ell}d\ell + \sqrt{\left(g_{0i}\dfrac{dx^i}{d\ell}\right)^2 - g_{00}g_{ij}\dfrac{dx^i}{d\ell}\dfrac{dx^j}{d\ell}}\,|d\ell|}{cg_{00}}$$

Time of flight difference

$$\Delta\tau = \tau_+ - \tau_- = -\frac{2}{c}\sqrt{g_{00}}\oint\frac{g_{0i}}{g_{00}}\frac{dx^i}{d\ell}d\ell$$

# A ringlaser on earth




Beat frequency

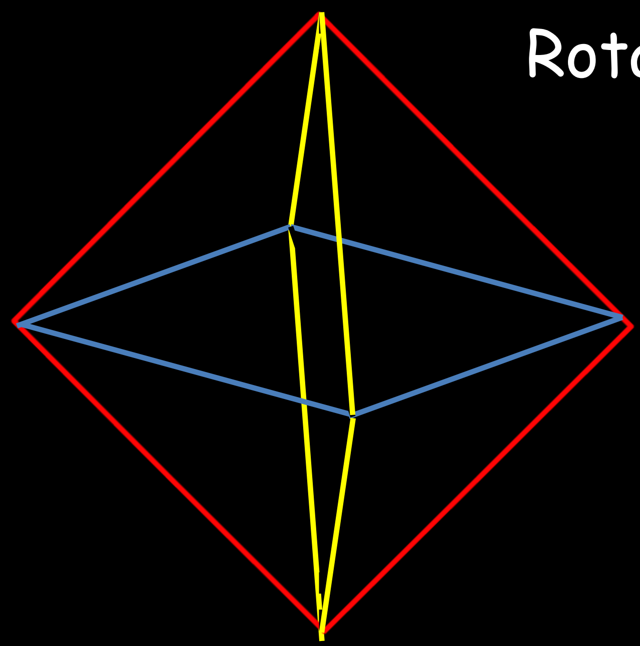
$$f_b = \frac{\Delta f}{2} = \frac{c^2}{2\lambda P}\Delta\tau$$

Length of the loop

Expected signal

$$f_b = 2\frac{A}{\lambda P}\left[\vec{\Omega} - 2G\frac{M}{c^2 R}\Omega\sin\theta\hat{u}_\theta + \frac{GJ_\oplus}{c^2 R^3}\left(2\cos\theta\hat{u}_r + \sin\theta\hat{u}_\theta\right)\right]\cdot\hat{u}_n$$

# GINGER
# Underground at Gran Sasso



Rotation speed of the Earth

$$\Omega = 7.2 \times 10^{-5}\, s^{-1}$$

GR terms

$$\Omega_G \approx 10^{-10}\, \Omega$$

$$\Omega_{LT} \approx 10^{-9}\, \Omega$$

Details in a poster presented at this conference

# PPN parameters

$$ds^2 = \left(1 - 2\frac{GM}{c^2 r}\right)d\tau^2 - \left(1 + 2\gamma\frac{GM}{c^2 r}\right)dr^2 - r^2 d\vartheta^2$$

$$- r^2 \sin^2\vartheta \, d\phi^2 + \left(4 + 4\gamma + \alpha_1\right)\frac{GJ}{2c^3 r}\sin^2\vartheta \, d\tau d\phi$$

$$2M \to (1+\gamma)M \qquad\qquad 2J \to \left(1 + \gamma + \frac{\alpha_1}{4}\right)J$$

# Changes in the GR effects induced by ppn terms

New expected results including the effects of $\gamma$ and $\alpha_1$

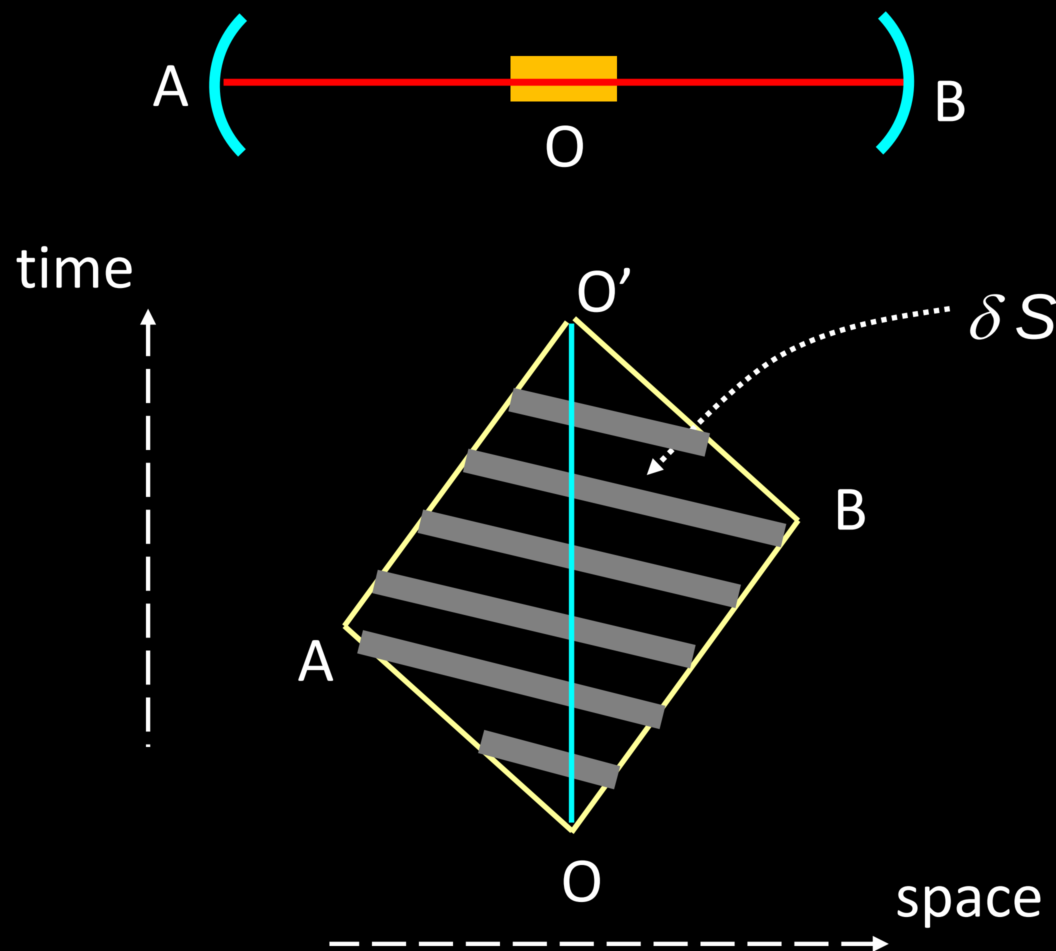
$$\Omega_G = -[2+(\gamma-1)]G\frac{M}{c^2R}\Omega\sin\theta\,\hat{u}_\theta\cdot\hat{u}_n$$

$$\Omega_{LT} = \frac{2+(\gamma-1)+\alpha_1/4}{2}G\frac{J}{c^2R^3}\left(2\cos\theta\,\hat{u}_r+\sin\theta\,\hat{u}_\theta\right)\cdot\hat{u}_n$$

GR: $\gamma=1$ $\alpha_1=0$

Present constraints on the parameters impose accuracies better than $10^{-22}$ rad/s.

# A linear cavity (and GW)



# The effect of curvature and anisotropy

$$\delta F^{\mu\nu} = \left(R^{\mu}{}_{\varepsilon 0i}F^{\varepsilon\nu} + R^{\nu}{}_{\varepsilon 0i}F^{\mu\varepsilon}\right)\delta S^{0i}$$

Electromagnetic tensor

Riemann tensor

Space-time area
spanned by the cavity

Depends on the orientation of the cavity

# Approximated Riemann tensor. An example:

Length of the cavity

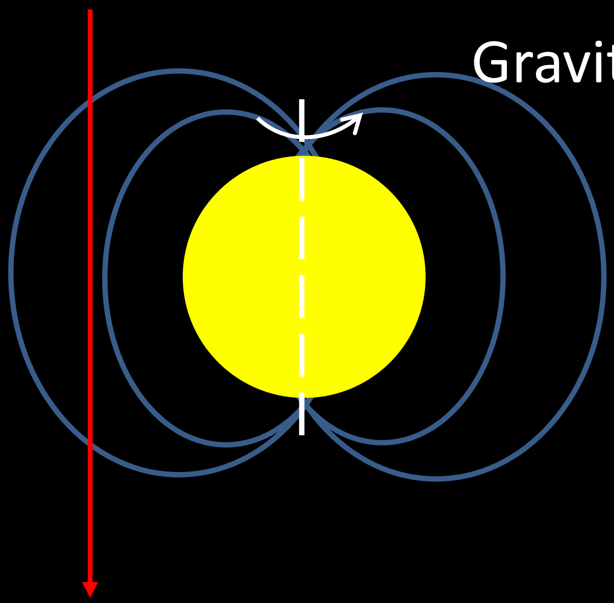
$R^{\phi}{}_{r0\theta}$

$$\delta F^{\vartheta\phi} \cong \left[\left(\frac{(GM)^{3/2}}{c^3 R^{7/2}} - 3\frac{GJ}{c^3 R^4}\right)\frac{\cos\vartheta}{\sin^2\vartheta}\right]\frac{l^2}{R} F^{\vartheta r}$$

Change in the radial component of the magnetic field

East-West component of the magnetic field

# Gravito-magnetic Faraday effect



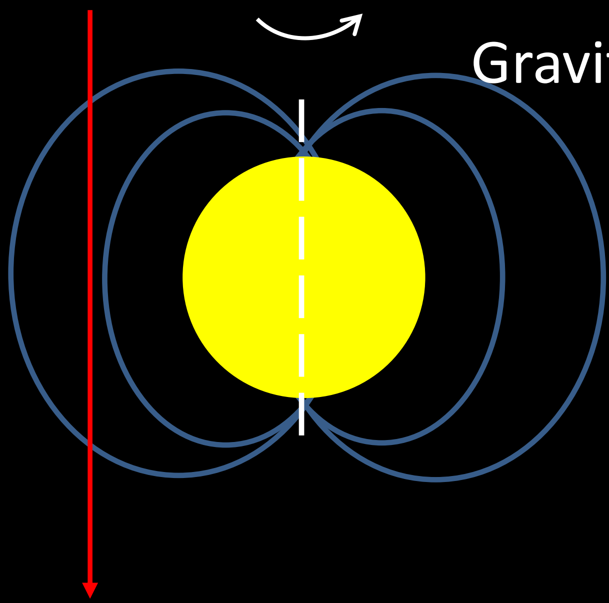

Rotation angle

$$\alpha = \frac{1}{2}\int_{emission}^{reception} \sqrt{g_{00}}\left(\vec{\nabla} \wedge \vec{h}\right)\cdot \hat{u}_k dl$$

$$h_i = \frac{g_{0i}}{g_{00}}$$

Gravito-magnetic potential

# Weak field approximation (lowest order in the angular momentum)



Gravitomagnetic field

Linearly polarized light

Approximately straight propagation parallel to the axis of the central mass

GJ/$c^2$

$$\alpha \cong \int_{-\infty}^{+\infty} \left( \vec{\nabla} \wedge \vec{g}_0 \right) \cdot \hat{u}_k \, dl = \frac{j}{4b^2}$$

Minimum approach distance

# Conclusion

- Light is an intrinsically relativistic probe of space-time
- Closed paths of light in space (and space-time) evidence general relativistic effects either by
  - Interferometric techniques
    or
  - Beat frequency measurements (ring lasers ↔ GINGER)
- Improved observation techniques may give access to gravito-magnetic effects on the polarization of light

# Collaboration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# References


- A. Di Virgilio, et al., *Comptes Rendus Physique*, vol. **15**, 866874 (2014)
- N. Beverini, et al., *Laser Phys.*, vol. **74**, 074005 (2014)
- M.L. Ruggiero and A. Tartaglia, *Eur. Phis. J. Plus*, vol. **129**, 126 (2014)
- F. Bosi, et al., *Phys. Rev. D*, vol. **84**, 122002/1-23 (2011)
- M.L. Ruggiero and A. Tartaglia, *MNRAS*, vol. 374, p. 847-851 (2007)