The effect of non-relativistic neutrino oscillations in cosmology

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Prologue

- Cosmological neutrinos contain distortions in their spectral distribution.
- Different distortions in each flavour allow the onset of flavour oscillation.
- Transition between relativistic regimes trigger new oscillation lengths.
- The change of mixed quantities increases the entropy inside the ensemble.
- Generation of entropy within the neutrino ensemble is related to a generation of bulk viscosity.
- The effective pressure (kinetic plus viscous) of neutrinos affects gravitational potentials after recombination.
- The changing potentials leave their imprint in the integrated Sachs-Wolfe effect.
- Non-relativistic neutrino oscillations could create a signal on the cosmic microwave background.

Neutrinos in equilibrium

- Cosmological neutrinos were created in thermal equilibrium.
- Decoupled while still in the relativistic regime, they kept the Fermi-Dirac distribution

$$n_\nu = \frac{3}{4}\frac{\zeta(3)}{\pi^2} g_{\star_\nu} T_\nu^3 \ . \tag{1}$$

- After decoupling, they stream as a free gas

$$a_{D\nu} T_\nu(a_{D\nu}) \ = \ a_0 T_\nu(a_0) \ . \tag{2}$$

- Anihilation of pairs electron-positron introduces a small diference in photons

$$T_\nu(a_0) \ = \ \left(\frac{4}{11}\right)^{1/3} T_\gamma(a_0) \cong 1.945 \text{ K} \cong 1.676 \times 10^{-4} \text{ eV} \ . \tag{3}$$

- The current number density

$$n_\nu = 0.67213 \ \mathrm{K}^3 \ \cong 320.6 \ \mathrm{cm}^{-3} \ . \tag{4}$$

- The current density parameter

$$\Omega_{\nu_0} = \frac{\sum m_{\nu_i}}{91.2 h^2 \mathrm{eV}} \ . \tag{5}$$

Spectral distortions

- Complete treatment of decoupling processes produces different spectra for ν_μ and ν_τ than for ν_e.
- Anihilation of $e^- e^+$ introduces distortions in the ν_e spectrum.
- Neutrino oscillations (plus refractive effect) opens a chanel to interchange distortions among flavours[1].

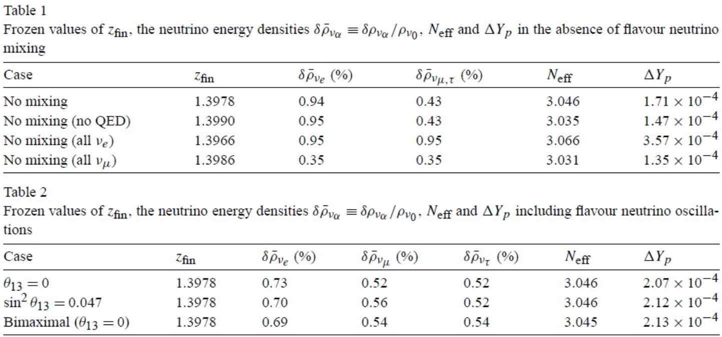

Table 1

Frozen values of z_{fin}, the neutrino energy densities $\delta\bar{\rho}_{\nu_\alpha} \equiv \delta\rho_{\nu_\alpha}/\rho_{\nu_0}$, N_{eff} and ΔY_p in the absence of flavour neutrino mixing

Case	z_{fin}	$\delta\bar{\rho}_{\nu_e}$ (%)	$\delta\bar{\rho}_{\nu_{\mu,\tau}}$ (%)	N_{eff}	ΔY_p
No mixing	1.3978	0.94	0.43	3.046	1.71×10^{-4}
No mixing (no QED)	1.3990	0.95	0.43	3.035	1.47×10^{-4}
No mixing (all ν_e)	1.3966	0.95	0.95	3.066	3.57×10^{-4}
No mixing (all ν_μ)	1.3986	0.35	0.35	3.031	1.35×10^{-4}

Table 2

Frozen values of z_{fin}, the neutrino energy densities $\delta\bar{\rho}_{\nu_\alpha} \equiv \delta\rho_{\nu_\alpha}/\rho_{\nu_0}$, N_{eff} and ΔY_p including flavour neutrino oscillations

Case	z_{fin}	$\delta\bar{\rho}_{\nu_e}$ (%)	$\delta\bar{\rho}_{\nu_\mu}$ (%)	$\delta\bar{\rho}_{\nu_\tau}$ (%)	N_{eff}	ΔY_p
$\theta_{13} = 0$	1.3978	0.73	0.52	0.52	3.046	2.07×10^{-4}
$\sin^2\theta_{13} = 0.047$	1.3978	0.70	0.56	0.52	3.046	2.12×10^{-4}
Bimaximal ($\theta_{13} = 0$)	1.3978	0.69	0.54	0.54	3.045	2.13×10^{-4}

- The distortions modify macroscopic thermal quantities: $N_{\text{eff}} = 3.046$ and $\Omega_{\nu_0} = \sum m_{\nu_i}/93.14h^2\text{eV}$.

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[1] G. Mangano et al., Nucl.Phys., B729, 221 (2005), arXiv:hep-ph/0506164 [hep-ph].

Neutrino oscillations

- The dynamics of neutrino oscillations by matrix density formalism is given by

$$(\partial_t - Hq\partial_q)\varrho_q = -\frac{i}{\hbar}[\mathcal{M}_0, \varrho_q] \;, \tag{6}$$

  where the matrix's entries are the particle occupation numbers in the flavour basis.
- Oscillation requires spectral distortions, different masses and non-orthogonal flavour and mass basis

$$\mathcal{M}_0^2 = U^\dagger \mathbf{M}^2 U + \mathbf{P}^2 \;. \tag{7}$$

- where the rotation is given by

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13} & -c_{12}s_{23} - s_{12}c_{23}s_{13} & c_{23}c_{13} \end{pmatrix}, \tag{8}$$

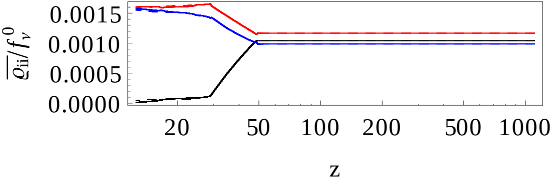
- Given the diagonality of M and P, oscillation among flavours will happen if
  - the matrix U mix the flavours, rotating M to a non-diagonal form
  - the matrix M although diagonal, has different entries, i.e. not reducible to a diagonal matrix ($M \not\propto I$)
  - the matrix ϱ_q has non-diagonal terms populated or the diagonal terms are not identical, i.e. not reducible to a diagonal matrix ($\varrho_q \not\propto I$)

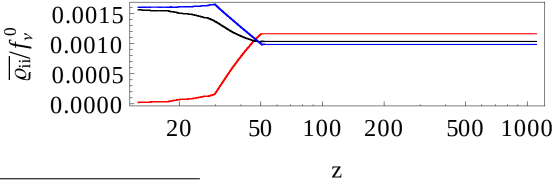
Neutrino oscillations

- The initial condition are the frozen asymptotical spectral distortions[2]

$$\begin{aligned} \delta f_{\nu_e}(q) &= \left(1 - 2.2q + 4.1q^2 - 0.047q^3\right) \times 10^{-4} \ , \\ \delta f_{\nu_{\mu,\tau}}(q) &= \left(-4 + 2.1q + 2.4q^2 - 0.019q^3\right) \times 10^{-4} \ . \end{aligned} \tag{9}$$

- Magnitude of distortions before and after oscillation are small, but its slope may be significative (normal and inverted hierarchy).





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[2] G. Mangano et al., Nucl.Phys., B729, 221 (2005), arXiv:hep-ph/0506164 [hep-ph].

Increase of entropy

- Entropy within the neutrino ensemble using the von Neumann entropy

$$S_\nu(\varrho) = -\int d\mathbf{q}\,\mathrm{Tr}\left[\varrho_q \ln\left(\varrho_q\right)\right] \ ,$$

- and its derivative

$$\dot{S}_\nu(\varrho) = -\int d\mathbf{q}\,\mathrm{Tr}\left[\dot{\varrho}_q \ln\left(\frac{\varrho_q}{\mathbf{I} - \varrho_q}\right)\right] \ . \tag{10}$$

- The entropy increases within a system of neutrinos that evolves from a configuration with pure to mixed states[3].
- Cosmological neutrinos are created in flavour pure states.
- Oscillation changes the ensemble from pure to mixed states, reaching a frozen difference of distortions, given by the oscillation probability for relativistic neutrinos.
- When neutrinos become non-relativistic, a new oscillation length is triggered along with a kinetic observable that discriminates mass eigenstates.

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[3] G. Sigl and G. Raffelt, Nucl.Phys., B406, 423 (1993).

The 2x2 case

- For oscillation between two states

$$\varrho_q = \frac{1}{2}\begin{pmatrix} B_0 + B_z & B_x - iB_y \\ B_x + iB_y & B_0 - B_z \end{pmatrix} \ , \qquad (11)$$

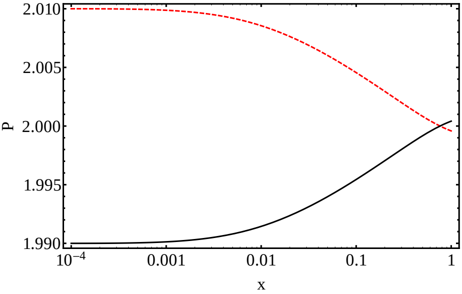
- where the flavour polarization vectors are given by[4]

$$\frac{dB_0}{dx} = 0 \ , \ \frac{dB_x}{dx} = \frac{\Delta m^2}{2EHx} cos(2\theta) B_y \ , \ \frac{dB_z}{dx} = \frac{\Delta m^2}{2EHx} sin(2\theta) B_y \ ,$$
$$\frac{dB_y}{dx} = -\frac{\Delta m^2}{2EHx} cos(2\theta) B_x - \frac{\Delta m^2}{2EHx} sin(2\theta) B_z \ , \qquad (12)$$

- and the difference in entropy

$$\Delta S_\nu = S_\nu(\varrho_f) - S_\nu(\varrho_i) \ , \qquad (13)$$

- For 1% of initial distortion, pure states, solar parameters and $(q = 1 \ k_B T_{\nu_0} c^{-1})$, the difference is an increase of 8%.



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[4] A. Dolgov et al., Nuclear Physics B, 632, 363 (2002), ISSN 0550-3213.

Generation of bulk viscosity

- Relativistic fluids may have dissipative effects

$$T^{\mu\nu} = \epsilon u^\mu u^\nu - p_k \left(g^{\mu\nu} - u^\mu u^\nu\right) + \Delta T^{\mu\nu} \ , \tag{14}$$

- that described by the Eckart phenomenological theory are[5]

$$\begin{aligned} \Delta T^{\mu\nu} &= \chi \left(h^{\mu\lambda} U^\nu + h^{\nu\lambda} U^\mu\right) \left(T_{,\lambda} - T U^\sigma U_{\lambda,\sigma}\right) \\ +\xi h^{\mu\sigma} h^{\nu\sigma} &\left(U_{\sigma,\rho} + U_{\rho,\sigma} - 2 g_{\sigma\rho} U^\lambda_{,\lambda}/3\right) + \zeta h^{\mu\nu} U^\lambda_{,\lambda} \ . \end{aligned} \tag{15}$$

- Any change in entropy for an isotropic fluid is parameterizable as bulk viscosity

$$\dot{S}_\nu = \frac{9\zeta H^2}{n_a k_b T_a} \tag{16}$$

- Effective pressure

$$p_\nu = p_k - 3H\zeta \ , \tag{17}$$

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[5] C. Eckart, Phys. Rev., 58, 919 (1940). N. Straumann, Helv. Phys. Acta, 49, 269 (1976).

The signal of neutrino oscillations on the CMB

- The viscous pressure affects the metric fluctuations

$$\ddot{\phi} + \frac{\dot{a}}{a}(\dot{\psi} + 2\dot{\phi}) + \left(2\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2}\right)\psi + \frac{k^2}{3}(\phi - \psi) = 4\pi G a^2 \delta T^i_i(Con) \; . \qquad (18)$$

- The contribution to the temperature deviation of photons is by the ISW

$$\Theta_l(k, \eta_0)_{\mathrm{ISW}} = \int_{\eta}^{\eta_0} \mathrm{d}\eta e^{-\tau} \left[\dot{\psi}(k, \eta) + \dot{\phi}(k, \eta)\right] j_l[k(\eta - \eta_0)] \; . \qquad (19)$$

Conclusions

- Standard model of particle physics/cosmology predict spectral distortions for cosmological neutrinos ($N_{\rm eff} = 3.046, \Omega_{\nu_0} = \sum m_{\nu_i}/93.14 h^2 {\rm eV}$).
- Parameters measured from oscillations are of high confidence and the standard procedure is to adopted them as well measured parameters ($\sum m_\nu \geq 0.06$).
- Including "runtime" oscillation effects in cosmology is a natural extension and only requires standard physics as input. No new parameters except hierarchy.
- Generation of entropy by oscillation is not a new topic, but mostly unexplored in cosmology[6].
- Viscosity of fluids with non-perfect behaviour may be relevant, since non-conventional pressure terms arise with negative values[7].
- Given its nature, oscillatory effects may have high slopes at times when changing potentials is captured by ISW at CMB.
- Neutrinos are good candidates to develop viscosity and could create an observable signal on the large scales of the CMB.
- "A superposition of relativistic and non-relativistic mass eigenstates has a rich structure"[8] with possible interesting effects in cosmology.

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[6] A. Bernardini, Europhys.Lett., 103, 30005 (2013), arXiv:1204.1504 [astro-ph.CO].

[7] Floerchinger et al, PRL 114, 091301 (2015).

[8] D. V. Ahluwalia and T. Goldman *Interplay of nonrelativistic and relativistic effects in neutrino oscillations* Phys. Rev. D 56, 1698

Issues

- As usual in neutrino oscillations, is not clear what “happens” along neutrino propagation. Possibly, any interpretation but wave package would introduce paradoxes.
- Since neutrinos are decoupled, it is not clear which importance oscillations have (no differentiation among flavours), and if kinetic terms are feasible operators of wave package colapse.
- The interpretation of neutrino states becoming mixed, with an inscrase in entropy, as different fluids that become mixed when new windows are open in previously separated ensemble lacks the quantum interpretation and may induce to misunderstanding. Possibly, the only reliable interpretation is the one that takes in account the density matrix evolution.
- Viscosity is not presumed to exist for free streaming particles as neutrinos. One should be limited to interpret it as a effective measure of deviation from perfect fluid behaviour that changes the kinetic pressure.
- The numerical calculation presented is not easily reproduced by semi-analitical tests.
- Final result on the effect over Cl’s requires the convergence on the integration over all the moments.