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- Conclusions



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Of interest for gw bar:

- DM due to very heavy particles with “strong” interaction (Quark Nuggets, MACROs, Nuclearites..) recently from astrophysical considerations $\sigma/M \sim 0.1-10 \text{ cm}^2 \text{ g}^{-1}$

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- DM due to very heavy particles with “strong” interaction (Quark Nuggets, MACROs, Nuclearites..) recently from astrophysical considerations $\sigma/M \sim 0.1-10 \text{ cm}^2 \text{ g}^{-1}$
- DM with **gravitational only interactions** (Newtorites if point like, or extended objects like walls, clumps..). Very big objects (MACHO's) already excluded by astronomical observations (micro-lensing experiments limit $10^{-7} M_{\text{sun}} < \text{Mass} < 10^{-3} M_{\text{sun}}$)

Tim M. P. Tait: Overview talk ICRC 2015 : we don't know very much. But DM should have gravitational interactions!

The Dark Matter Questionnaire

Mass

Spin

Stable?

Yes

No

Couplings:

Gravity

Weak Interaction?

Higgs?

Quarks / Gluons?

Leptons?

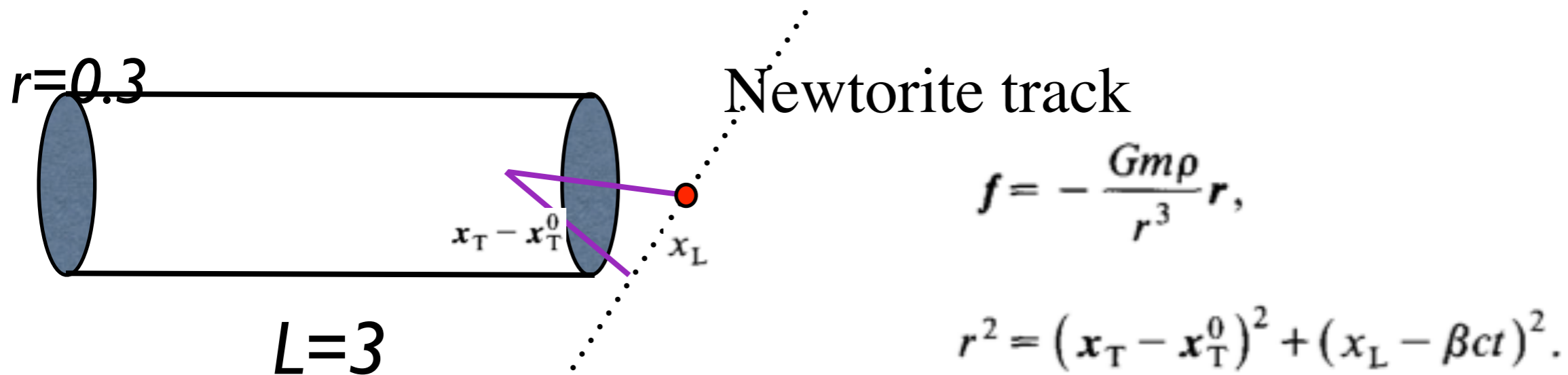
Thermal Relic?

Yes

No

Newtorite resonant bar response

Bernard DeRujula Lautrup Nucl Phys B242 93 (1984)



Integrating on time from $-\infty$ to ∞ the amplitude of the n oscillation mode is

$$A_n = -2 \frac{Gm}{V\beta c} \int_V \frac{\mathbf{u}_n \cdot (\mathbf{x}_T - \mathbf{x}_T^0)}{(\mathbf{x}_T - \mathbf{x}_T^0)^2} d^3x.$$

\mathbf{u}_n is the bar oscillation normal n mode; for a thin bar in cylindrical coordinates:

$$\begin{aligned} u_n^r &= \sqrt{2}\sigma_P\pi(r/L)\sin(n\pi z/L) \\ u_n^z &= \sqrt{2}\cos(n\pi z/L) \end{aligned}$$

Search of events $E > 0.1$ K in NAUTILUS and EXPLORER

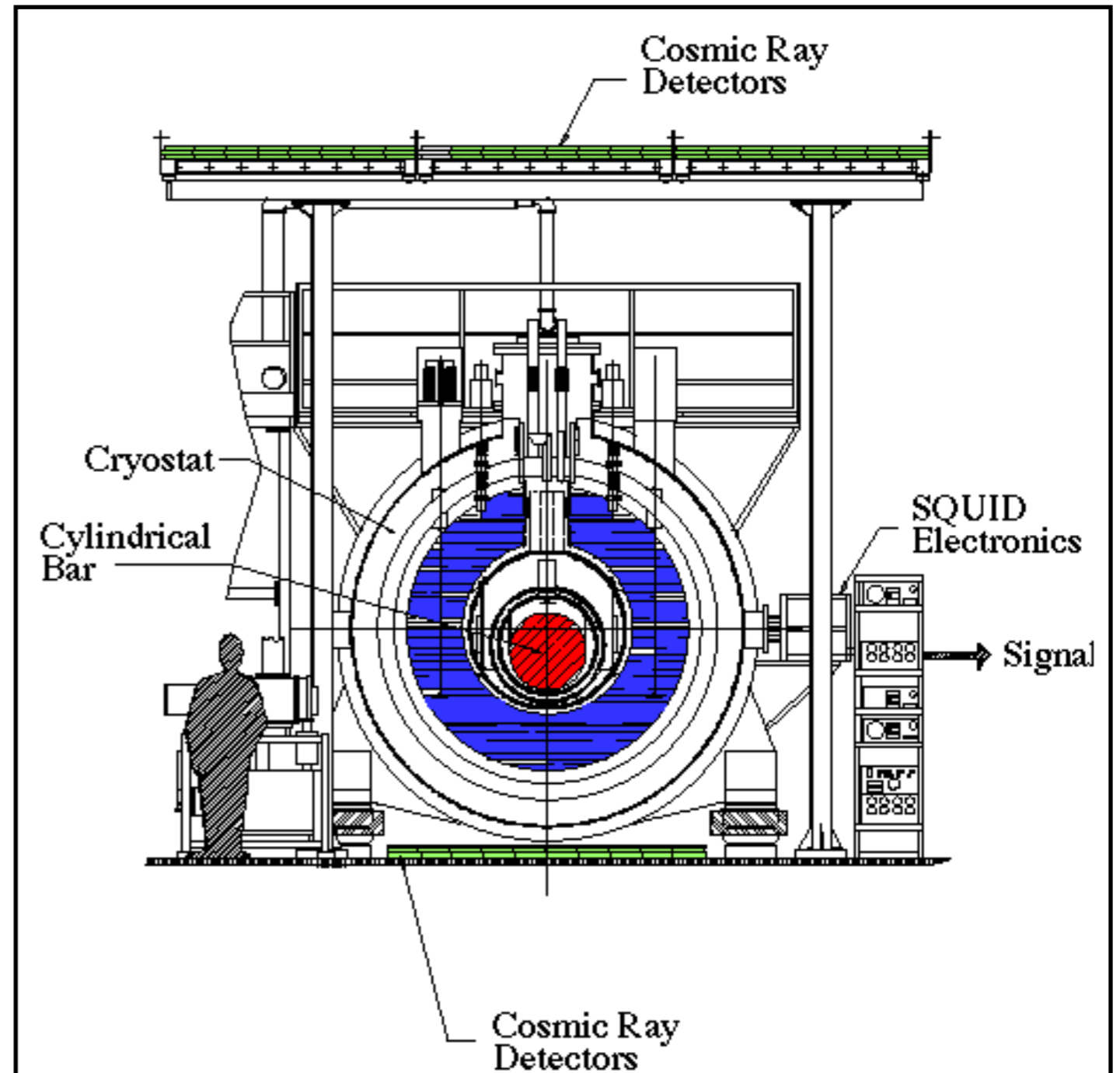
M. Bassan^{b,c}, E. Coccia^{b,c}, S. D'Antonio^b, V. Fafone^b, G. Giordano^a,
A. Marini^a, Y. Minenkov^b, I. Modena^c, G.V. Pallottino^d, G. Pizzella^a,
A. Rocchi^b, F. Ronga^{a,*}, M. Visco^e

Two similar detectors,
 $T \sim 2-3$ K:

EXPLORER (CERN)
run ended in JUNE 2010

NAUTILUS (Frascati)
running until end of 2015

Al 2036 bar **2300 Kg**
 $L=3$ m $r=0.3$ m



Events $E > 0.1$: selection

- In this analysis we restrict the search to events with excitation times of the order of 1 msec or less, and we use the standard searches of gw burst events with an optimal filter **optimized to short signals**
- standard cuts: calibration, seismic noise, cosmic rays, T_{eff} (NAUTILUS < 2.5 mK, EXPLORER 5mK)
- the usual gw searches are done using coincidences between antennas. **additional selection criteria are necessary in this analysis to remove spurious events**

1) remove periods with high rate of events $E > 0.1K$

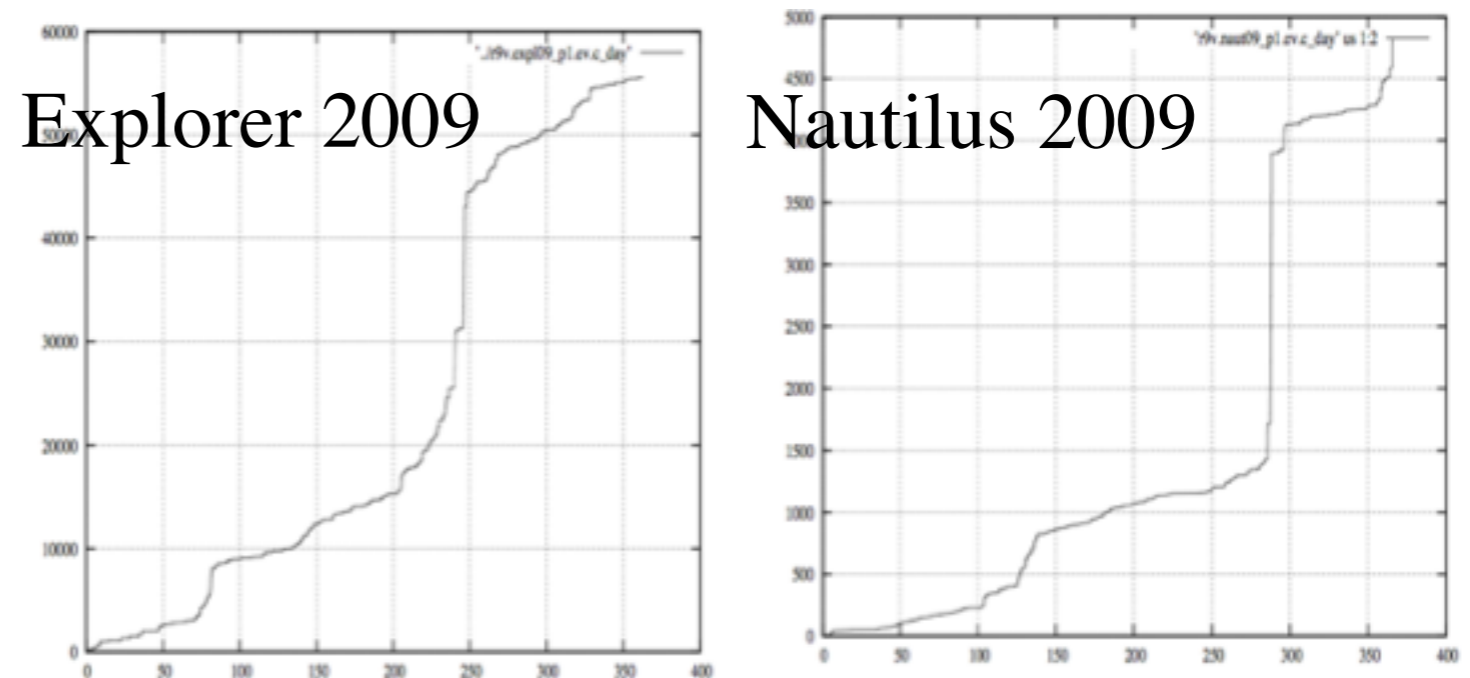
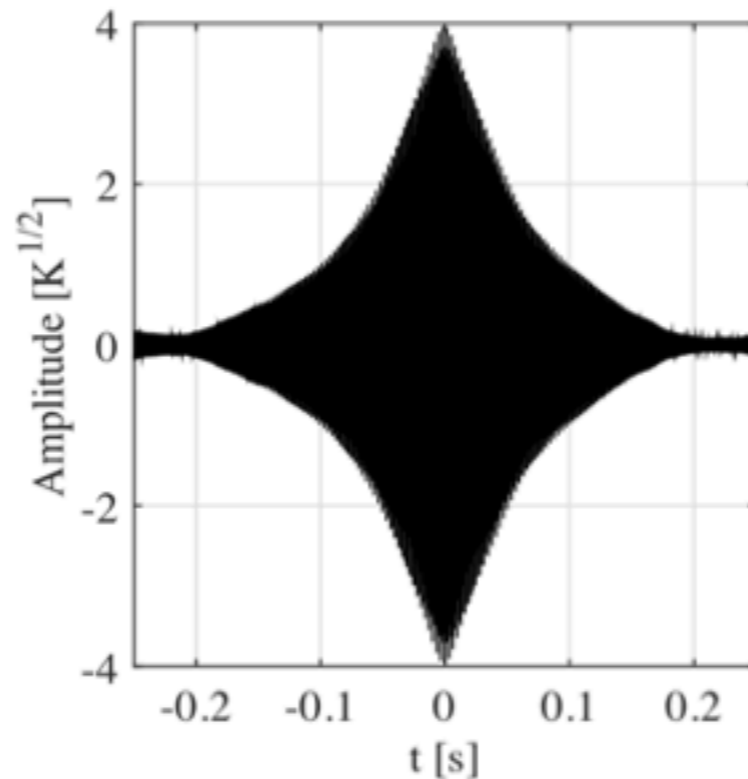


Figure 4: Cumulative distribution of the EX o NA events during year 2009. One can clearly see that there are periods with a rate of events orders of magnitude larger than in the quiet periods of operation.

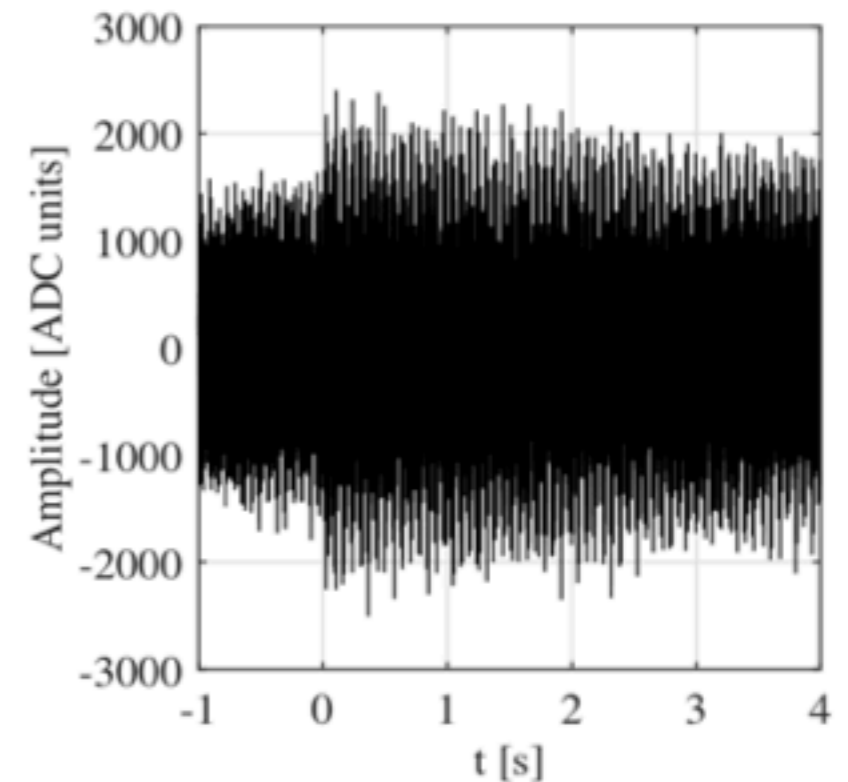
Events $E > 0.1$ K : selection

2) analyze each single event and select those compatible with the assumption to be produced by a very short excitation, such as we expect to be caused by a newtorite. **Cosmic rays are used as reference**

very short excitation
in NAUTILUS
produced by a cosmic
rays extensive air
shower



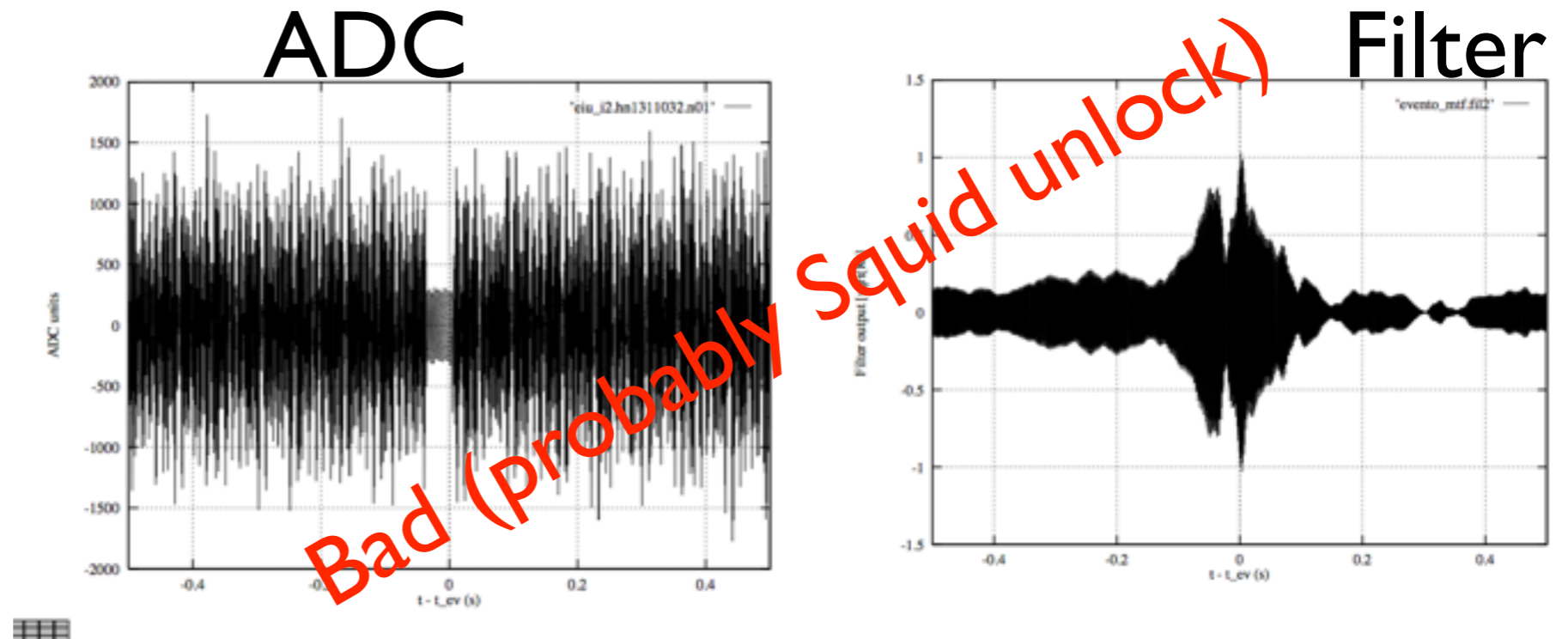
Output of the optimal filter



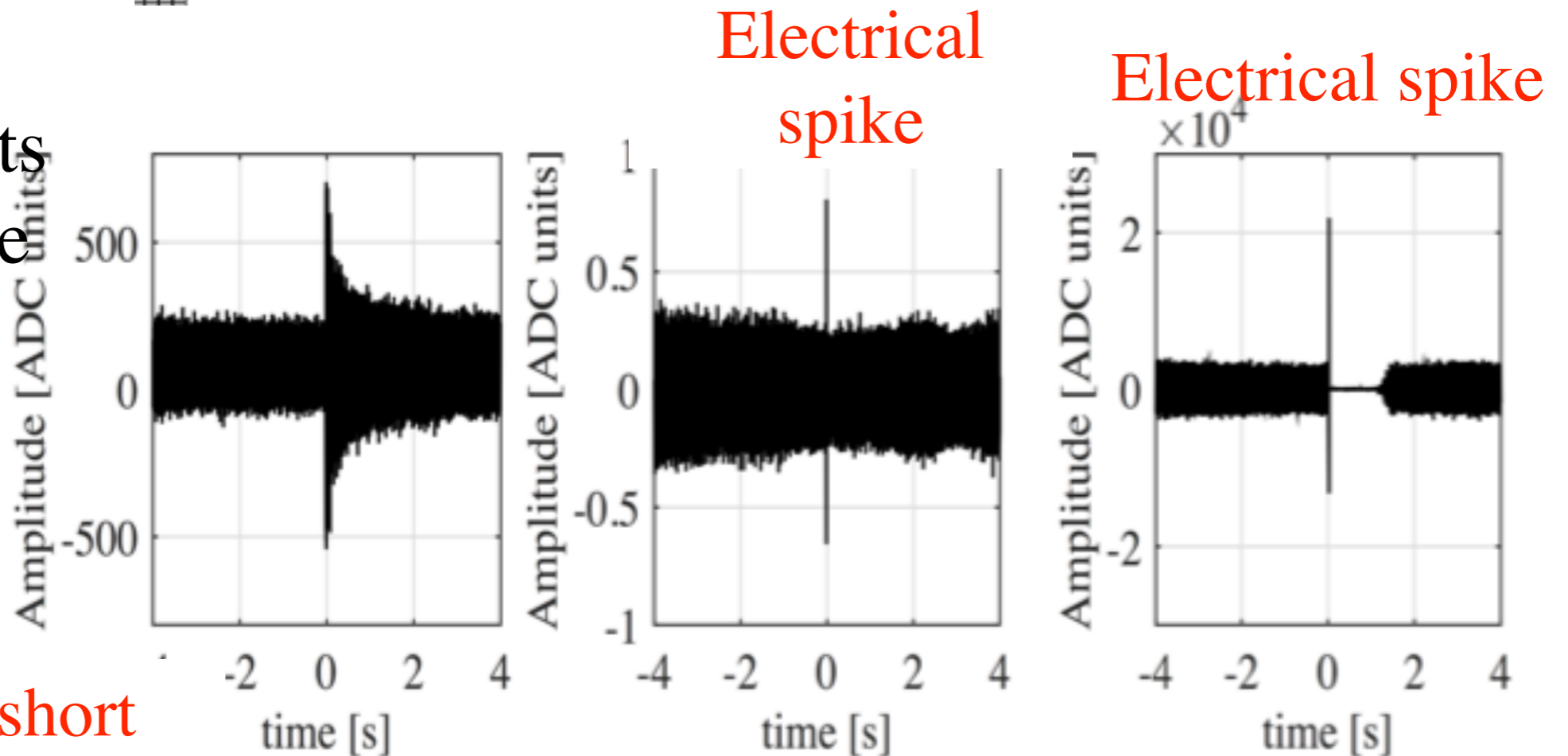
ADC raw data

Events $E > 0.1$ K : bad events examples

Examples of events rejected due ADC and shape of the optimal filter



Examples of events rejected due to the raw ADC signal



Too short decay time

Events $E > 0.1$ K : results

Nautilus rates 2005

2014 : note that rates changes with year

Ev/year

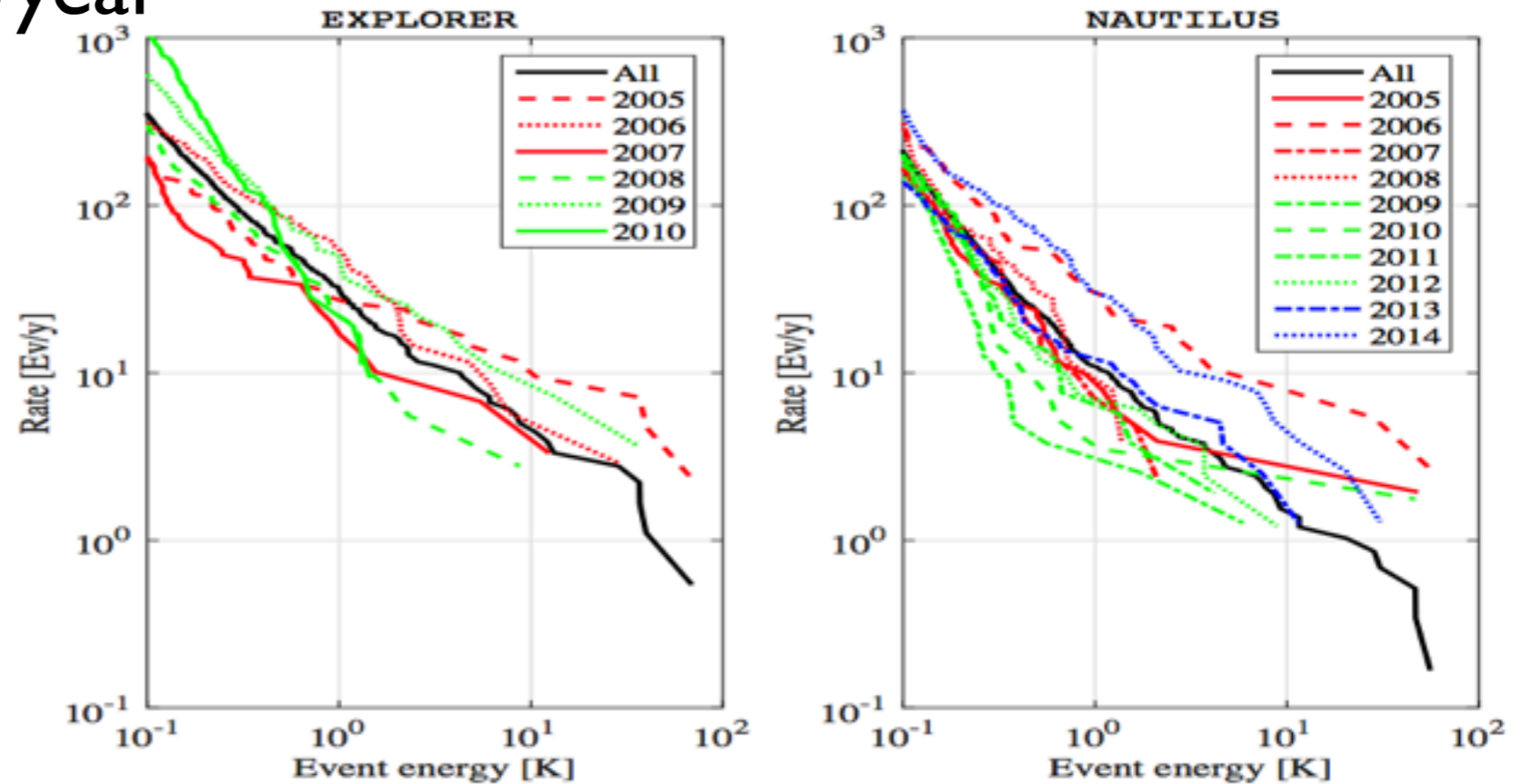
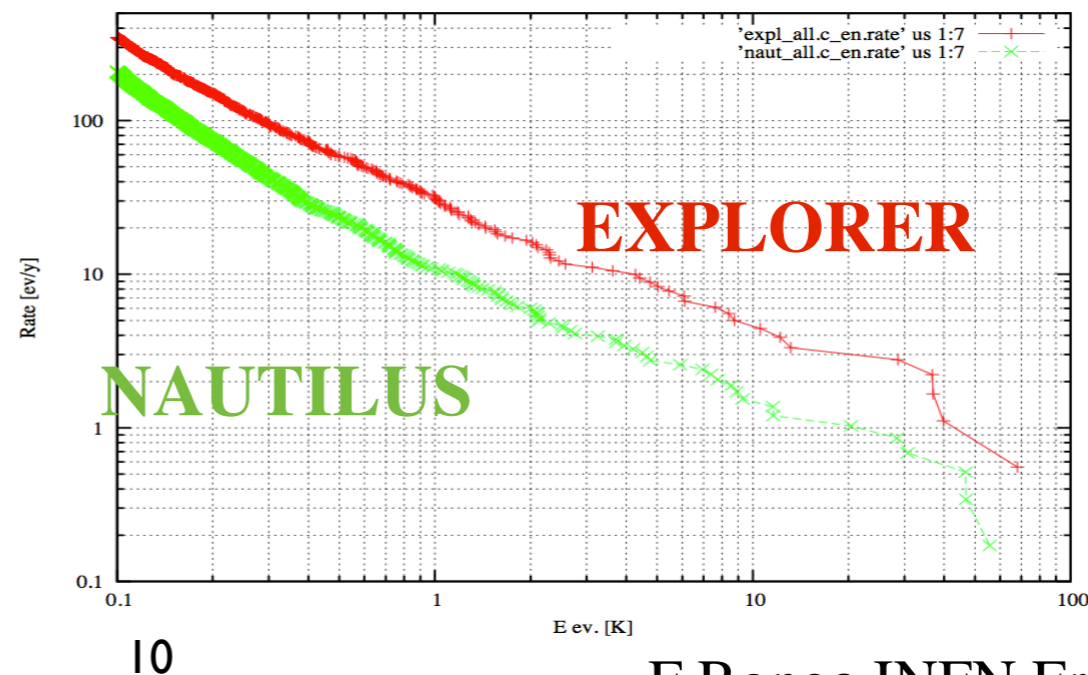


Figure 9: Rate of events of energy larger than the abscissa value for Explorer (left) and Nautilus (right). The different lines are the results at each different year, the overall result is shown with linespoints.

Nautilus rates 2005
2014 compared with
the Explorer rate 2005

2010 : **NAUTILUS**
much better



Events $E > 0.1$ K : detection efficiencies

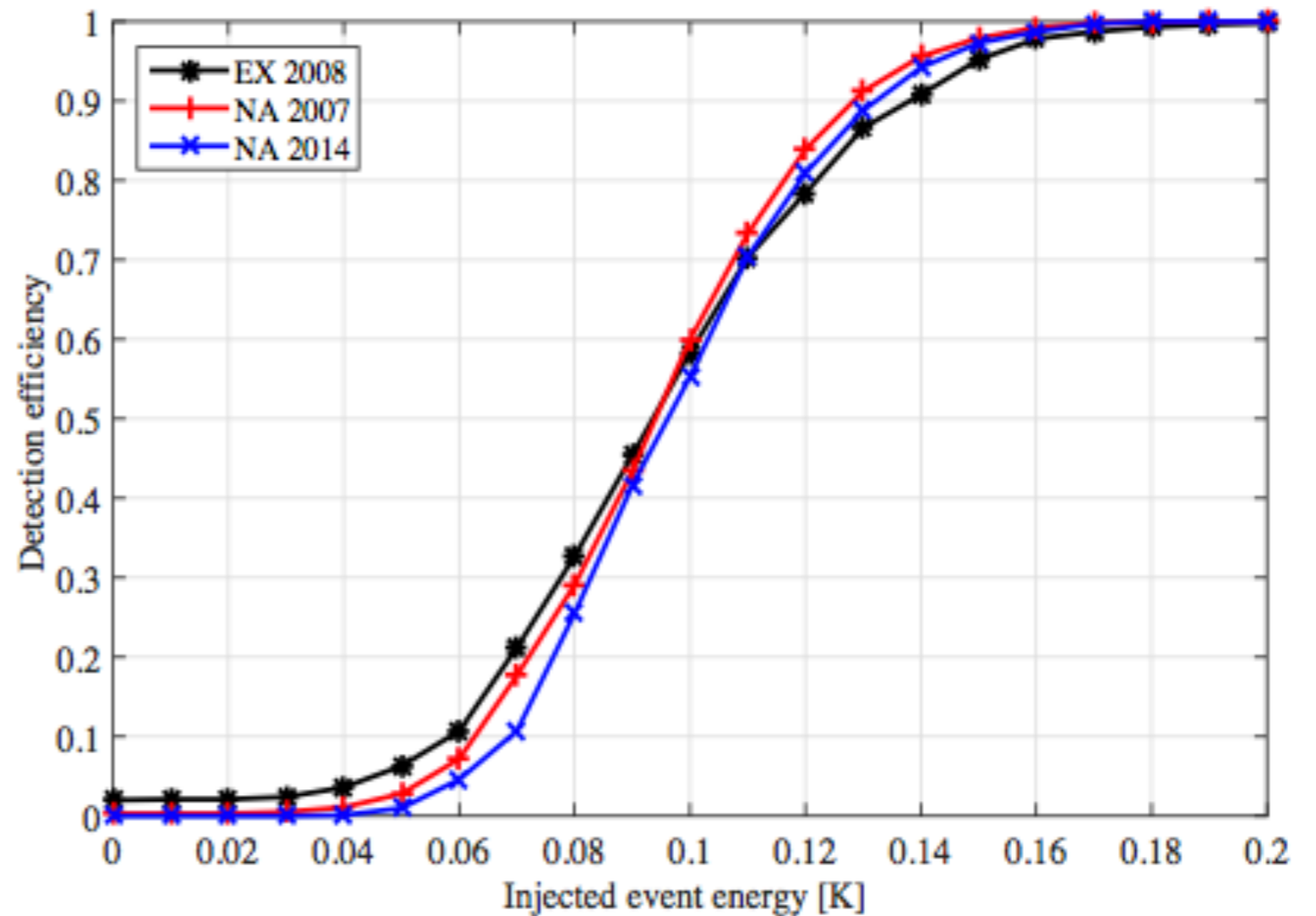


Figure 10: Detection efficiency in three data sets

E[K]

Events $E > 0.1$ K : detection efficiencies

- randomly choose N time stamps inside the good operation periods and extract from the filtered data a time segment around those times

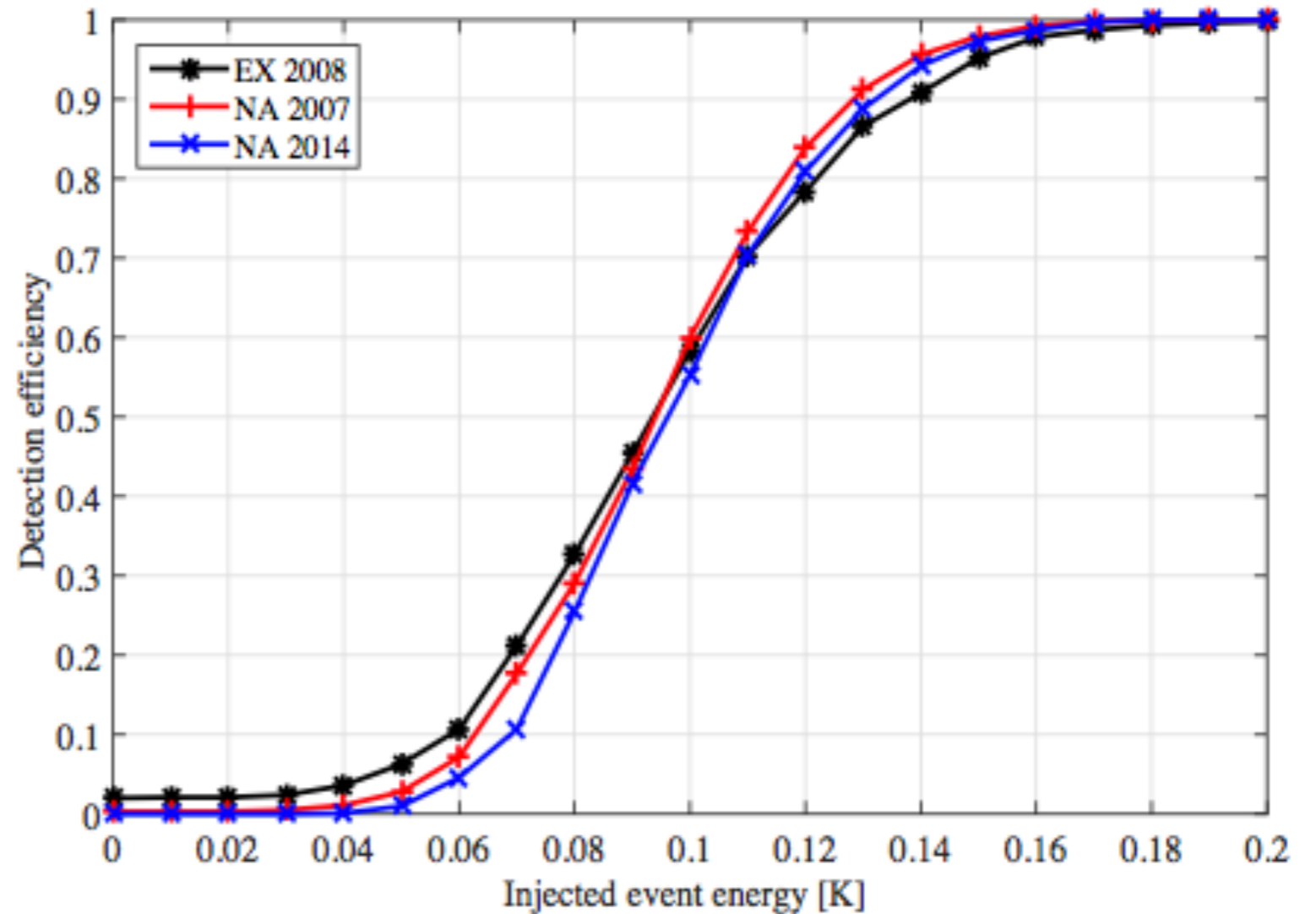


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Events $E > 0.1$ K : detection efficiencies

- randomly choose N time stamps inside the good operation periods and extract from the filtered data a time segment around those times
- inject in those time segments a copy of a real good event (cosmic ray shower) scaled to a given value of energy, repeating this for different energies

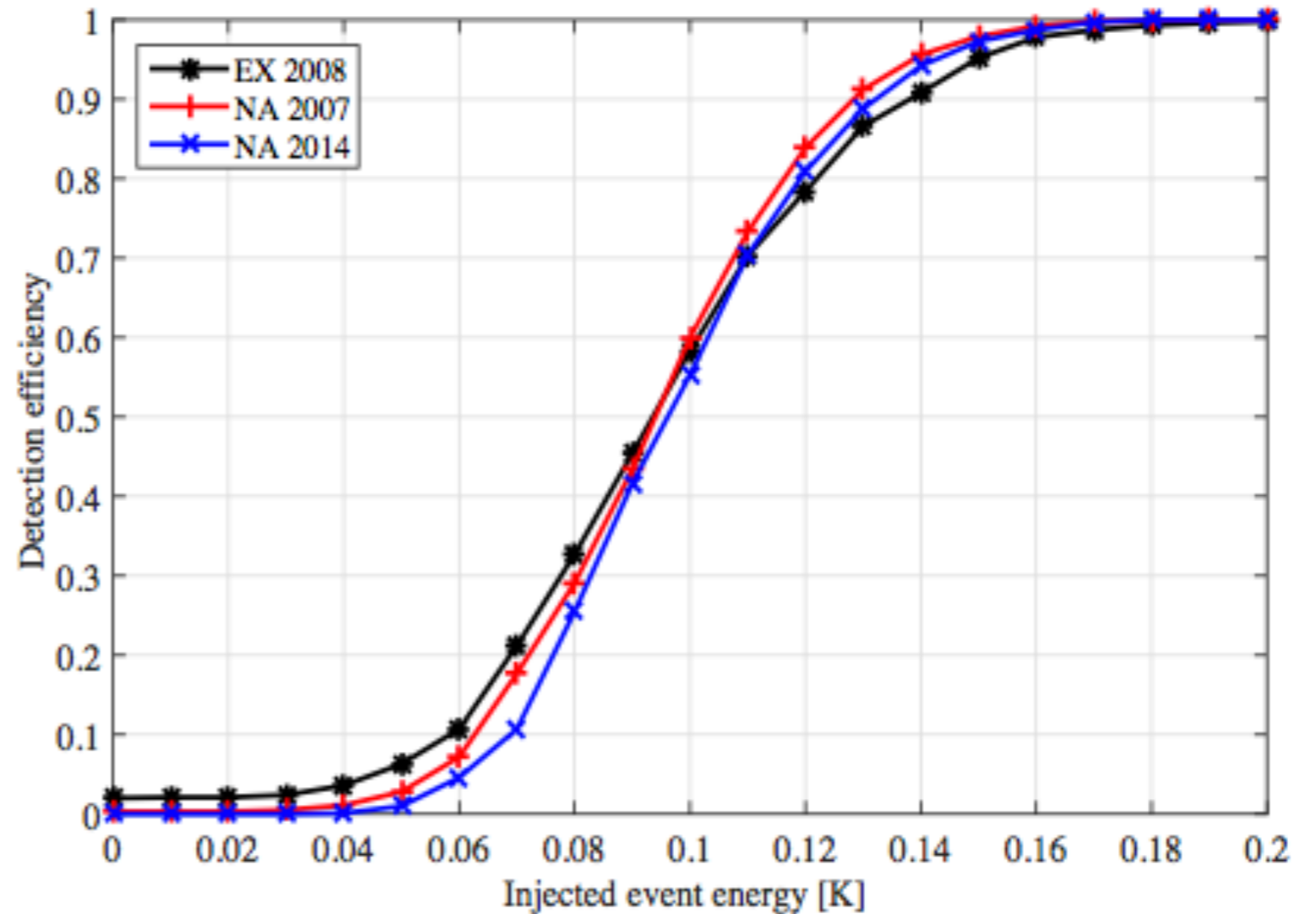


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- inject in those time segments a copy of a real good event (cosmic ray shower) scaled to a given value of energy, repeating this for different energies
- apply to this new time segments (data+event) the selection procedures used to identify bad events and count how many we would reject, at any value of injected energy

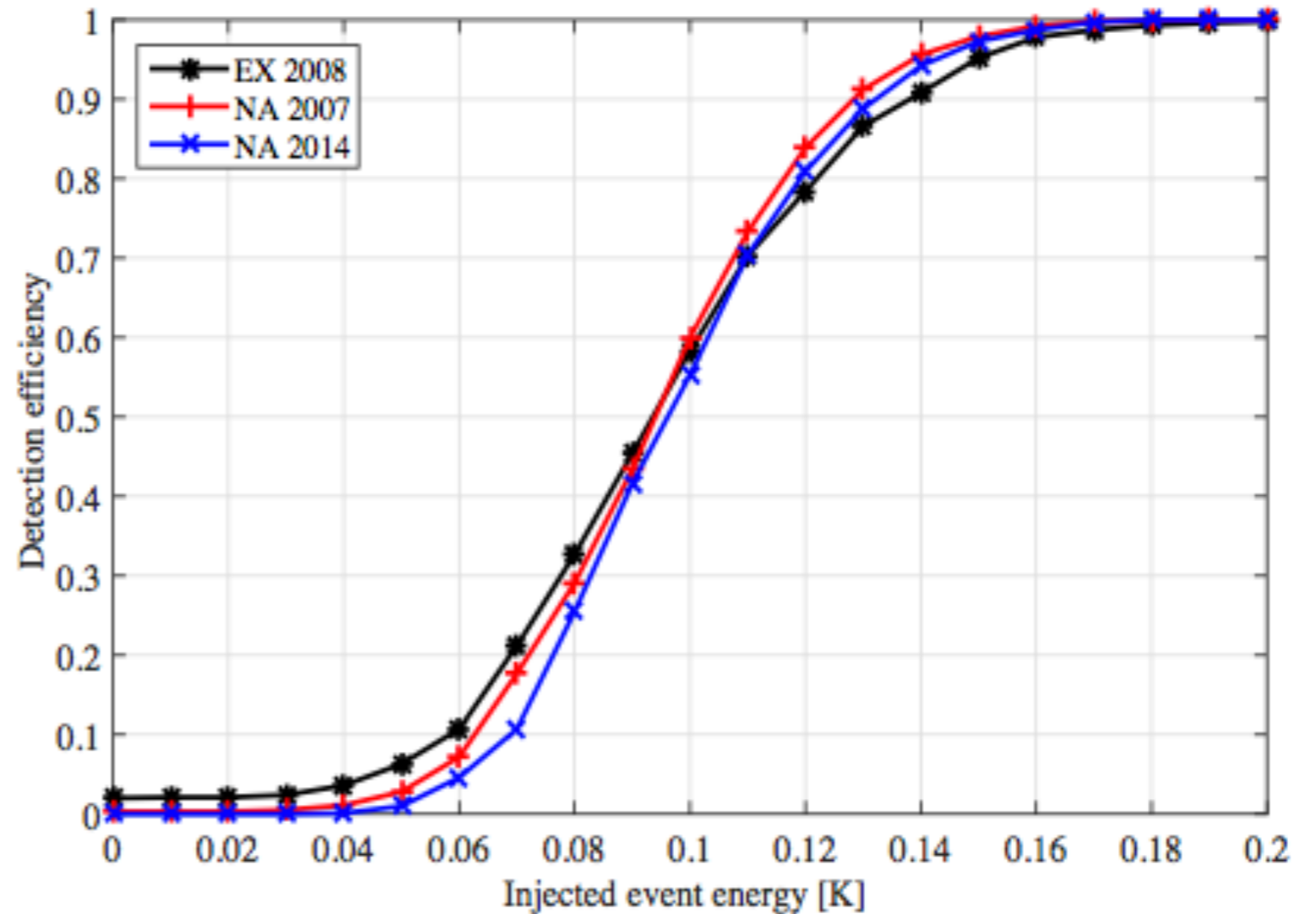


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$E[K]$

Events $E > 0.1\text{K}$: data Montecarlo comparison

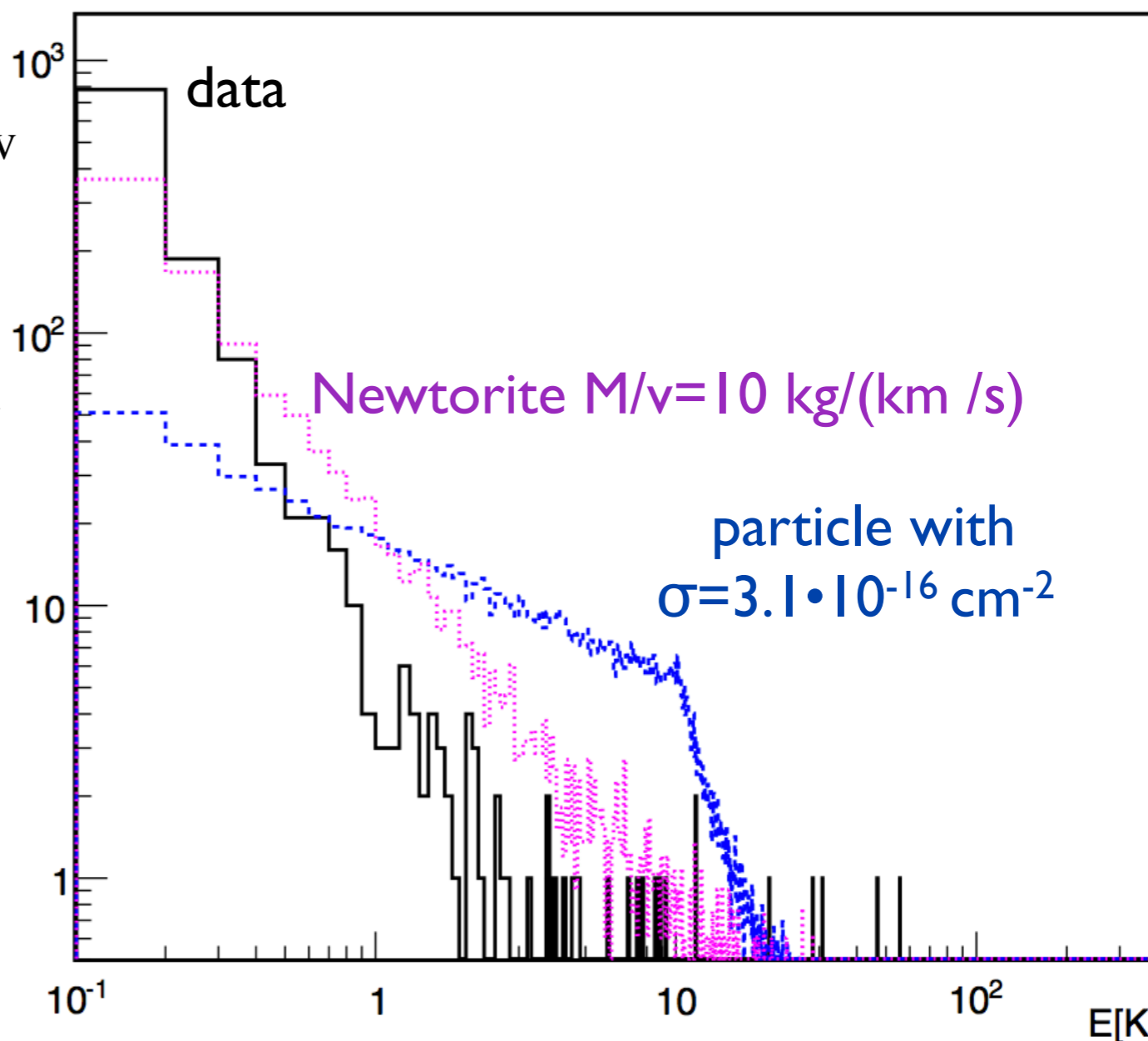
- For the Newtorite upper limit we use only the NAUTILUS data (more noise in Explorer)

- The shape of the newtorite energy distribution is not very different from the one of the noise. This does not depend very much from the particular M/v . As M/v increases increases the sensitive volume and therefore the fraction of small events.

- This mean that is important for newtorite to select data with low noise, **best data NAUTILUS 2011 (largest event 5.9 K)**

- For a particle with hadronic interaction (MACRO, nuclearite ecc.) the shape in general is different and increasing σ increases the differences with the data distribution. **For this kind of particle we can use the full EXPLORER NAUTILUS data set**

Energy(K) Nautilus 2005-2014



Newtorite 90% C.L. Upper limits

Nautilus 2005-2014 Livetime=2129.1 days
Note the linear increase of acceptance with M/v

- Upper limits computed with **the optimum interval method.**
 Used in the Dark matter community
 =Maximum number of events with the expected Montecarlo energy distribution compatible with data

M/v ($kg\ km^{-1}\ s$)	acceptance ($m^2\ sr$)	events upper limit	flux upper limit ($cm^{-2}\ s^{-1}\ sr^{-1}$)
1	33.4	545	$8.9 \cdot 10^{-12}$
2	85.3	365	$2.4 \cdot 10^{-12}$
5	209	253	$6.5 \cdot 10^{-13}$
10	426	257	$3.5 \cdot 10^{-13}$
20	888	244	$1.5 \cdot 10^{-13}$
40	1652	248	$8.3 \cdot 10^{-14}$
60	2398	242	$5.5 \cdot 10^{-14}$

Table 5: Nautilus 2005-2014, newtorites upper limits. Note the linear increase of the acceptance with M/v .

- Since the energy distribution doesn't change very much with M/v the events upper limit is ~ constant with M/v

Nautilus 2011 Livetime=278.8 days

Used software in the web site:

<http://titus.stanford.edu/Upperlimit/>.

M/v $kg\ km^{-1}\ s$	events upper limit	flux upper limit ($cm^{-2}\ s^{-1}\ sr^{-1}$)
1	37	$4.5 \cdot 10^{-12}$
2	23	$1.1 \cdot 10^{-12}$
5	22	$4.3 \cdot 10^{-13}$
10	22	$2.1 \cdot 10^{-13}$
20	22	$1.0 \cdot 10^{-13}$
40	22	$5.5 \cdot 10^{-14}$
60	21	$3.5 \cdot 10^{-14}$

Table 6: Nautilus 2011. newtorites upper limits. Livetime=278.8 days, 160 events $\geq 0.1K$

Newtorite Upper limits vs mass

- Adler obtained a direct upper limit 0.13 kg/km^3 **of the mass of Earth-bound dark matter** lying between the radius of the moon orbit and the geodetic satellite orbits s. L.

Adler, J. Phys. A 41, 412002 (2008) [arXiv:0808.0899 [astro-ph]]

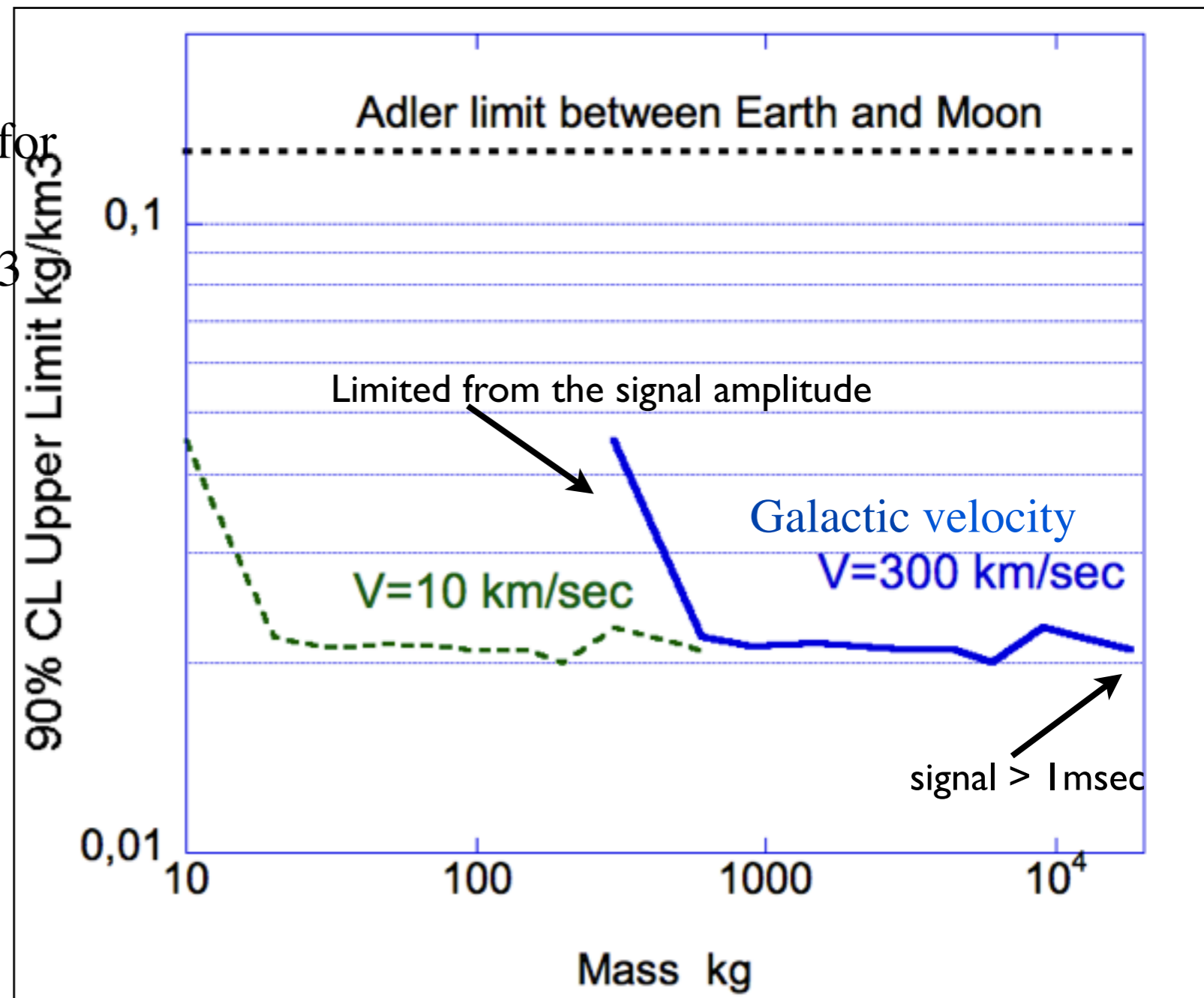
- Our limit is better than the Adler limit in the range of sensitivity of the bar

- Pitjev has found a much better limit for possible DM inside the Earth-Sun orbits of the order of $1.4 \cdot 10^{-7} \text{ kg/km}^3$

N. P. Pitjev and E. V. Pitjeva, Astron. Lett. 39, 141 (2013) [Astron. Zh. 39, 163 (2013)] [arXiv:1306.5534 [astro-ph.EP]].

- **very far from DM = $5 \cdot 10^{-13} \text{ kg/km}^3$**

- this upper limit is on the Earth : trapping effect from the earth gravitational field could increase the DM density



Newtorite limits improvements

-

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- **Coincidence** ~ a factor 300 with two bar with the performance of NAUTILUS at distance 1.5 meter (Montecarlo simulation using the real data) in 10 years of data taking

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- **Large array** : to increase the geometrical acceptance, to reach the Earth-Sun bound 1000 or more bar are needed. Too much!
- **Better sensitivity**: the intrinsic quantum limit of a bar noise is $\Delta E = \hbar\omega_0 = 6 \cdot 10^{-31}$ Joules **very far** from the current noise $\sim 1\text{mK} \sim 1.38 \cdot 10^{-26}$ Joules. But a very large R/D is needed. With a much lower noise could be possible to increase the distance between bars and therefore increase the geometrical acceptance



Signal in aLIGO

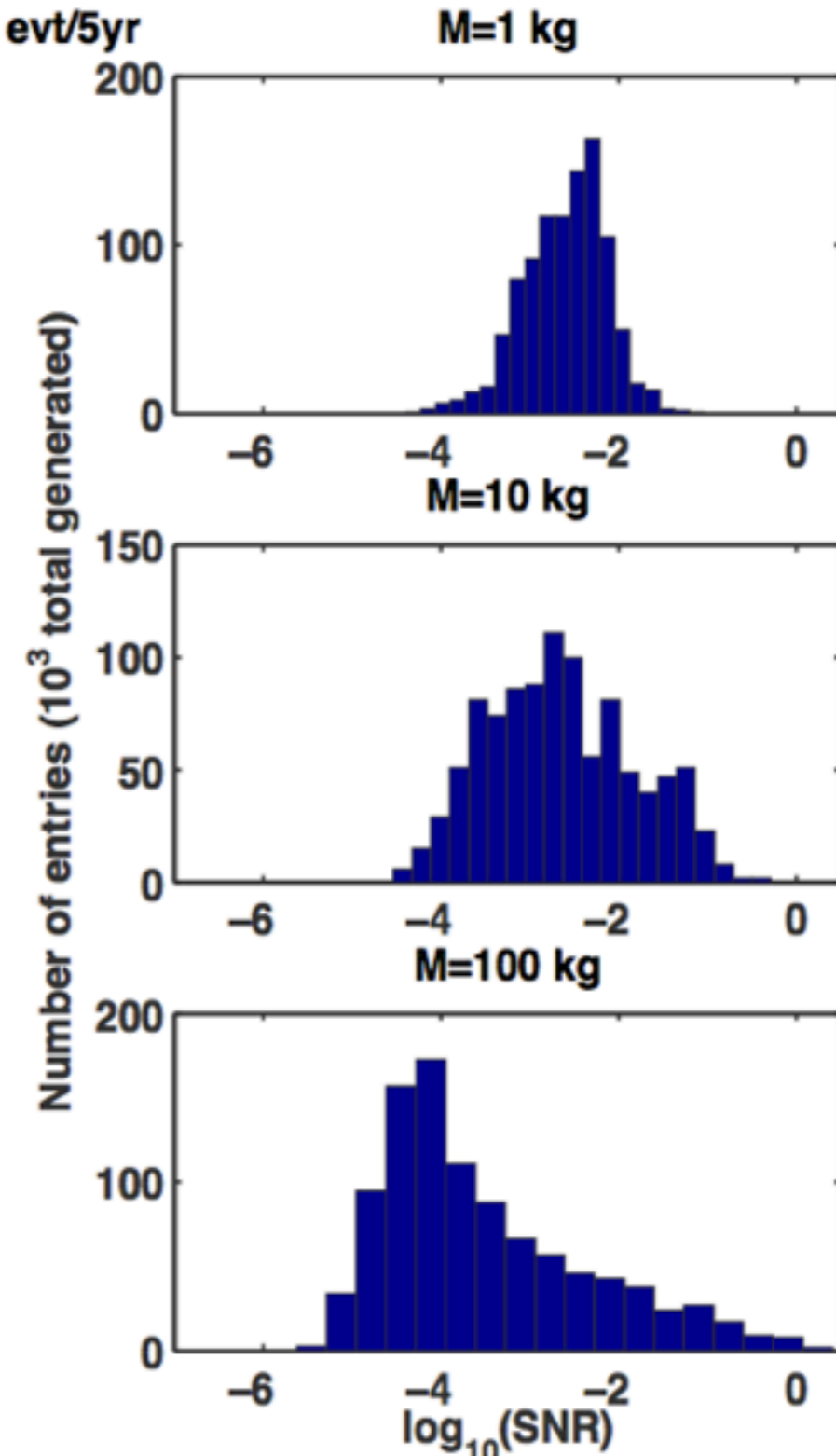
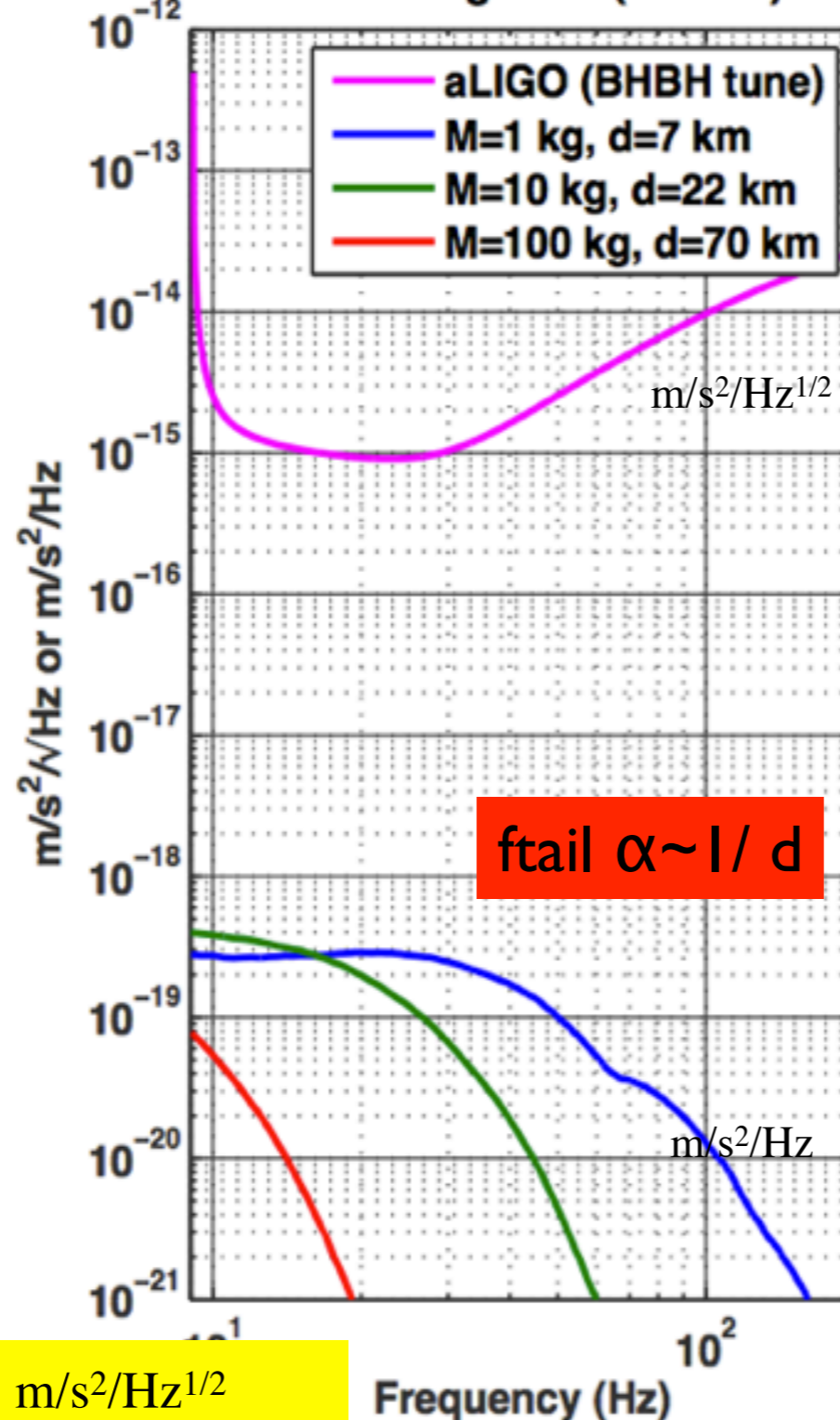
Random angle and position

Signal from gravitational interaction is small but DM may have long range non-minimal interaction - lot of room for model building and testing

SNR was obtained by integration. Searches should be done using templates.

No coincidences between non-located interferometers due to solid angle

aLIGO noise and DM signal FT(median) for 1 evt/5yr



Bar sensitivity 1 kHz $\sim 4E^{-14}$ m/s²/Hz^{1/2}

$\sim 2E^{-16}$ at the quantum limit

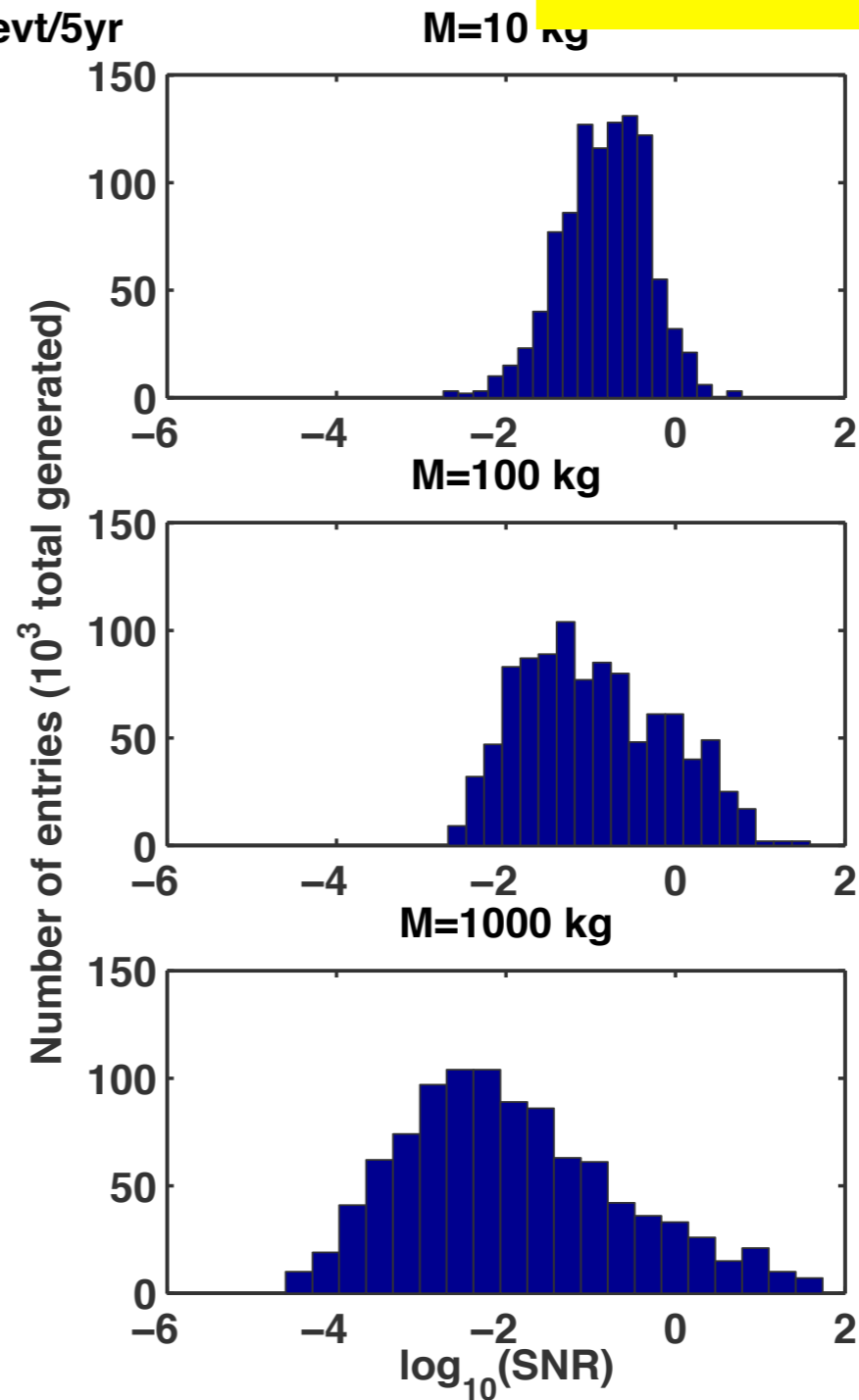
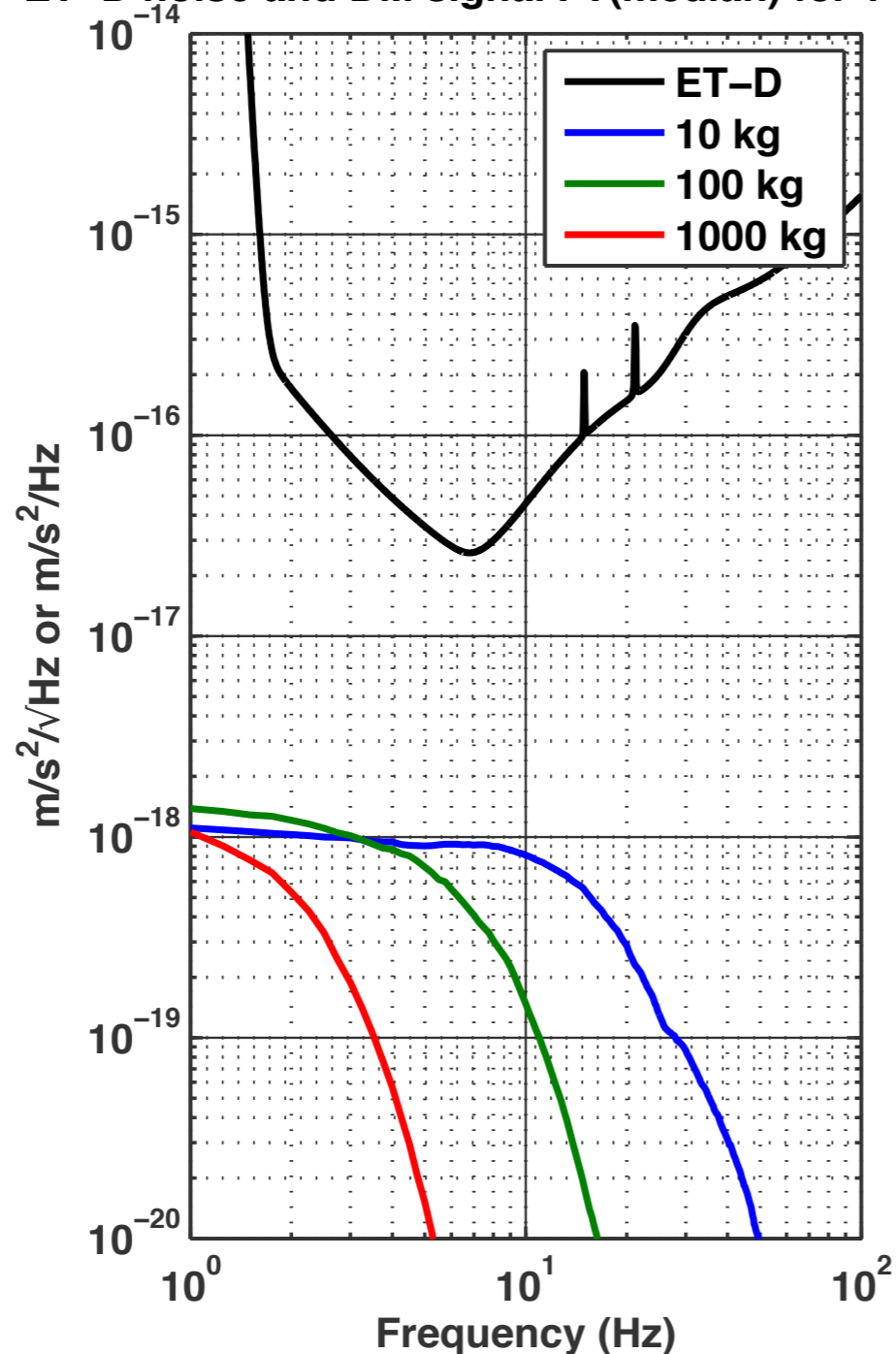


Signal in ET

Three interferometers are co-located. Coincidence and wave form analysis allows background rejection.

Correlated noise?

ET-D noise and DM signal FT(median) for 1 evt/5yr



GW bars are interesting
for other DM
candidates and limits
constraints models ..

Interest of bar results for other kind of DM candidate #1 MACRO

[Resonant bar detector constraints on macro dark matter](#)

David M. Jacobs ([Cape Town U., ACGC](#) & [Cape Town U., Dept. Math.](#)), Glenn D. Starkman ([Case Western Reserve U.](#) & [Case Western Reserve U., CERCA](#)), Amanda Weltman ([Cape Town U., ACGC](#) & [Cape Town U., Dept. Math.](#)). Apr 10, 2015. 5 pp.

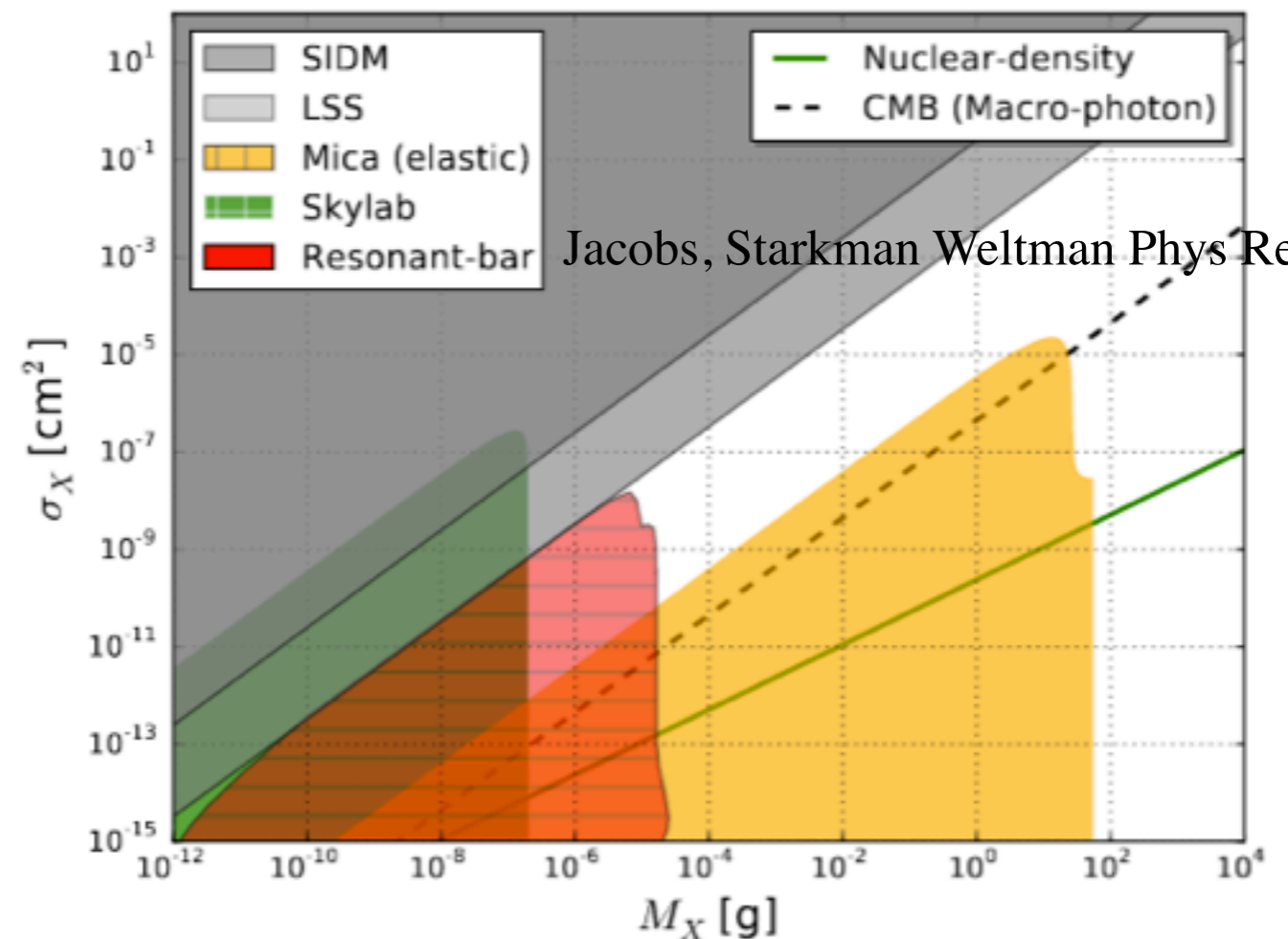
Published in *Phys.Rev. D*91 (2015) 11, 115023 e-Print: [arXiv:1504.02779](#) [astro-ph.CO] | [PDF](#)

The authors of this paper uses our limits for “nuclearites” presented at ICRC 2013 ([arXiv:1306.516](#), *Quark nuggets search using 2350 Kg gravitational waves aluminum bar detectors*)

to evaluate limits on their “MACRO” model (MACRO a generic massive particle with cross section of the order of the particle area, no ionization). In the bar the signal is due to the heating.

astrophysical considerations

$$\sigma/M \sim 0.1-10 \text{ cm}^2 \text{ g}^{-1}$$

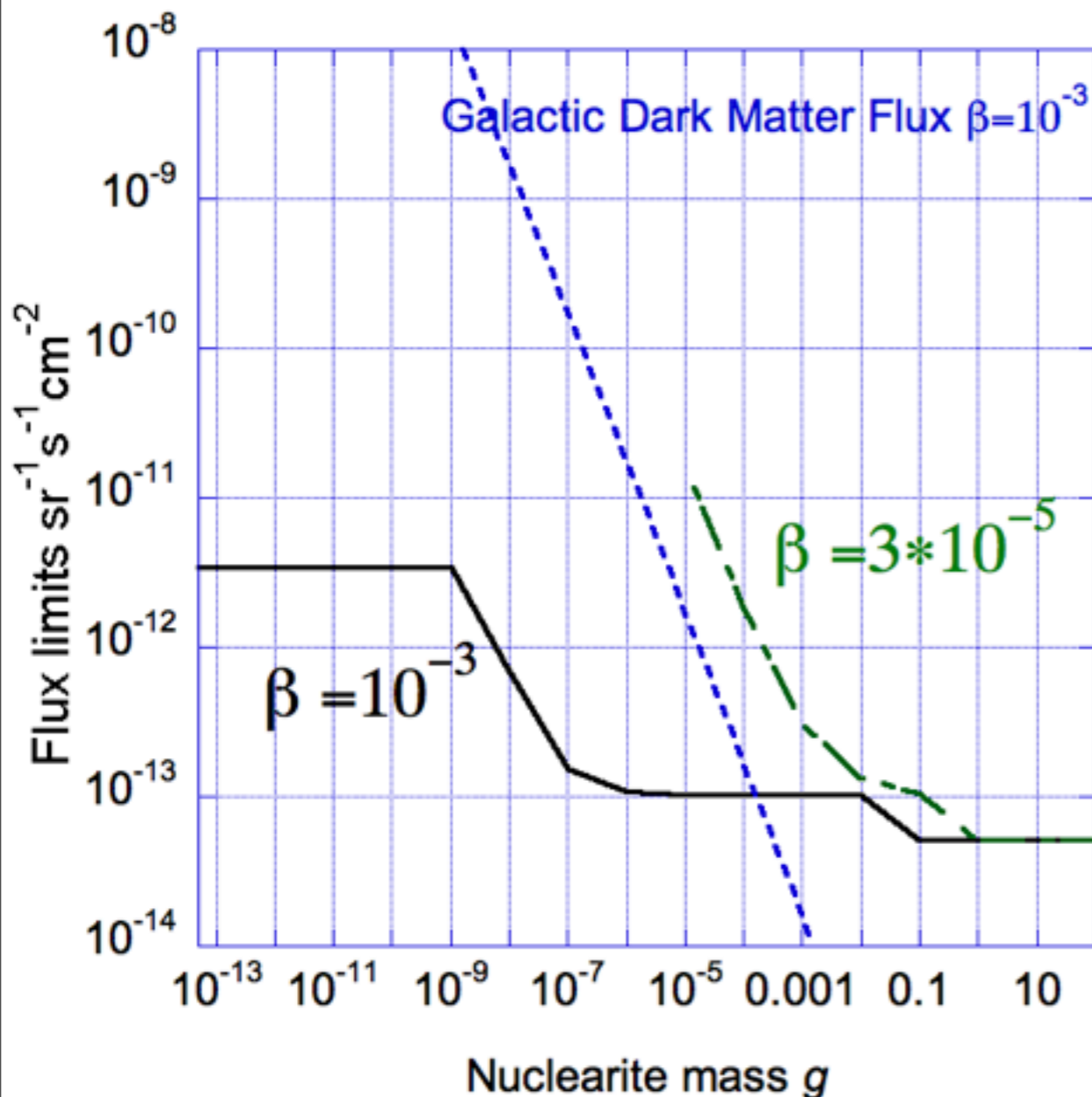


Interest of bar results for other kind of DM candidate #2 Nuclearite

particular case of MACRO with strange quarks. Mass cross section link

$$\sigma = A = \begin{cases} \pi \cdot 10^{-16} \text{ cm}^2 & \text{for } M < 1.5 \text{ ng} \\ \pi \left(\frac{3M}{4\pi\rho_N} \right)^{2/3} & \text{for } M > 1.5 \text{ ng} \end{cases}$$

where $\rho_N = 3.6 \cdot 10^{14} \text{ g/cm}^3$ is the nuclearite density and M its mass



- Other experiments above sea level using track etch detectors have obtained lower limits.
- The interest in this search is the “more robust” calorimetric technique (calibrated on a beam)

Interest of bar results for other kind of DM candidate #3

[The Sound of Dark Matter: Searching for Light Scalars with Resonant-Mass Detectors](#)

[Asimina Arvanitaki](#), [Savas Dimopoulos](#), [Ken Van Tilburg](#). Aug 7, 2015. 5 pp.

e-Print: [arXiv:1508.01798](#) [hep-ph] | [PDF](#)

Abstract (arXiv)

The fine structure constant and the electron mass in string theory are determined by the values of scalar fields called moduli. If the dark matter takes on the form of such a light modulus, it oscillates with a frequency equal to its mass and an amplitude determined by the local dark matter density. This translates into an oscillation of the size of a solid that can be observed by resonant-mass antennae. Existing and proposed resonant-mass detectors can probe dark matter moduli with frequencies between 1 kHz and 1 GHz, with much better sensitivity than force measurements

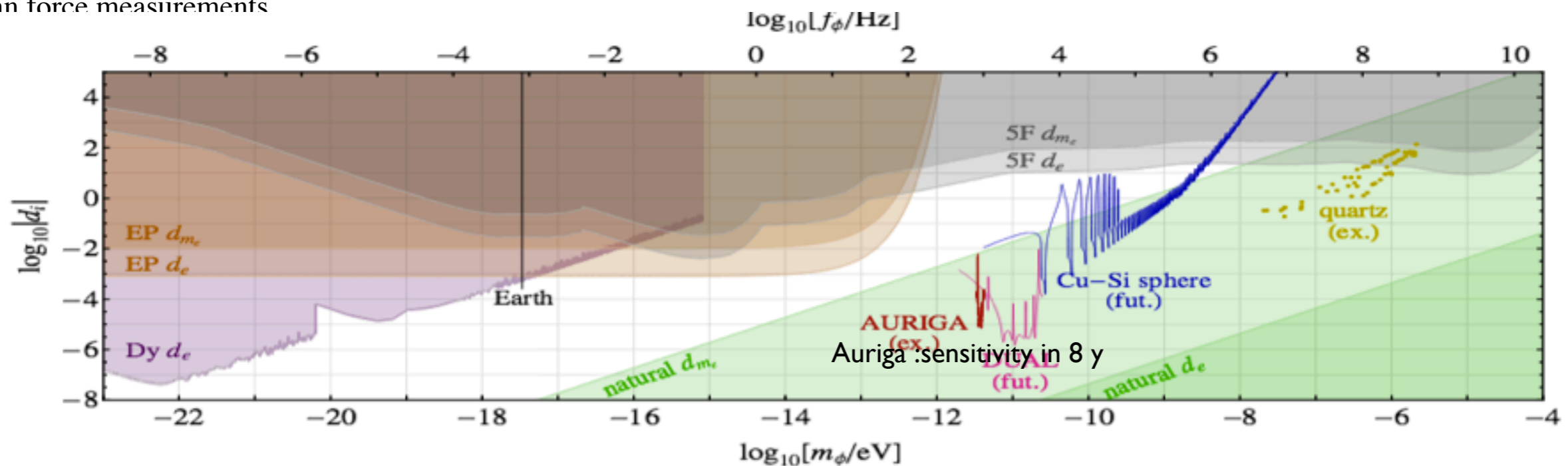


FIG. 1. Scalar field parameter space, with mass m_ϕ and corresponding DM oscillation frequency $f_\phi = m_\phi/2\pi$ on the bottom and top horizontal axes, and couplings of both an electron mass modulus ($d_i = d_{m_e}$) and electromagnetic gauge modulus ($d_i = d_e$) on the vertical axis. Natural parameter space for a 10 TeV cutoff is depicted by the green regions, while the other regions represent 95% CL limits from fifth-force tests (“5F”, gray), equivalence-principle tests (“EP”, orange), atomic spectroscopy in dysprosium (“Dy”, purple), and low-frequency terrestrial seismology (“Earth”, black). The blue curve shows the projected SNR = 1 reach of a proposed resonant-mass detector—a copper-silicon (Cu-Si) sphere 30 cm in radius—after 1.6 y of integration time, while the red curve shows the reach for the current AURIGA detector with 8 y of recasted data. Rough estimates of the 1-year reach of a proposed DUAL detector (pink) and several harmonics of two piezoelectric quartz resonators (gold points) are also shown.

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- bar improvements: coincidences between many near antennas, better sensitivity: it will possible to reach the experimental bounds on Earth-Sun, but not the expected DM signal.
- interferometer: Advanced Ligo, Virgo , ET, Lisa (space) **Much better prospects particularly for the Einstein Telescope (ET), up to a three-fold coincidence!**

Backup

De Rujula idea (1984); but for newtorites is not necessary to have a compact detector..

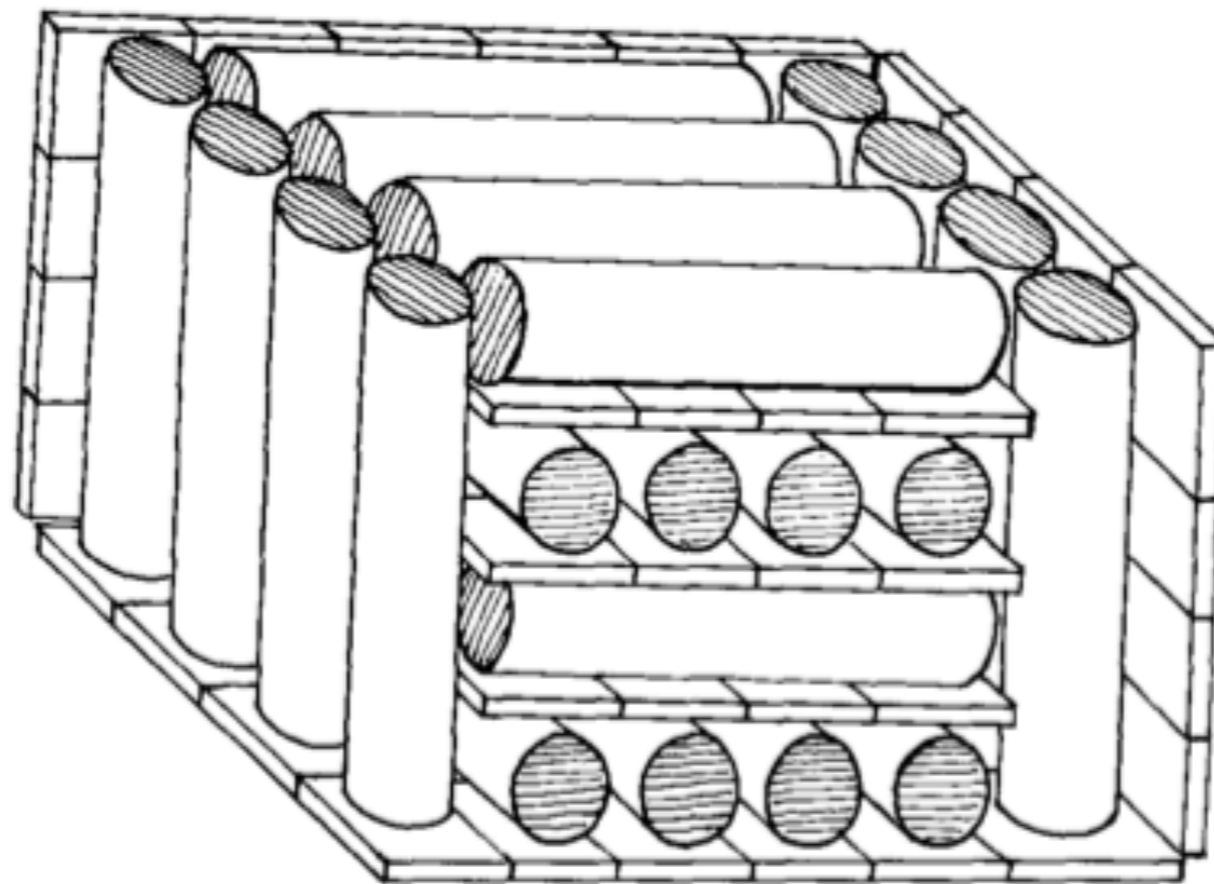


Fig. 12. A possible geometry for a direction-sensitive gravitational wave antenna with the capability of detecting monopoles and newtorites.

GW Bar as particle detector, cosmic ray showers

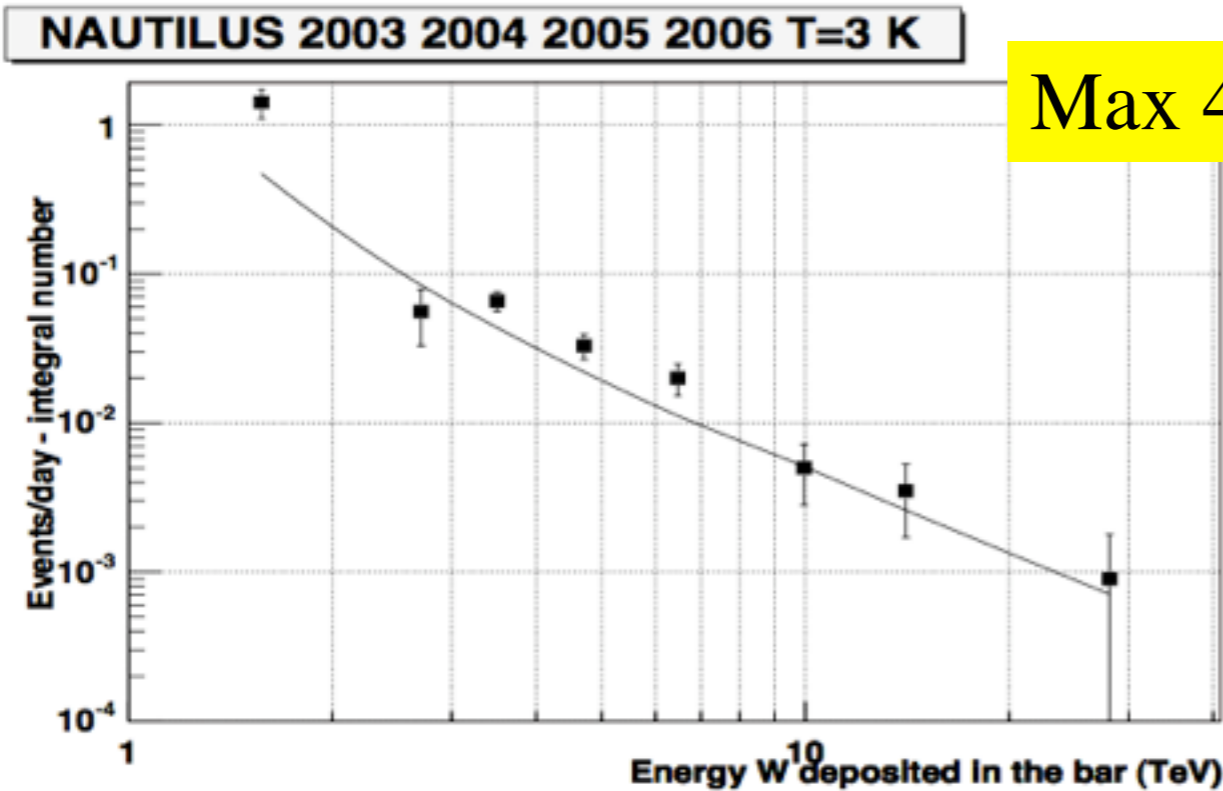


Fig. 8. NAUTILUS 2003-2006 : The integral distribution of the event rate after the background unfolding, as in fig.5, for the four years 2003-2006, compared with the expected distribution (continuous line). The prediction is computed using the data of Table 1.

In Explorer biggest event
 ~ 360 TeV in 1022 days - expected ~ 0.1
 from the extrapolation at lower energies

Max 670 K ~ 360 TeV in the bar

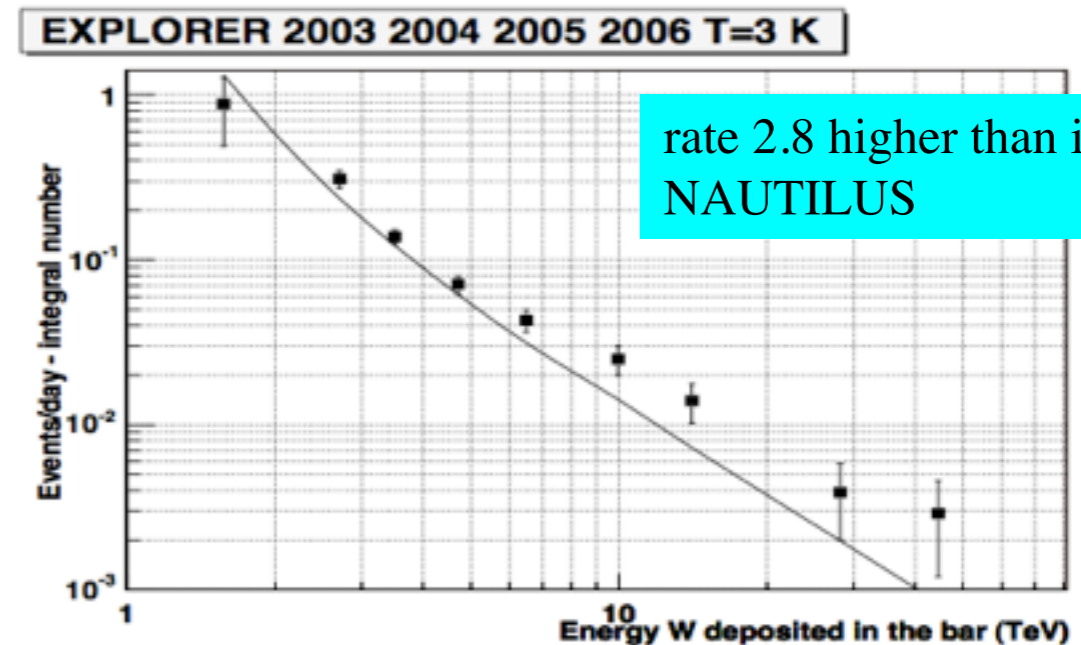


Fig. 11. EXPLORER 2003-2006 : The integral distribution of the event rate after the background unfolding, as in Fig.5, compared with the expected distribution (continuous line). The prediction is computed using Table 1 multiplied by a factor 2.8 (see text).

Introduction: Energy loss of a point-like particle only gravitationally interacting (Newtorite)

- at a microscopic level the process is similar to the energy loss of a *charged particle* like a muon in a media (the main difference is that the main contribution for a charged particle is due to the electrons)

$$\frac{dE}{dx} = 4\pi N G^2 \frac{(M)^2 m}{V^2} \log\left(\frac{b_{\max}}{b_{\min}}\right)$$

N = Avogadro number, M, V particle mass and velocity, m nucleus mass, $b_{\max} \sim 1$ meter, $b_{\min} \sim 10^{-15}$ m

very small numbers! for aluminum and $V=1$ km/sec $M=1$ kg
 $dE/dx \sim 5 \cdot 10^{-21}$ Joules/m directly in mechanical oscillation.

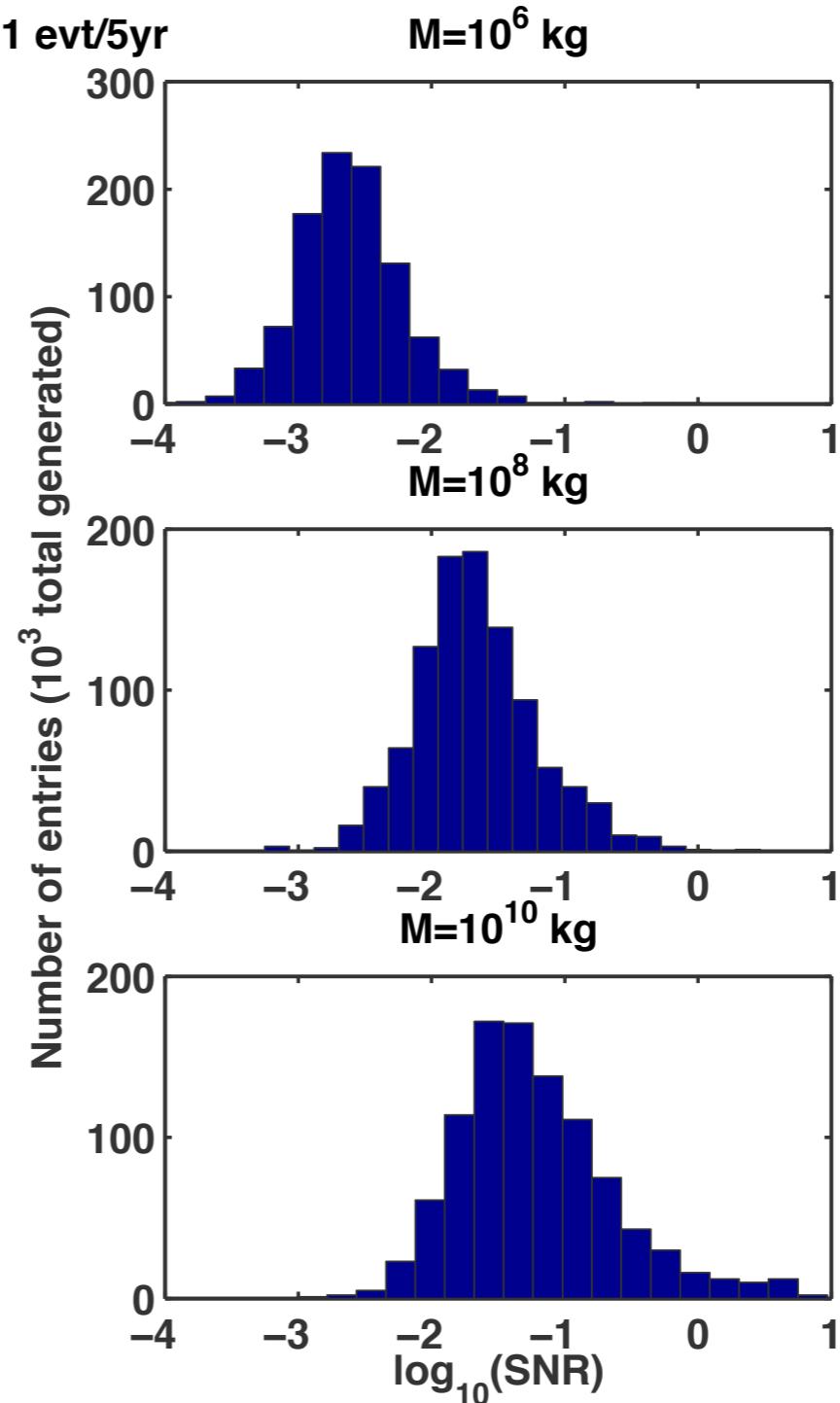
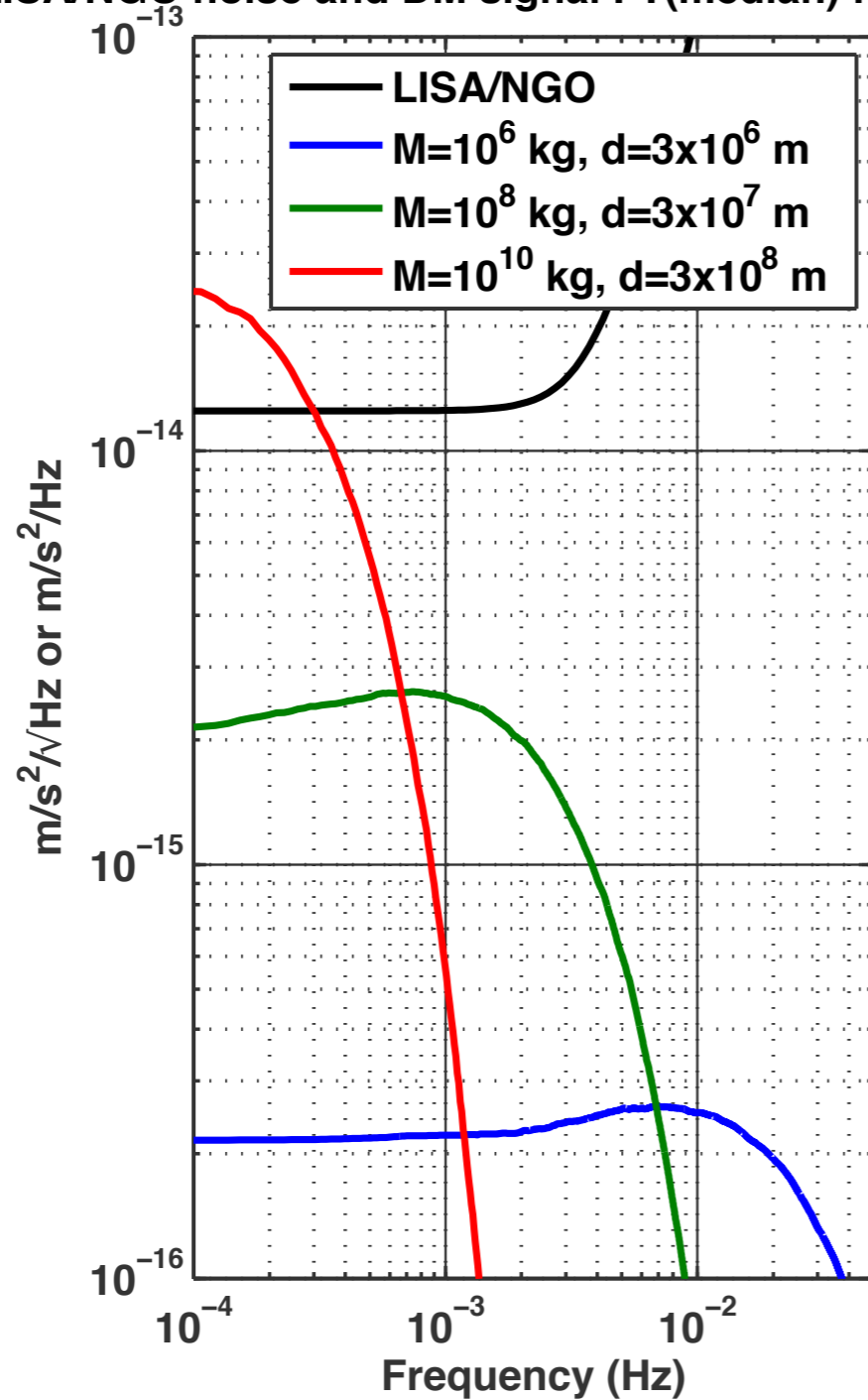
We survive to the collision because this number is very small!

The bar detectors are sensitive to this small amount of energy!



Signal in LISA

LISA/NGO noise and DM signal FT(median) for 1 evt/5yr



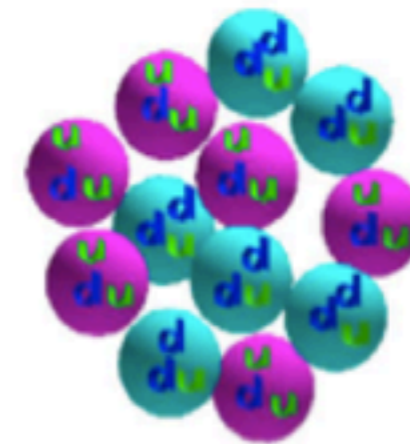
9

Nuclearite : a neutral quark nugget with strange quarks

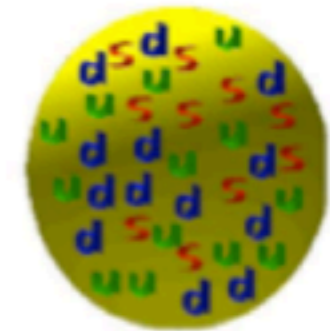
Strange Quark Matter (nuclearites, strangelets)

E. Witten, Phys. Rev. D30 (1984) 272A. De Rujula, L. Glashow, Nature 312 (1984) 734

- Aggregates of **u, d, s** quarks + **electrons** of ~ equal number, density: $3.5 \times 10^{14} \text{ g cm}^{-3}$.
- Ground state of nuclear matter ($E/A < 930 \text{ MeV}$).
- Stable for any baryon number A ($\text{few} < A < 10^{57}$).

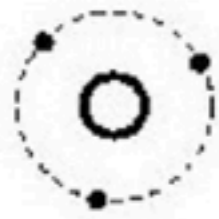


NUCLEAR MATTER

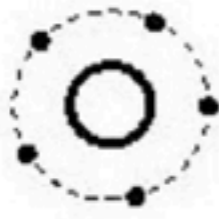


STRANGE MATTER

..a qualitative picture...



10^6



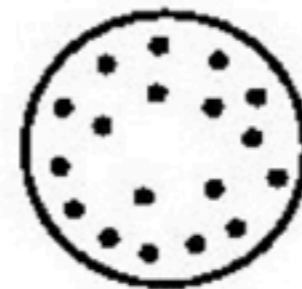
10^9



10^{12}



10^{15}

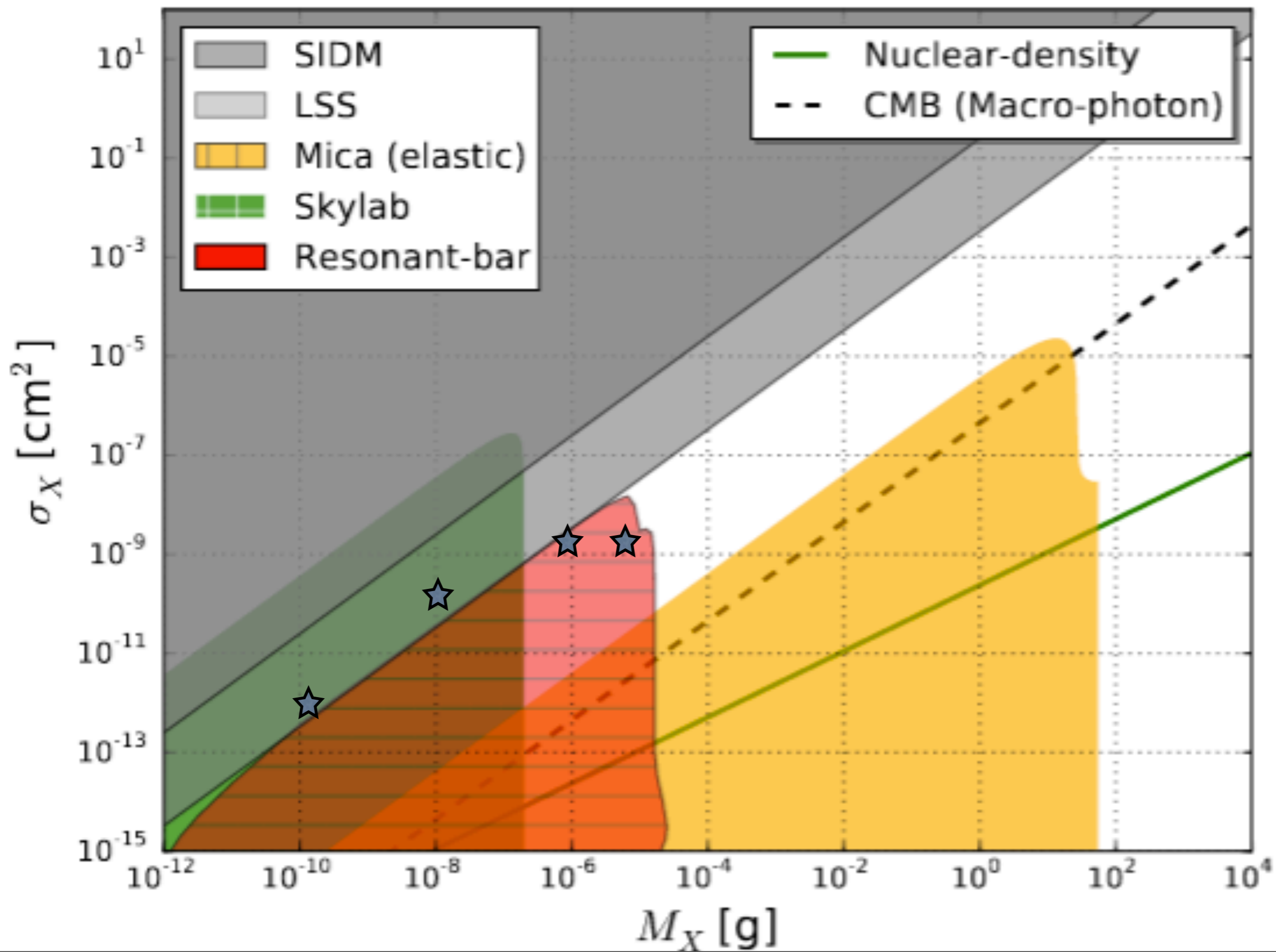


10^{18}

(black dots are electrons)

M (GeV)

- **Nuclearites** : core + electrons , neutral, $A > 10^6$ **CDM candidate**
- **Strangelets** : positively charged, $A < 10^6$ **Cosmic ray component**



Interest of bar results for other kind of DM candidate #1 MACRO

our results with this analysis

Our evaluation NAUTILUS + EXPLORER
full data set 2788 days

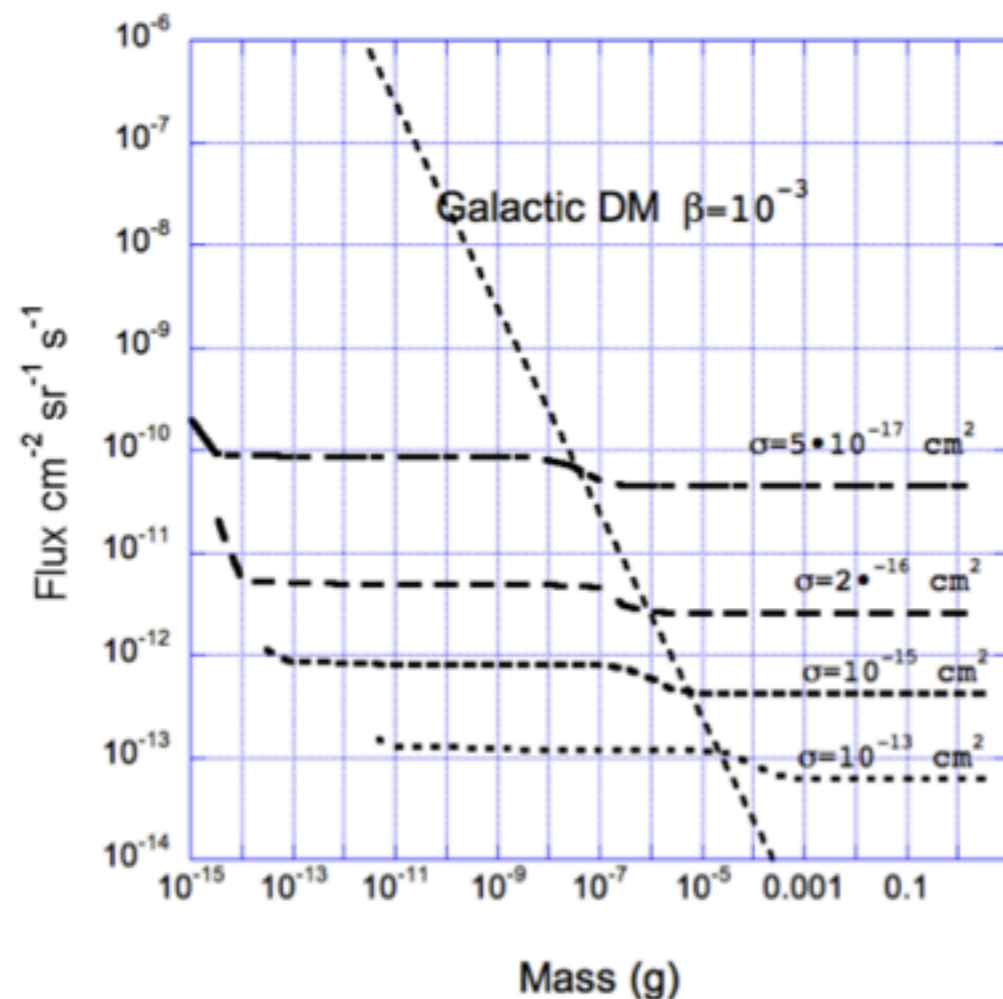
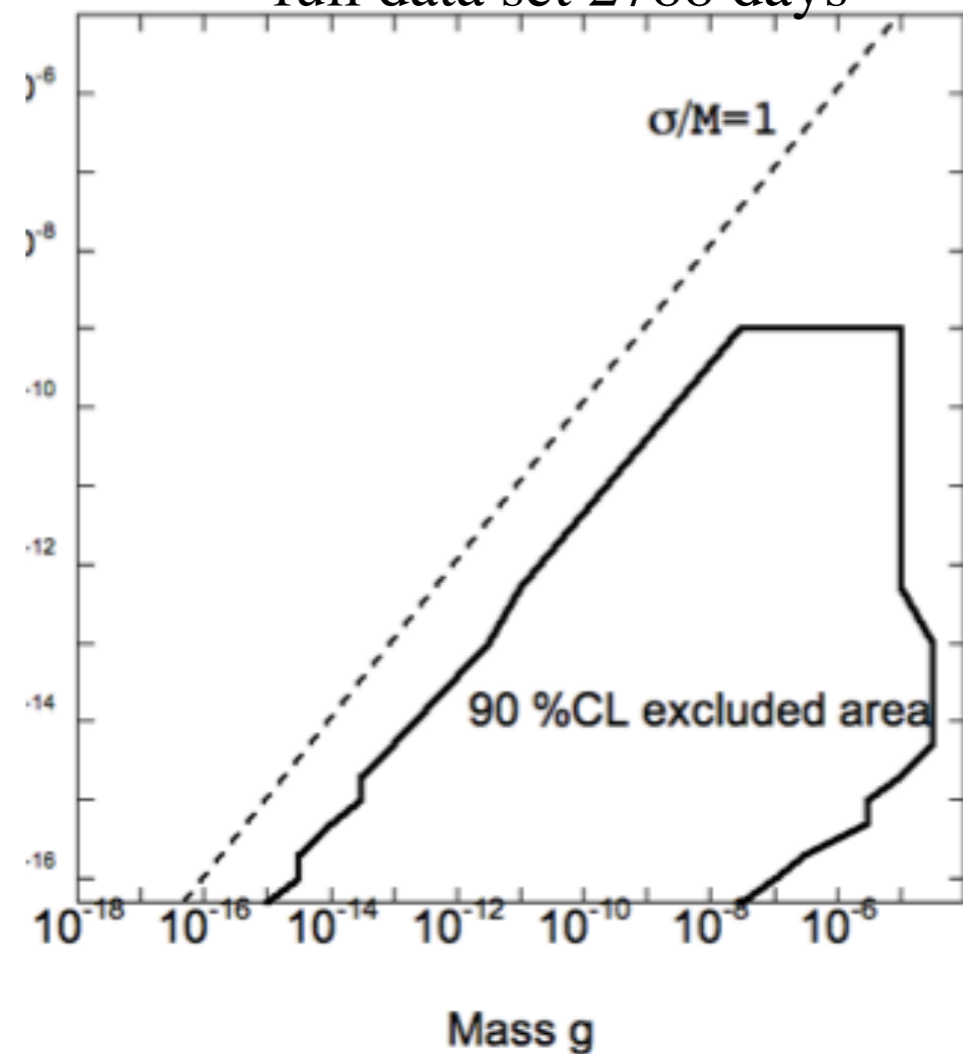


Figure 14: 90 % CL flux upper limits vs MACRO mass for different cross sections. The lowest mass limit on abscissa is set by a mass large enough to cross the atmosphere.

b

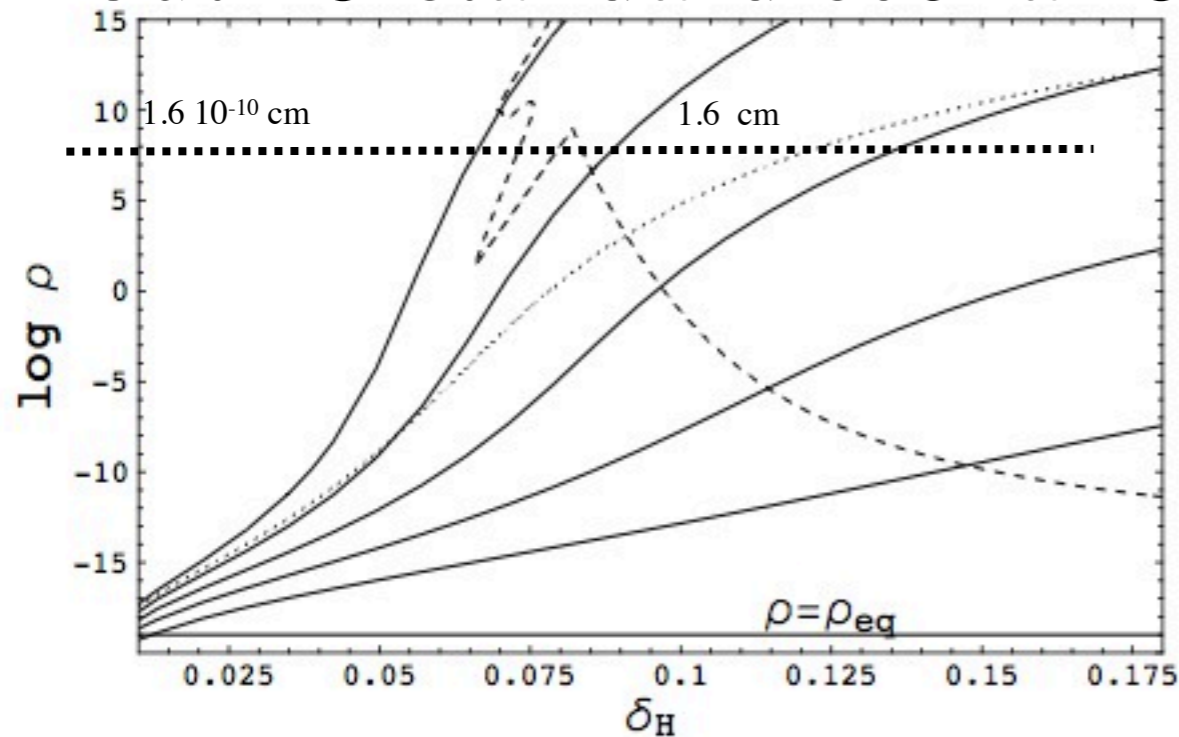


90 % CL excluded regions in the plane cross section - mass. The line reference. The study of cosmological halos may suggest a value $\sigma / M=0.1$.

Dark matter could be in “clumps”

Berezinski et al. [arXiv:1002.3445v2](https://arxiv.org/abs/1002.3445v2) and PRD

- in the standard cosmological scenario and for neutralinos the minimum clump mass is quite large (10^{-7} – 10^{-5} Msun)
- but non-standard scenarios are possible



according to this plot clumps radius could range from very small ($\ll 10^{-10}$) cm to very big dimensions of astrophysical interest

Clumps of astrophysical dimensions are of interest for LISA

FIG. 6: The mean density ρ (in g cm^{-3}) of DM clumps as function of the perturbation δ_H in the radiation density on the horizon scale; solid lines from top to bottom are for clump masses $M = 10^{-10}, 1, 10^{10}, 10^{20}, 10^{30}$ g. The dashed line is the bound on the clump density from primordial black holes overproduction with threshold $\delta_c = 0.7$. The time of two-body gravitational relaxation inside the clump core is less than the age of the Universe for clumps above the dotted lines, if the DM particle mass is $m \geq 10^{11}$ GeV.

