Neutrino in the Standard Model and beyond

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I will discuss

1. Status of neutrino masses and mixing (briefly).
2. Role of neutrinos in SM. Simplicity is a way of nature.
3. The simplest possibility of the generation of neutrino masses and mixing.

Basic relation

$$\nu_{lL}(x) = \sum_{i=1}^{3} U_{li}\, \nu_{iL}(x)$$

$\nu_{lL}(x)$ is the "mixed" flavor neutrino field in CC and NC

U is PMNS mixing matrix ($U^{\dagger} U = 1$)

$\nu_i(x)$ is the field of neutrino with mass m_i

Two possibilities for $\nu_i(x)$

1. Dirac field . Total Lepton number $L = L_e + L_\mu + L_\tau$ is conserved. $L(\nu_i) = 1, L(\bar{\nu}_i) = -1$
2. Majorana field. No conserved lepton numbers, $\nu_i \equiv \bar{\nu}_i$

In the total Lagrangian enter a nondiagonal neutrino mass term

The Dirac and Majorana mass terms

$$\mathcal{L}^{\mathrm{D}} = -\sum_{l'l} \bar{\nu}_{l'L} \, M^{\mathrm{D}}_{l'l} \, \nu_{lR} + \mathrm{h.c.}$$

$$\mathcal{L}^{\mathrm{L}} = -\frac{1}{2} \sum_{l',l} \bar{\nu}_{l'L} \, M^{L}_{l'l} (\nu_{lL})^c + \mathrm{h.c.}$$

M^{D} and $M^L = (M^L)^T$ are nondiagonal complex 3×3 matrices

The most general Dirac and Majorana mass term

$$\mathcal{L}^{\mathrm{D+M}} = \mathcal{L}^{\mathrm{D}} + \mathcal{L}^{\mathrm{L}} + \mathcal{L}^{\mathrm{R}}$$

Mixing has the form

$$\nu_{lL} = \sum_{i=1}^{6} U_{li}\, \nu_{iL}, \quad (\nu_{lR})^c = \sum_{i=1}^{6} U_{\bar{l}i}\, \nu_{iL}$$

Different possibilities

Seesaw: m_1, m_2, m_3 (small) M_1, M_2, M_3 (large)

Sterile: $m_1, m_2, m_3, m_4, ...$ (small)

Neutrino oscillations

1. Different Δm^2 can not be resolved in weak processes (Heisenberg uncertainty relation). Flavor neutrinos ν_e, ν_μ, ν_τ are described by coherent states

$$|\nu_l\rangle = \sum_i U^*_{li}\ |\nu_i\rangle$$

$|\nu_i\rangle$ is the state of neutrino with mass m_i, momentum $\vec{p}$ and energy $E_i \simeq E + \frac{m_i^2}{2E}$

2. Small Δm^2 can be resolved in propagation if $\Delta m^2 \frac{L}{E} \geq 1$ (time-energy uncertainty relation)
3. $P(\nu_l \to \nu_{l'}) = |\sum_i U_{l'i}\ e^{-iE_i\ t}\ U^*_{li}|^2$

In the case of the three-neutrino mixing oscillations are characterized by two mass-squared differences, three mixing angles and CP phase

Two possibilities for the neutrino mass spectrum

1. Normal spectrum $m_1 < m_2 < m_3$,
   $\Delta m^2_{12} \ll \Delta m^2_{23}, \quad \Delta m^2_{12} \equiv \Delta m^2_S, \;\; \Delta m^2_{23} \equiv \Delta m^2_A$
2. Inverted spectrum $m_3 < m_1 < m_2$,
   $\Delta m^2_{12} \ll |\Delta m^2_{13}| \quad \Delta m^2_{12} \equiv \Delta m^2_S, \quad |\Delta m^2_{13}| \equiv \Delta m^2_A$

$$\Delta m^2_{ki} = m^2_i - \; m^2_k$$

General (convenient) expression for the transition probability in the three-neutrino case

$$P^{NS(IS)}(\overset{(-)}{\nu}_l \to \overset{(-)}{\nu}_{l'}) = \delta_{l'l} - 4|U_{l3}|^2(\delta_{l'l} - |U_{l'3}|^2)\sin^2\Delta_A -$$
$$4|U_{l1(2)}|^2(\delta_{l'l} - |U_{l'1(2)}|^2)\sin^2\Delta_S - 8\ [\mathrm{Re}\ \mathrm{A}^{31(2)}_{l'l}\cos(\Delta_A + \Delta_S)$$
$$\pm(\mp)\ \mathrm{Im}\ \mathrm{A}^{31(2)}_{l'l}\sin(\Delta_A + \Delta_S)]\sin\Delta_A\sin\Delta_S$$

$$\mathcal{A}^{ik}_{l'l} = U_{l'i}U^*_{li}U^*_{l'k}U_{lk}$$

$$\Delta_{S,A} = \frac{\Delta m^2_{S,A}L}{4E}$$

Results of a global analysis of the data

Parameter	Normal Spectrum	Inverted Spectrum
$\sin^2\theta_{12}$	$0.304^{+0.013}_{-0.012}$	$0.304^{+0.013}_{-0.012}$
$\sin^2\theta_{23}$	$0.452^{+0.052}_{-0.028}$	$0.579^{+0.025}_{-0.037}$
$\sin^2\theta_{13}$	$0.0218^{+0.0010}_{-0.0010}$	$0.0219^{+0.0011}_{-0.0010}$
δ (in $^\circ$)	(306^{+39}_{-70})	(254^{+63}_{-62})
Δm^2_S	$(7.50^{+0.19}_{-0.17})\cdot 10^{-5}\ \mathrm{eV}^2$	$(7.50^{+0.19}_{-0.17})\cdot 10^{-5}\ \mathrm{eV}^2$
Δm^2_A	$(2.457^{+0.047}_{-0.047})\cdot 10^{-3}\ \mathrm{eV}^2$	$(2.449^{+0.048}_{-0.047})\cdot 10^{-3}\ \mathrm{eV}^2$

Upper bounds for the absolute values of neutrino masses

$m_\beta < 2.3\ (2.05)\ \mathrm{eV}\ (\mathrm{Mainz},\ \mathrm{Troitsk})\quad m_\beta = (\sum_i |U_{ei}|^2 m_i^2)^{1/2}.$

From the Planck and other cosmological measurements

$\sum_i m_i < 2.3\cdot 10^{-1}\ \mathrm{eV}.$

If $m_0 \ll \sqrt{\Delta m^2_S}$, $\sum_i m_i \simeq 5\cdot 10^{-2}\ \mathrm{eV}$ (NH)

$\sum_i m_i \simeq 10^{-1}\ \mathrm{eV}$ (IH). Cosmology is moving to these numbers

Neutrino and the Standard Model

After the discovery of the Higgs boson at LHC the SM acquire the status of a theory of elementary particles in the electroweak range (up to ~ 300 GeV)

What the SM teaches us?

The Standard Model is based on the following principles

- Local gauge symmetry
- Unification of the electromagnetic and weak interactions
- Spontaneous breaking of the electroweak symmetry

In the framework of these principles nature choose the simplest, most economical possibilities

We will try to apply this observation to neutrinos

The Standard Model started with the theory of the two-component neutrino.

A bit of history. In 1929 soon after Dirac proposed relativistic four-component equation for a spin 1/2 particle H. Weil proposed the two-component equation for a relativistic spin 1/2 a particle. Weil introduced two-component spinors

$$\psi_{L,R} = \frac{1}{2}(1 \mp \gamma_5)\psi$$

For a particle with a mass m only coupled equations for $\psi_L(x)$ and $\psi_R(x)$ can be obtained

For $m = 0$ Weil found two decoupled equations for $\psi_L(x)$ and $\psi_R(x)$

$$i\gamma^\alpha \partial_\alpha \, \psi_L(x) = 0, \quad i\gamma^\alpha \partial_\alpha \, \psi_R(x) = 0$$

These equations, however, are not invariant under the space inversion

$$\psi'_{R,L}(x') = e^{i\alpha}\gamma^0 \psi_{L,R}(x), \quad x' = (x^0, -\vec{x})$$

At that time people believed that the conservation of parity is a low of nature. The Weil equations were rejected.

"...because the equation for $\psi_L(x)$ ($\psi_R(x)$) is not invariant under space reflection it is not applicable to the physical reality". (Pauli "Quantum Mechanics")

Weyl: "In my work I always tried to unite the truth with the beautiful, but when I had to choose one or the other, I usually chose the beautiful."

After it was discovered that parity (and C) is not conserved in the weak interaction (1957) Landau and Lee and Yang and Salam proposed the two-component neutrino theory (1958)

They had different arguments. Lee and Yang applied to neutrino the Weil theory.

According to the two-component neutrino theory neutrino is massless particle and neutrino field is $\nu_L(x)$ or $\nu_R(x)$. The theory predicted

1. Large violation of the parity in the β-decay and other processes
2. The helicity of ν ($\bar{\nu}$) is equal to -1(+1) in the case of the field $\nu_L(x)$ and +1(-1) in the case of the field $\nu_R(x)$

The crucial test of the two-component neutrino theory was the measurement of the neutrino helicity in the classical GGS experiment (1958)

Goldhaber et al concluded "... our result is compatible with 100% negative helicity of neutrino..." This result confirmed the two-component theory with neutrino field $\nu_L(x)$

The two-component neutrino is the most economical possibility (2 dof instead of 4 dof in the general spin 1/2 Dirac case).

Local gauge symmetry

The simplest symmetry group which allows to include neutrinos, leptons and quarks is $SU_L(2)$ with doublets

$$\psi^{lep}_{eL} = \begin{pmatrix} \nu'_{eL} \\ e'_L \end{pmatrix}, \ \psi^{lep}_{\mu L} = \begin{pmatrix} \nu'_{\mu L} \\ \mu'_L \end{pmatrix}, \ \psi^{lep}_{\tau L} = \begin{pmatrix} \nu'_{\tau L} \\ \tau'_L \end{pmatrix}, ...$$

To insure symmetry lepton (and quarks) fields must be massless two-component Weyl left-handed fields like neutrino fields

The local gauge invariance requires that all fermion fields interact with massless vector field $\vec{A}_\alpha(x)$

Interaction is the minimal (compatible with the local gauge invariance) interaction

$$\mathcal{L}_I(x) = -g\ \vec{j}_\alpha \vec{A}^\alpha, \ \ \vec{j}_\alpha = \sum_{l=e,\mu,tau} \bar{\psi}^{lep}_{lL} \gamma_\alpha \frac{1}{2} \vec{\tau} \psi^{lep}_{lL}$$

Unification of the weak and electromagnetic interactions

Electromagnetic current

$j^{\mathrm{EM}}_\alpha = \sum_l (-1)\ \bar{l}'_L \gamma_\alpha l'_L + \sum_l (-1)\ \bar{l}'_R \gamma_\alpha l'_R$

In order to include electromagnetic interaction we need to include right-handed $SU_L(2)$ singlets l'_R

The minimal enlargement is the local $SU_L(2) \times U_Y(1)$ group

$U_Y(1)$ is the group of the hypercharge Y

Hypercharges of the fields are determined by the Gell-Mann-Nishijima relation $Q = T_3 + \frac{1}{2} Y$

Neutrinos have no electromagnetic interaction. Unification of the weak and electromagnetic interactions does not require right-handed neutrino fields.

The minimal Lagrangian compatible with the local $SU_L(2) \times U_Y(1)$ invariance is the sum of the CC and NC and EM terms $\mathcal{L}_I = \left(-\frac{g}{2\sqrt{2}} j^{CC}_\alpha W^\alpha + \mathrm{h.c}\right) - \frac{g}{2\cos\theta_W} j^{\mathrm{NC}}_\alpha Z^\alpha - j^{EM}_\alpha A^\alpha$

$$j^{\mathrm{NC}}_\alpha = 2\, j^3_\alpha - 2\, \sin^2\theta_W\, j^{\mathrm{EM}}_\alpha$$

The weak and EM interactions are fundamentally different: the weak interaction violates P and C (and T) and EM interaction conserve P and C and T

Nonconservation of P and C in the CC and NC is originated from two-component Weil neutrino

It looks that nature chooses simplicity and economy by the prise of the non conservation of P and C

The SM mechanism of the mass generation is the Brout-Englert-Higgs mechanism based on the assumption of the existence of scalar Higgs fields

In order to generate masses of $W^{\pm}$ and Z^0 bosons we need to have three (Goldstone) degrees of freedom

Minimal possibility is a doublet of complex Higgs fields (four degrees of freedom)

After the spontaneous breaking of the symmetry we left with one hermitian scalar field (one neutral scalar Higgs boson)

Such a picture was confirmed by LHC experiments

Masses of $W^{\pm}$ and Z^0 are proportional to the vacuum expectation value v

$$m_W = \frac{1}{2} g v, \quad m_Z = \frac{1}{2}\sqrt{g^2 + g'^2}\, v = \frac{g}{2\cos\theta_W} v$$

The parameter v(dimension M) is given by

$$v = (\sqrt{2} G_F)^{-1/2} = 246 \ \ \mathrm{GeV}$$

Lepton (and quark) masses and mixing are generated by the $SU_L(2) \times U_Y(1)$ invariant Yukawa interaction

$$\mathcal{L}_Y^{lep} = -\sqrt{2} \sum_{l_1, l_2} \bar{\psi}_{l_1 L}^{lep} Y_{l_1 l_2} l'_{2R} \ \phi + \mathrm{h.c}$$

After the spontaneous breaking of the symmetry the Dirac mass term is generated

$$\mathcal{L}_Y^{lep} = -\sum_l m_l \bar{l} \ l$$

Lepton (and quark) masses are proportional to v

$$m_l = y_l \ v$$

y_l is the eigenvalue of the matrix Y

Neutrinos after spontaneous breaking of the symmetry remain two-component massless Weyl particles

In such a picture neutrino masses and mixing can be generated only by a beyond the SM mechanism

The method of the effective Lagrangian is a powerful, general method which allows to describe beyond the SM effects

For neutrino mass term we need to built a Lagrangian which is quadratic in neutrino fields. The only possibility is the Weinberg Lagrangian

$$\mathcal{L}_{I}^{\mathrm{eff}} = -\frac{1}{\Lambda} \sum_{l_1,l_2} (\bar{\psi}^{lep}_{l_1 L}\tilde{\phi})\; Y_{l_1 l_2}\; (\tilde{\phi}^{T}(\psi^{lep}_{l_2 L})^{c}) + \mathrm{h.c.}, \quad Y_{l_1 l_2} = Y_{l_2 l_1}$$

1. The dimension M parameter Λ characterizes a scale of a beyond the SM physics; $\Lambda \gg v$.
2. The Lagrangian $\mathcal{L}_{I}^{\mathrm{eff}}$ does not conserve the total lepton number

This is the only Lagrangian of the dimension five

Neutrinos are the most sensitive probe of a new physics.

After spontaneous symmetry breaking we come to the Majorana mass term

$$\mathcal{L}^{\mathrm{M}} = -\frac{1}{2}\,\frac{v^2}{\Lambda}\sum_{l_1,l_2}\bar{\nu}'_{l_1 L}\;Y_{l_1 l_2}(\nu'_{l_2 L})^c + \mathrm{h.c.}$$

The matrix Y can be easily diagonalized:

$\mathcal{L}^{\mathrm{M}} = -\frac{1}{2}\sum_{i=1}^{3}\bar{\nu}_i\nu_i$

$\nu_i = \nu_i^c$ is the field of the neutrino Majorana with the mass m_i

$m_i = \frac{v^2}{\Lambda}y_i = \frac{v}{\Lambda}(y_i v)$

$y_i v$ is a "typical" fermion mass in SM

$$\frac{v}{\Lambda} = \frac{\text{scale of SM}}{\text{scale of a new physics}} \ll 1$$

is a suppression factor

Neutrino masses are much less that lepton and quark masses.

To estimate Λ assume hierarchy of neutrino masses $m_1 \ll m_2 \ll m_3$. In this case $m_3 \simeq \sqrt{\Delta m_A^2} \simeq 5 \cdot 10^{-2}\mathrm{eV}$. Assume also that $y_3 \simeq 1$. $\Lambda \simeq 10^{15}$ GeV

General consequences

- Neutrino with definite masses ν_i must be Majorana particles
- The number of neutrinos with definite masses must be equal to the number of lepton-quark generations (three)

The most practical way to reveal the nature of neutrinos with definite masses (Majorana or Dirac?) is to look for neutrinoless double β-decay of some even-even nuclei

$$(A, Z) \to (A, Z+2) + e^- + e^-$$

Due to $V - A$ interaction the probability of the process is proportional to square of neutrino masses and have the following general form

$$\frac{1}{T^{0\nu}_{1/2}} = |m_{\beta\beta}|^2 \; |M^{0\nu}|^2 \; G^{0\nu}(Q, Z)$$

$m_{\beta\beta} = \sum_i U_{ei}^2 m_i$, $M^{0\nu}$ is nuclear matrix element and $G^{0\nu}(Q, Z)$ is known phase factor

There are many experiments on the search for $0\nu\beta\beta$ of different nuclei. The process was not observed. Very large lower bounds for half-lives were obtained

Some recent data

EXO-200.

$$T^{0\nu}_{1/2}(^{136}\mathrm{Xe}) > 1.1 \cdot 10^{25}\ \mathrm{y}\ \ (90\% CL)\ \ |m_{\beta\beta}| < (1.9 - 4.5) \cdot 10^{-1}\ \mathrm{eV}$$

KamLAND-Zen

$$T^{0\nu}_{1/2}(^{136}\mathrm{Xe}) > 2.6 \cdot 10^{25}\ \mathrm{y}\ \ (90\% CL) |m_{\beta\beta}| < (1.4 - 2.8) \cdot 10^{-1}\ \mathrm{eV}$$

GERDA, Heidelberg-Moscow, IGEX

$$T^{0\nu}_{1/2}(^{76}\mathrm{Ge}) > 3.0 \cdot 10^{25}\ \mathrm{y}\ \ (90\% CL)\ \ |m_{\beta\beta}| < (2 - 4) \cdot 10^{-1}\ \mathrm{eV}$$

$|m_{\beta\beta}|$ can be predicted only if we make assumption about neutrino spectrum. If there is the inverted hierarchy ($m_3 \ll m_1 < m_2$) and neutrinos are Majorana particles from oscillation data it follows

$$|m_{\beta\beta}| \simeq \mathrm{a\ few} \cdot 10^{-2}\ \mathrm{eV}$$

Next experiments on the search for $0\nu\beta\beta$ are planned to reach this region

If the number of ν_i is more than 3, transition into sterile neutrinos become possible.

There are indications in favor of transitions into sterile neutrinos obtained in the LSND, MiniBooNE, reactor and source experiments. From analysis of the data of these experiments it follows that massive neutrino(s) with mass $\sim 1\mathrm{eV}$ exist. Many experiments with the aim to check these indications are in preparations.

Dimension five effective Weinberg Lagrangian can be generated by the exchange of heavy Majorana right-handed leptons with masses of the order Λ between lepton-Higgs pairs (seesaw mechanism of the neutrino mass generation). CP violating decays of heavy Majorana leptons in the early Universe is a plausible mechanism of the generation of the barion asymmetry of the Universe which is some indication in favor of the scheme considered

CONCLUSION

The Standard Model teaches us that the simplest possibilities are more likely to be correct. Massless two-component left-handed Weyl neutrinos and absence of the right-handed neutrino fields in the Standard Model is the simplest, most elegant and most economical possibility

(it took, however, about 50 years of enormous efforts of many people to come to the "simple" SM)

Majorana neutrino mass term generated by the beyond the SM dimension 5 effective Lagrangian is the simplest possibility for neutrinos to be massive, naturally light and mixed