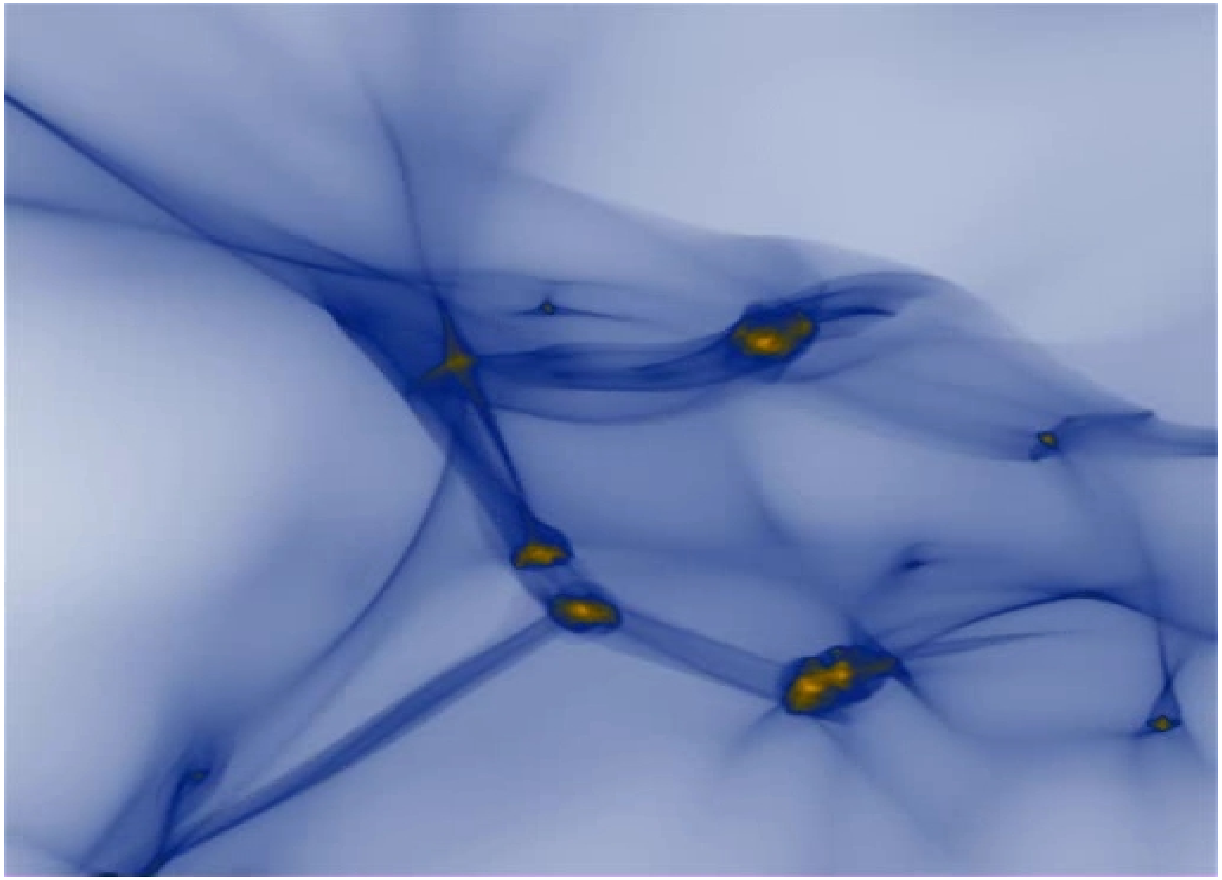
Cosmological Structure Formation in Numerical Simulations



Angulo, Hahn & Abell 2013

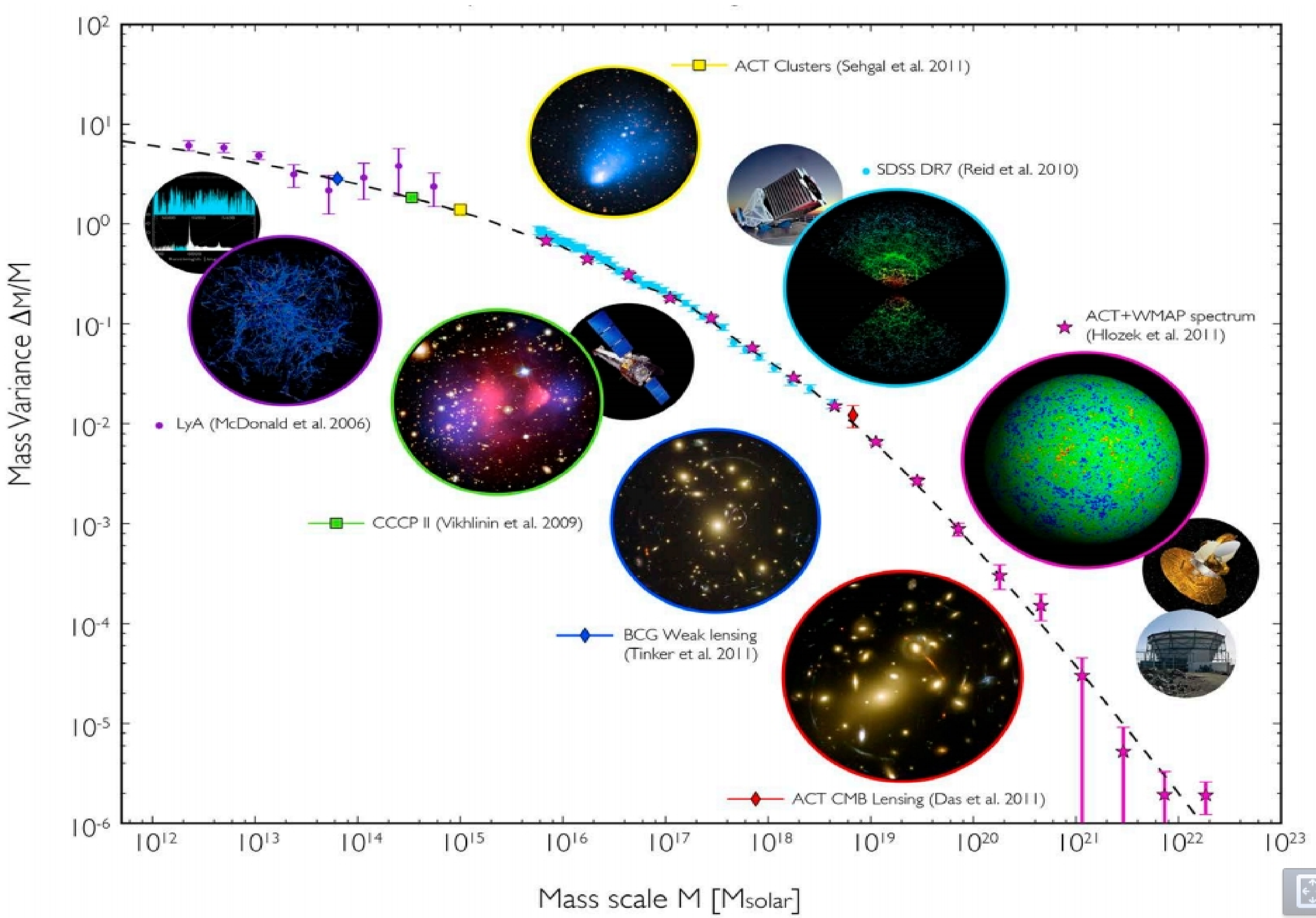
CEFCA
Centro de Estudios de Física del Cosmos de Aragón

Raul E. Angulo

Torino
September 2015

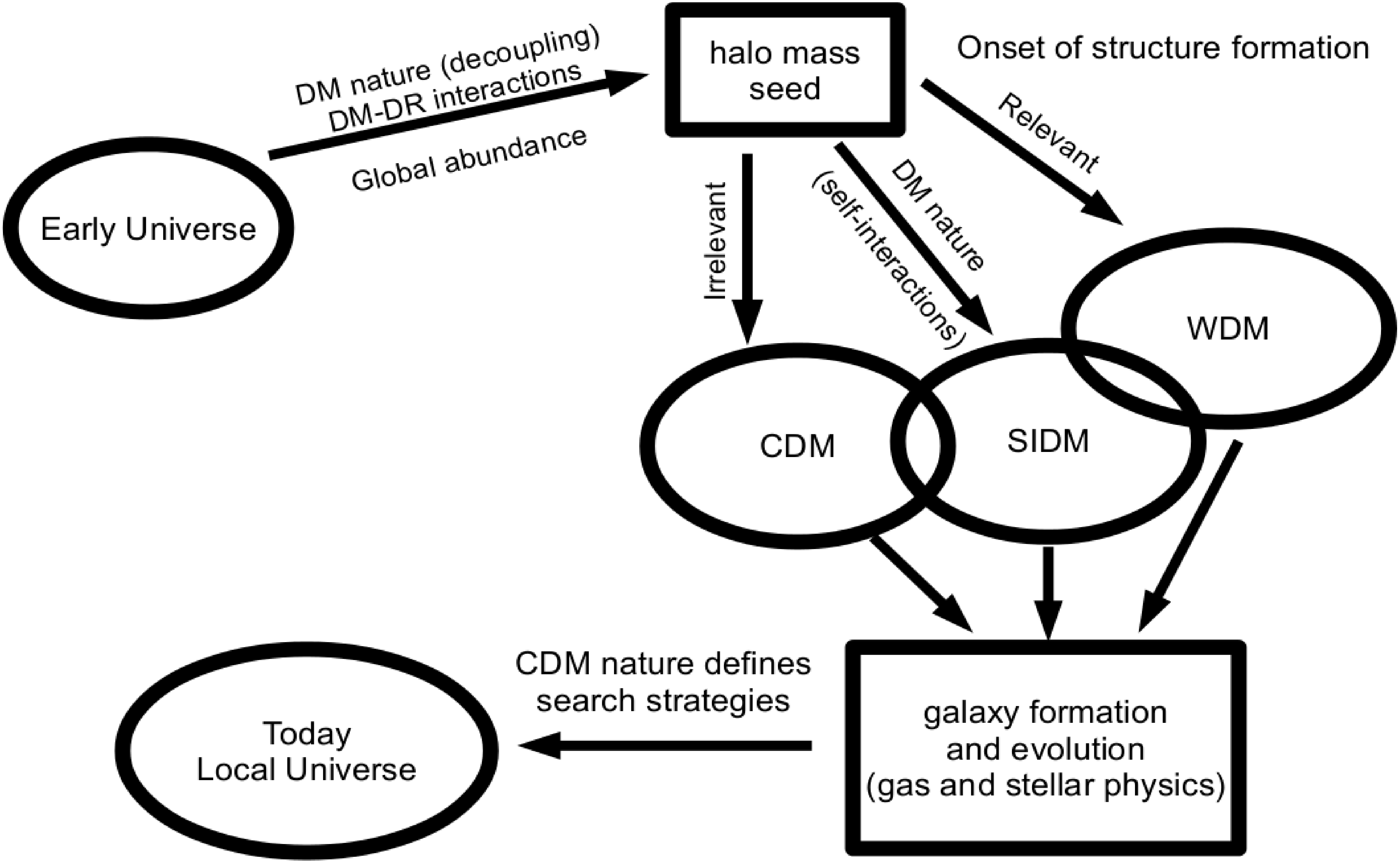
Simulating structure formation in the Universe

Most of the mass in the Universe is in the form of an unknown elementary particle: the Cold Dark Matter



Primack 2015

Structure formation in a nutshell



halo mass seed

Onset of structure formation

DM nature (decoupling)
DM-DR interactions

Global abundance

Early Universe

Irrelevant

DM nature
(self-interactions)

Relevant

WDM

CDM

SIDM

CDM nature defines search strategies

Today
Local Universe

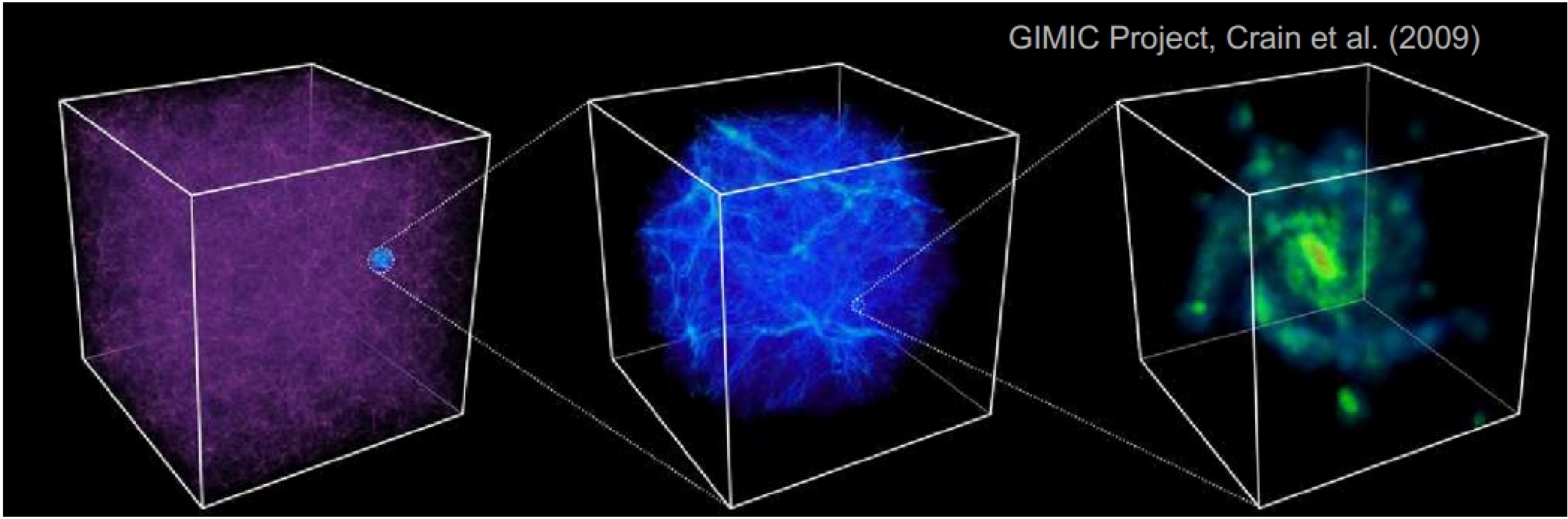
galaxy formation
and evolution
(gas and stellar physics)

From J. Zavala

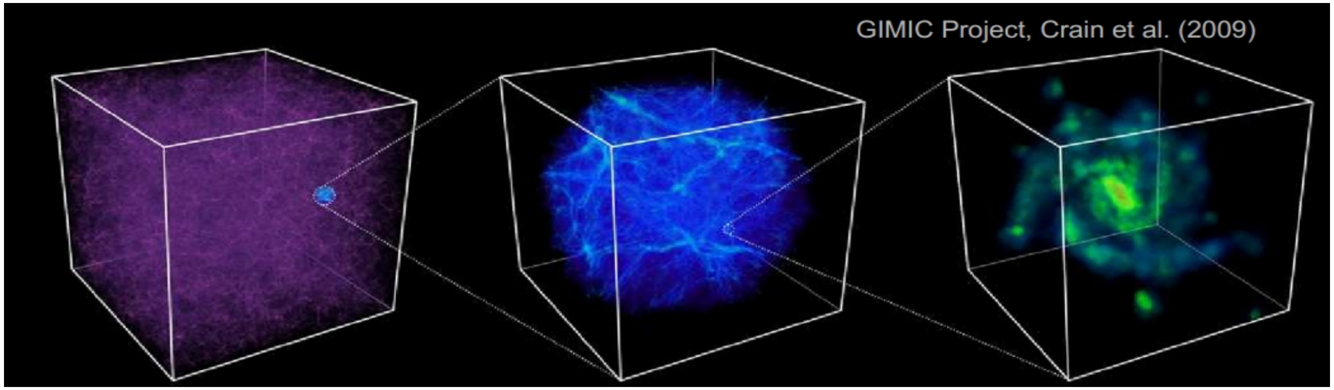
The signatures of the DM depend sensitively on:

- → the detailed distribution of dark matter
- → the precise impact of DM properties on cosmic structure
- → the physics of galaxy formation

All this from gigaparsecs down to subgalactic scales

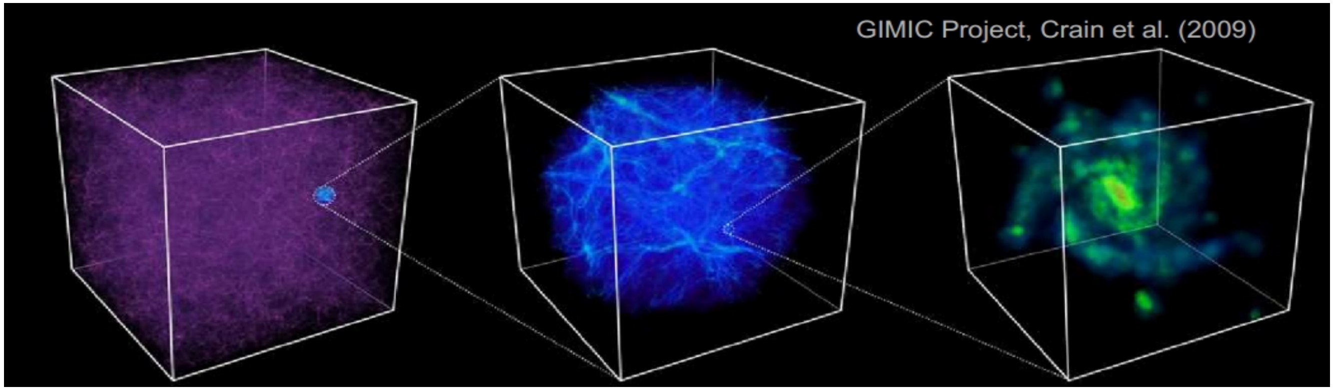

GIMIC Project, Crain et al. (2009)

Computer simulations offer the most accurate path towards these goals



GIMIC Project, Crain et al. (2009)

Domain	**LSS**	**Halos**	**Substructures**	**Local**
DM Detection	Extra-galactic DGRB Lensing Clusters Collisions	Stellar Streams Dwarf Galaxies Galactic DFRB Galactic Center	Clusters Dark Subhalos Boost factors	Direct Detections



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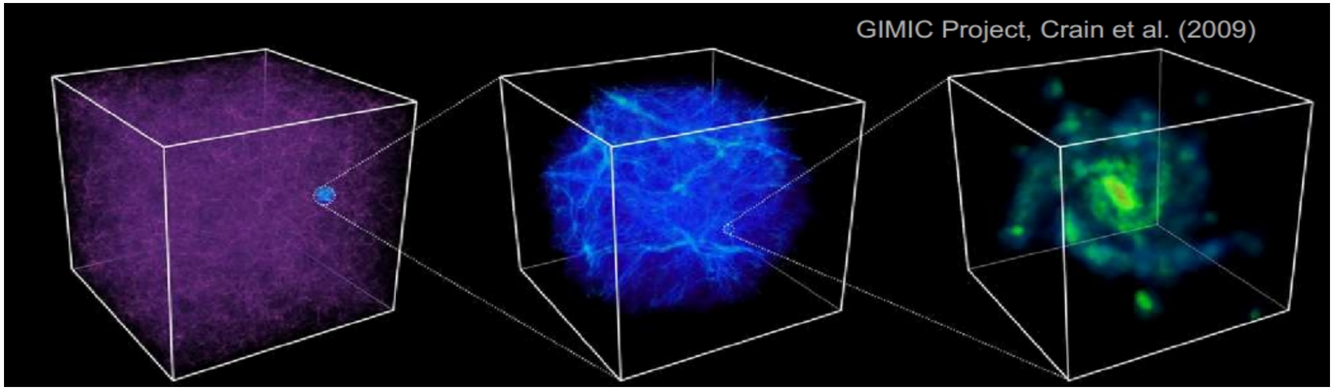

GIMIC Project, Crain et al. (2009)

Domain

LSS

Halos

Substructures

Local

DM Detection

Extra-galactic DGRB

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Clusters Collisions

Stellar Streams

Dwarf Galaxies

Galactic DFRB

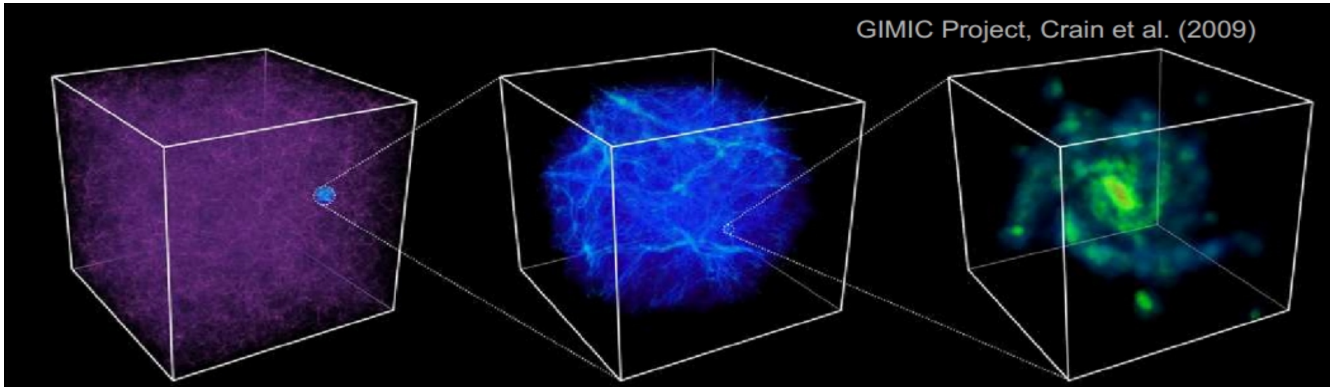
Galactic Center

Clusters

Dark Subhalos

Boost factors

Direct Detections



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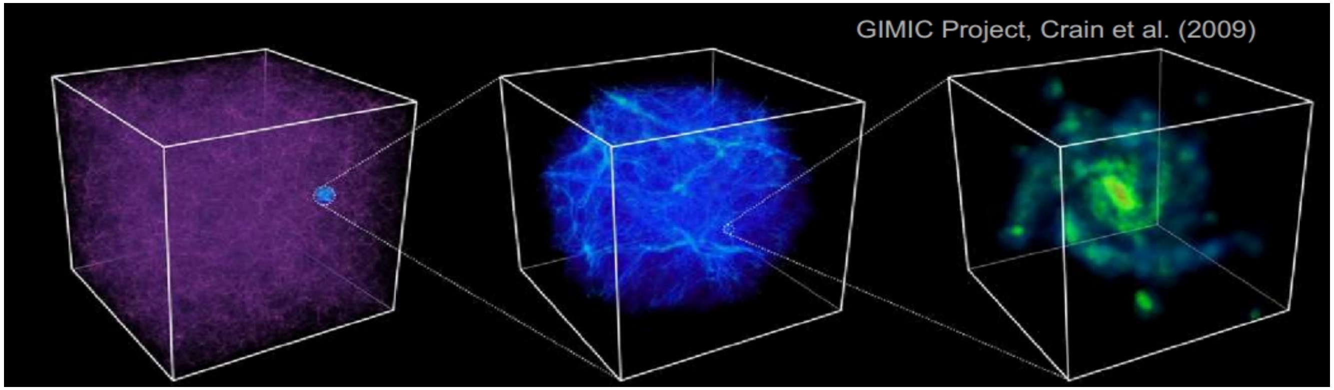

GIMIC Project, Crain et al. (2009)

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DM Detection	Extra-galactic DGRB Lensing Clusters Collisions	Stellar Streams Dwarf Galaxies Galactic DFRB Galactic Center	Clusters Dark Subhalos Boost factors	**Direct Detections**

All these require sophisticated numerical simulations

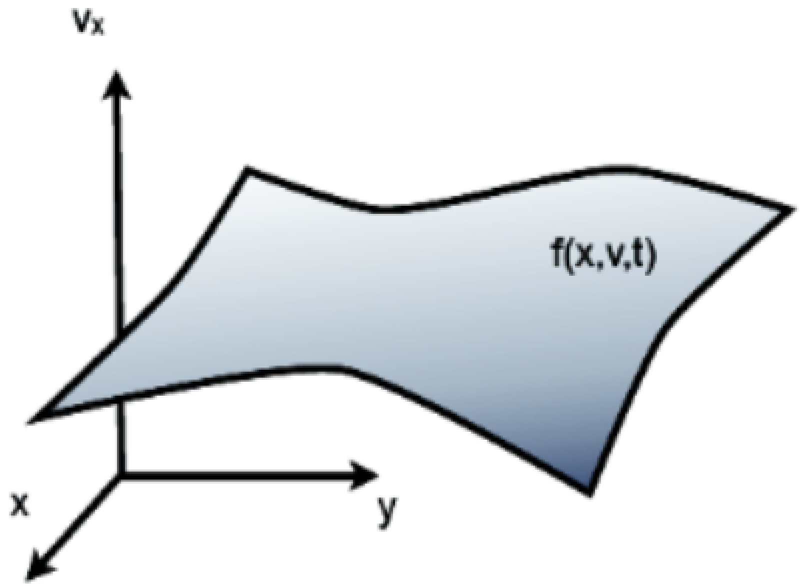
→ Dark Matter Simulations

→ Current State of the Art

→ The next decade

Simulating structure formation in the Universe

Most of the mass in the Universe is in the form of an unknown elementary particle: the Cold Dark Matter



Properties of CDM

- → Almost no extent in the velocity direction
- → Interacts mainly gravitationally
- → Tiny primordial fluctuations

...but simulating trillions of micro-physical CDM particles is impossible

CDM forms a "sheet": A continuous 3D surface embedded in a 6D space

The Vlassov-Poisson Equation

$$0 = \frac{\mathrm{d}f}{\mathrm{d}t} = \frac{\partial f}{\partial t} + \frac{\mathbf{v}}{a^2} \cdot \frac{\partial f}{\partial \mathbf{v}} - \frac{\partial f}{\partial \mathbf{x}} \frac{\partial \Phi}{\partial \mathbf{x}}$$

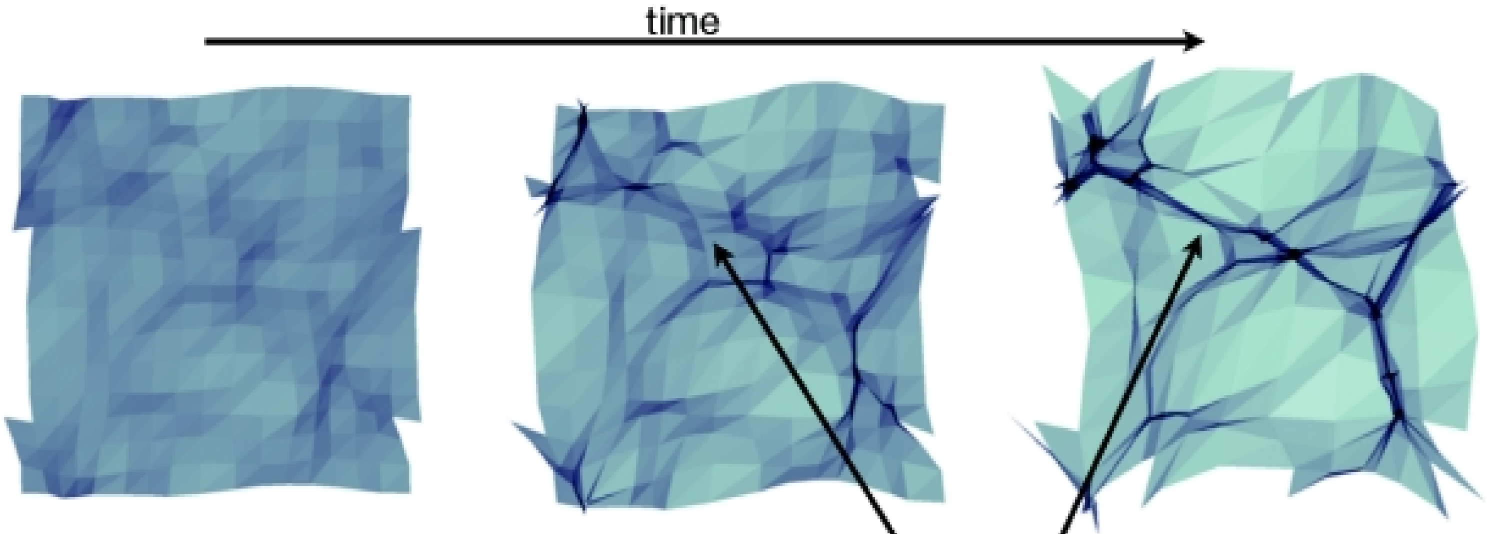
$$\nabla^2 \Phi = \frac{4\pi G}{a} \int f \mathrm{d}^3 v.$$

CDM Sheet Properties

→ phase-space is conserved along characteristics

→ It can never tear

→ It can never intersect



shell crossing

multi-stream regions appear

density, velocity = sum over many cells

The Vlassov-Poisson Equation

$$0 = \frac{\mathrm{d}f}{\mathrm{d}t} = \frac{\partial f}{\partial t} + \frac{\mathbf{v}}{a^2} \cdot \frac{\partial f}{\partial \mathbf{v}} - \frac{\partial f}{\partial \mathbf{x}} \frac{\partial \Phi}{\partial \mathbf{x}}$$

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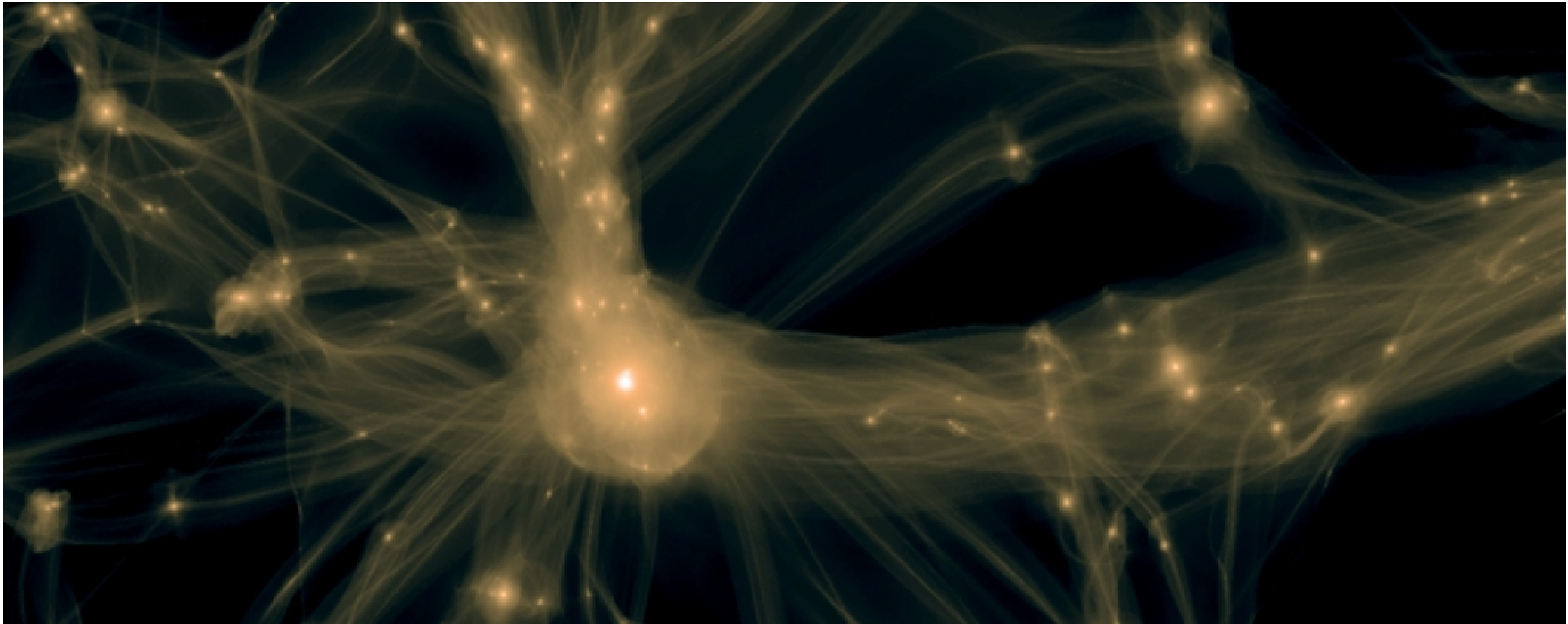
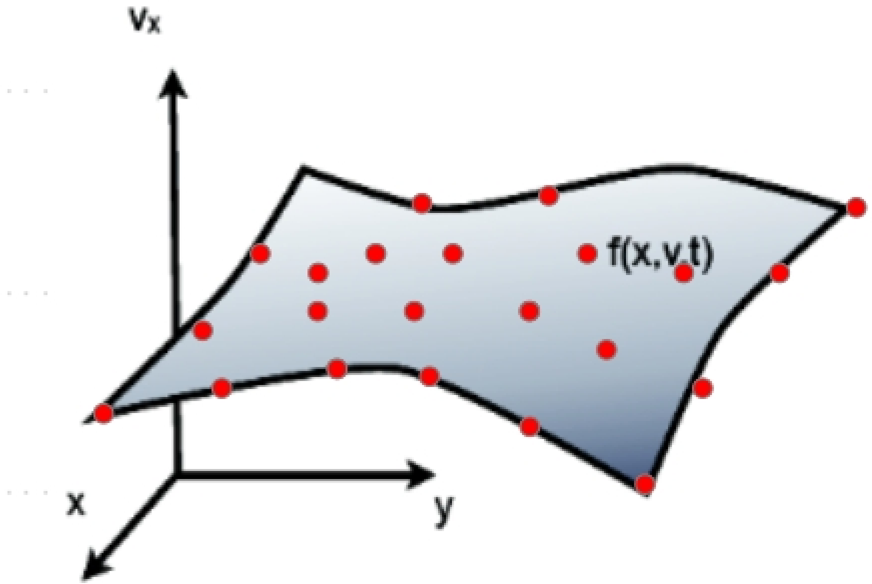


Standard approach to solving the VP equation:

Montecarlo Sampling and coarse graining the CDM distribution function

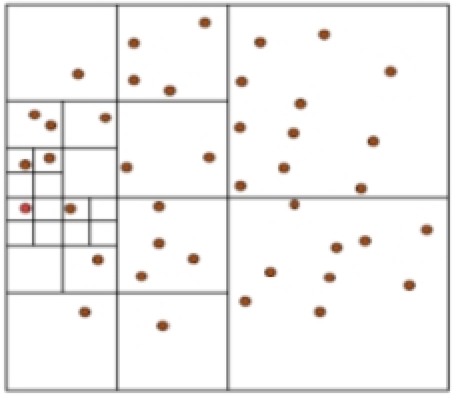


$$\frac{\mathrm{d}^2\mathbf{x}_i}{\mathrm{d}^2 t} = \nabla_i \Phi(\mathbf{x}_i),$$

$$\Phi(\mathbf{x}) = -G \sum_i \frac{m_i}{[(\mathbf{x}_i - \mathbf{x})^2 + \varepsilon^2]}$$

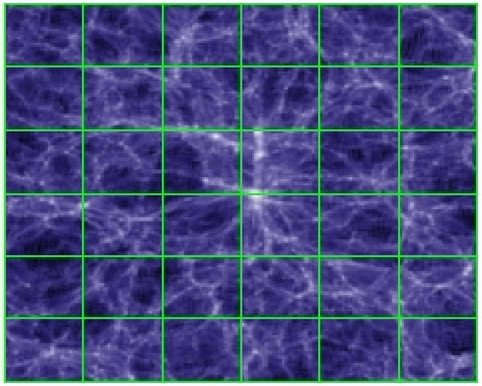
Tree Algorithms

Multipole decomposition



Particle-Mesh

Poisson equation



The evolution of the fine and coarse grained distribution functions are NOT equivalent.

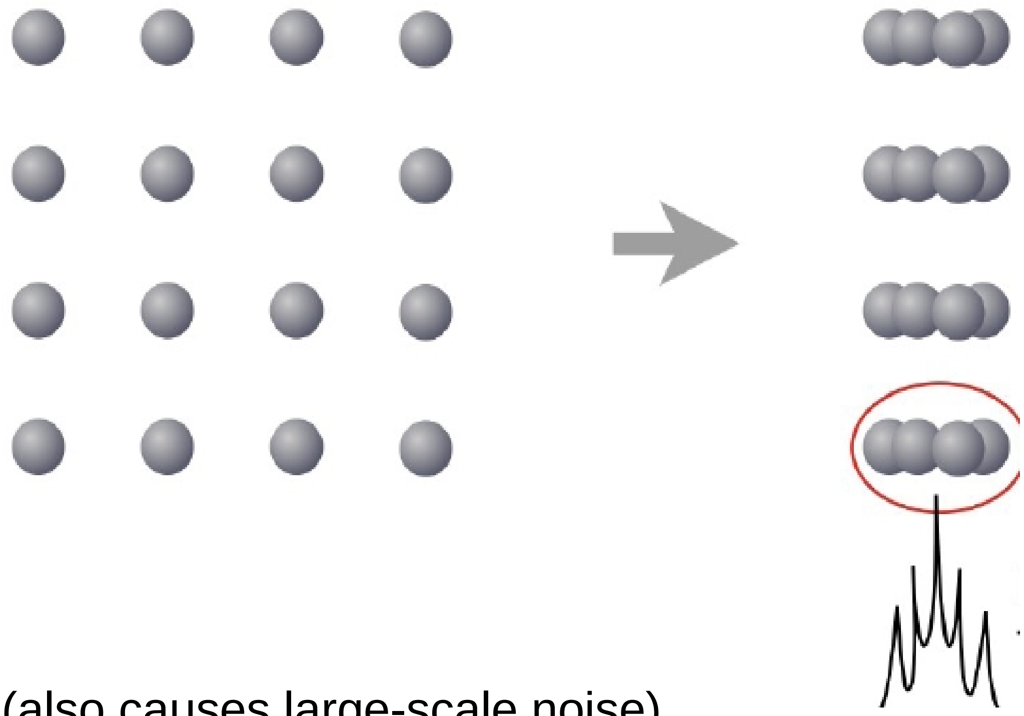
Collisionless Relaxation

Phase Mixing
Chaotic Mixing
Violent Relaxation
Landau Damping

Softening Length

It prevents forces to diverge, which would lead to large-angle scattering events

Anisotropic compression in triaxial collapse

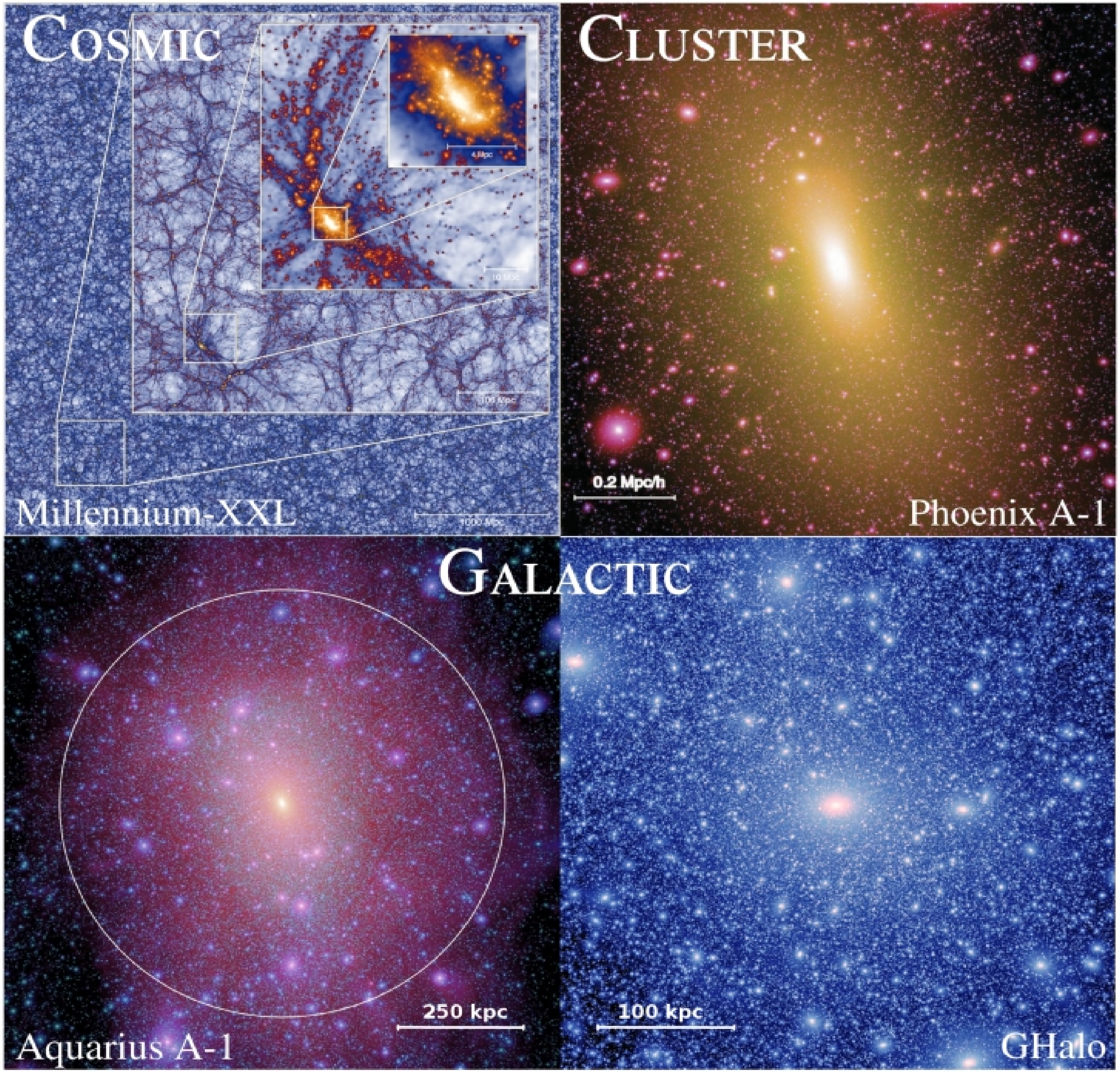


(also causes large-scale noise)

→ Dark Matter Simulations

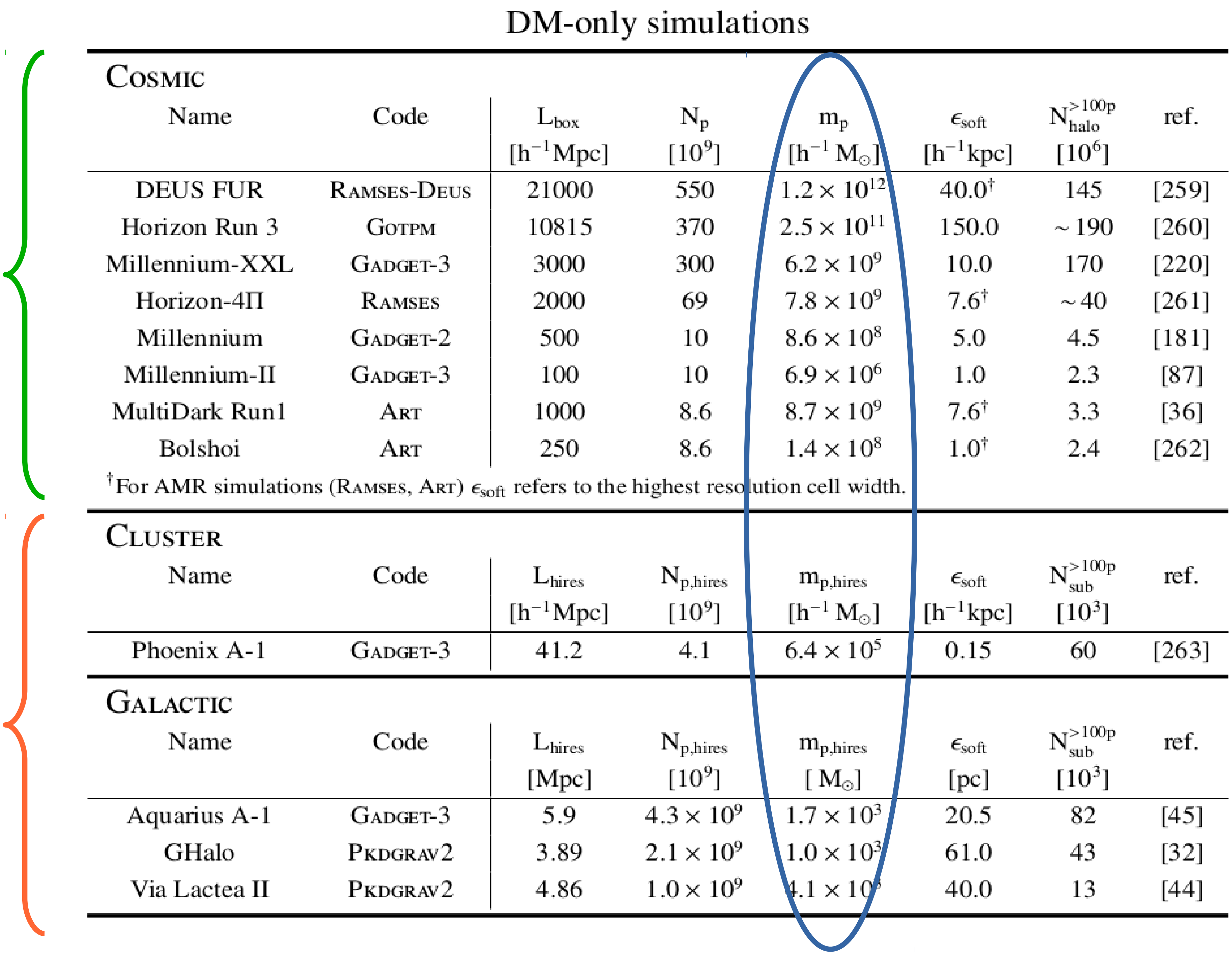
→ Current State of the Art

→ The next decade



Full Box

Zoom In



DM-only simulations

COSMIC

Name	Code	L_{box} $[h^{-1}Mpc]$	N_p $[10^9]$	m_p $[h^{-1} M_\odot]$	ϵ_{soft} $[h^{-1}kpc]$	$N_{halo}^{>100p}$ $[10^6]$	ref.
DEUS FUR	RAMSES-DEUS	21000	550	1.2×10^{12}	$40.0^\dagger$	145	[259]
Horizon Run 3	GOTPM	10815	370	2.5×10^{11}	150.0	~ 190	[260]
Millennium-XXL	GADGET-3	3000	300	6.2×10^9	10.0	170	[220]
Horizon-4Π	RAMSES	2000	69	7.8×10^9	$7.6^\dagger$	~ 40	[261]
Millennium	GADGET-2	500	10	8.6×10^8	5.0	4.5	[181]
Millennium-II	GADGET-3	100	10	6.9×10^6	1.0	2.3	[87]
MultiDark Run1	ART	1000	8.6	8.7×10^9	$7.6^\dagger$	3.3	[36]
Bolshoi	ART	250	8.6	1.4×10^8	$1.0^\dagger$	2.4	[262]

†For AMR simulations (RAMSES, ART) ϵ_{soft} refers to the highest resolution cell width.

CLUSTER

Name	Code	L_{hires} $[h^{-1}Mpc]$	$N_{p,hires}$ $[10^9]$	$m_{p,hires}$ $[h^{-1} M_\odot]$	ϵ_{soft} $[h^{-1}kpc]$	$N_{sub}^{>100p}$ $[10^3]$	ref.
Phoenix A-1	GADGET-3	41.2	4.1	6.4×10^5	0.15	60	[263]

GALACTIC

Name	Code	L_{hires} $[Mpc]$	$N_{p,hires}$ $[10^9]$	$m_{p,hires}$ $[M_\odot]$	ϵ_{soft} $[pc]$	$N_{sub}^{>100p}$ $[10^3]$	ref.
Aquarius A-1	GADGET-3	5.9	4.3×10^9	1.7×10^3	20.5	82	[45]
GHalo	PKDGRAV2	3.89	2.1×10^9	1.0×10^3	61.0	43	[32]
Via Lactea II	PKDGRAV2	4.86	1.0×10^9	4.1×10^3	40.0	13	[44]

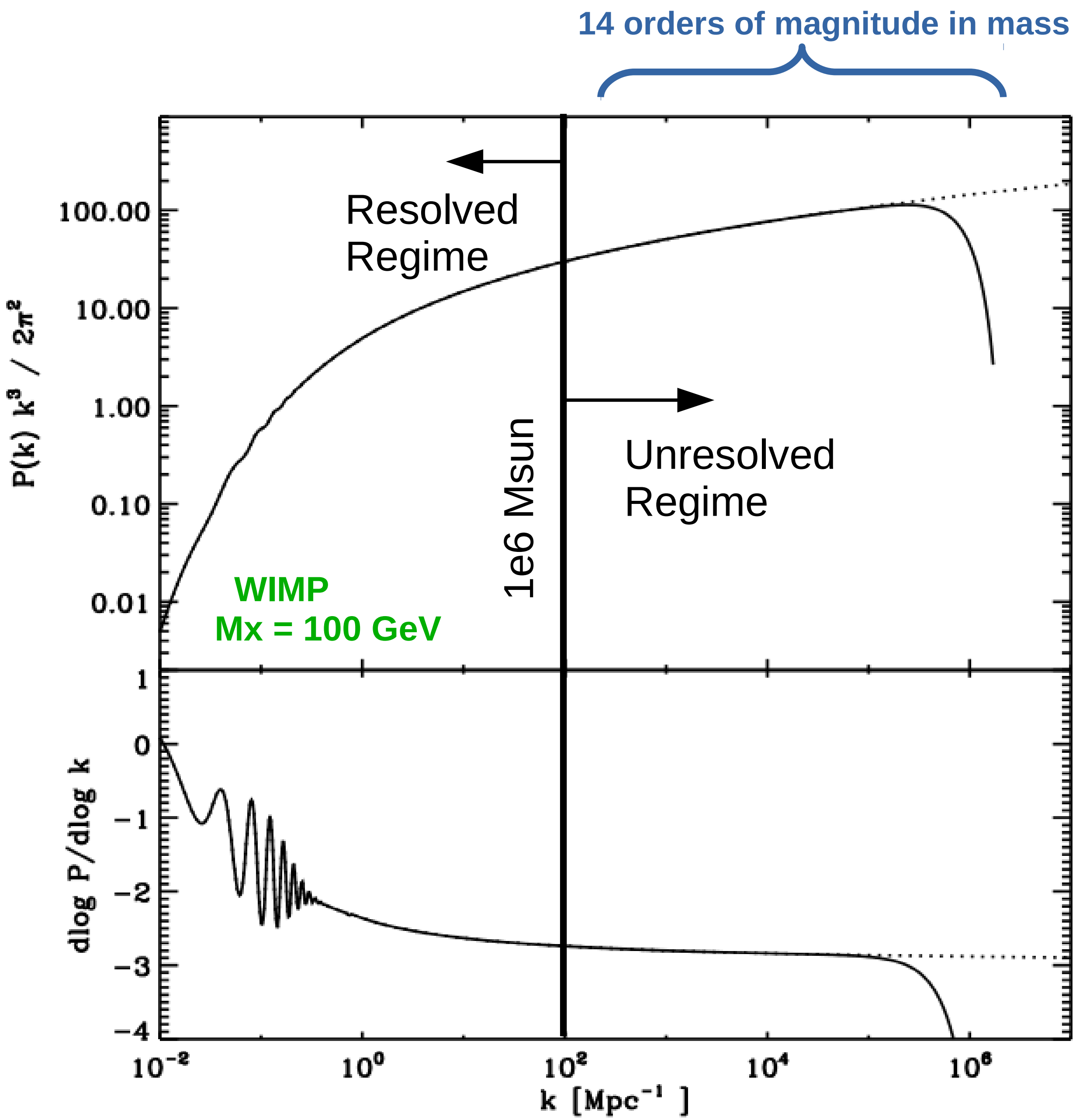
> 1 trillion particles

> 1 billion particles

Kuhlen, Vogelsberger, Angulo 2012

Current State of the Art

A simulation of the full DM hierarchy might require 1e21 particles



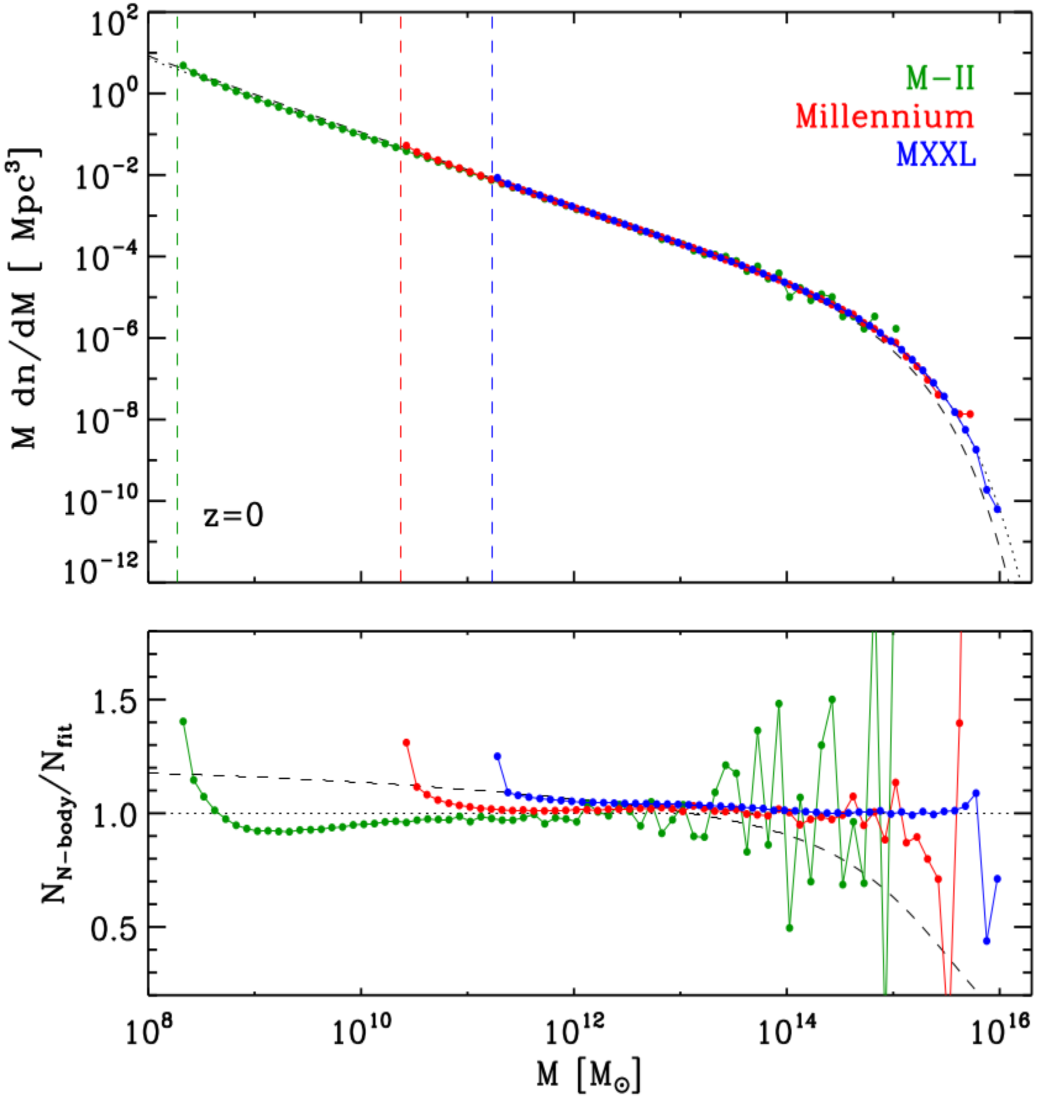
Angulo & White 2010

- → Dark Matter Simulations
- → Current State of the Art
  What we know
- → The next decade

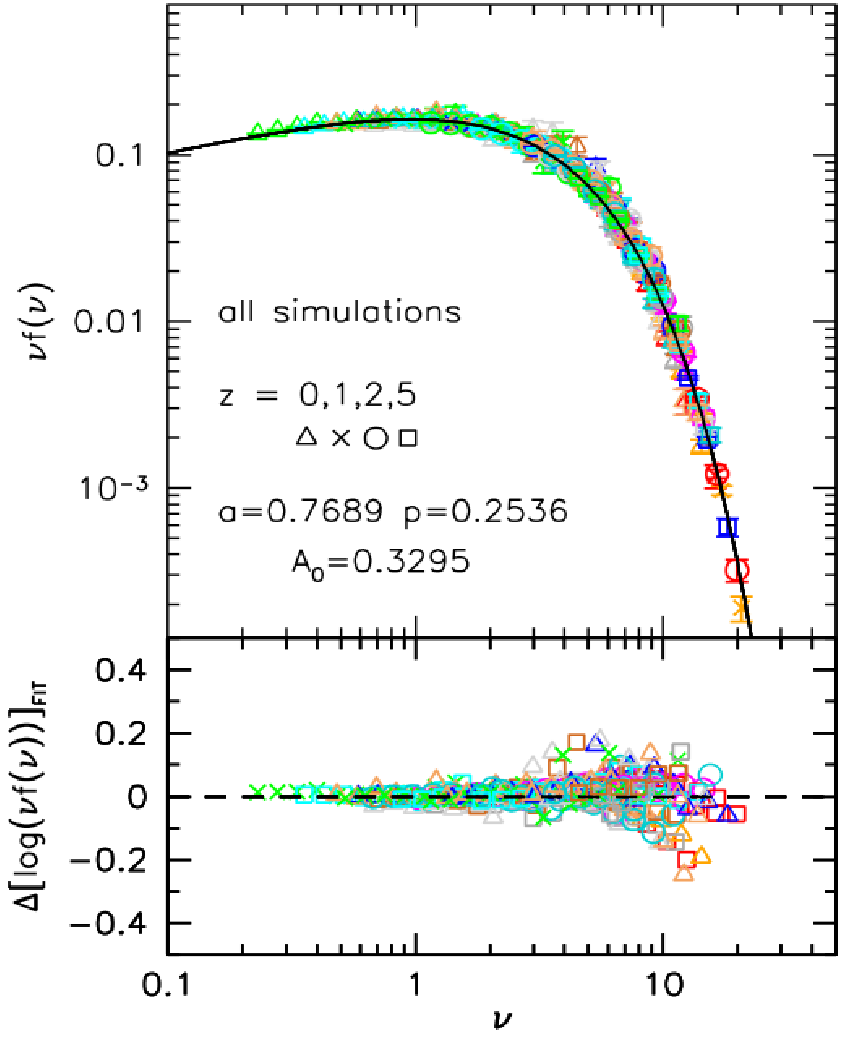
The abundance of CDM collapsed structures

Simulations resolve the mass range relevant for galaxy formation
If written in the adequate variables, the abundance is universal

dn/dM ~ $M^{-1.9}$ (at small masses)

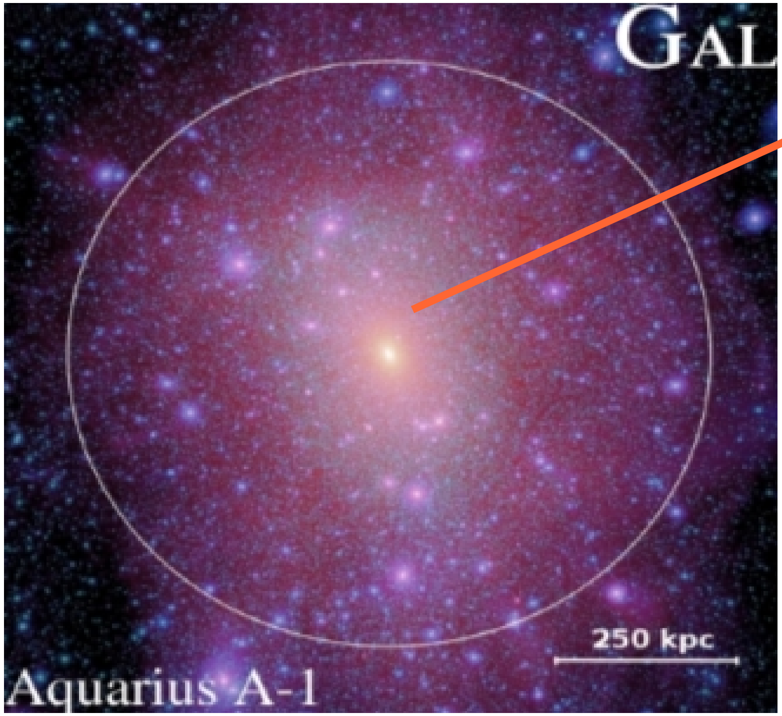


Angulo et al 2012



Despali et al 2015

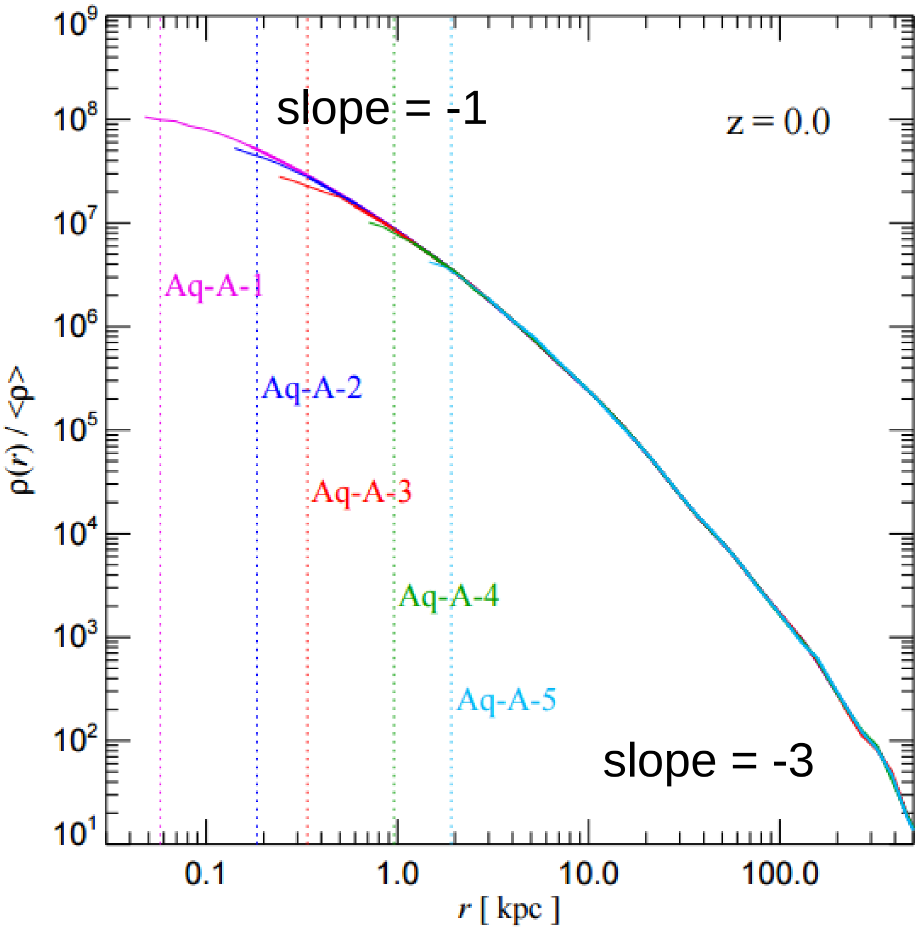
The inner structure of Dark Matter halos



Springel et al 2008

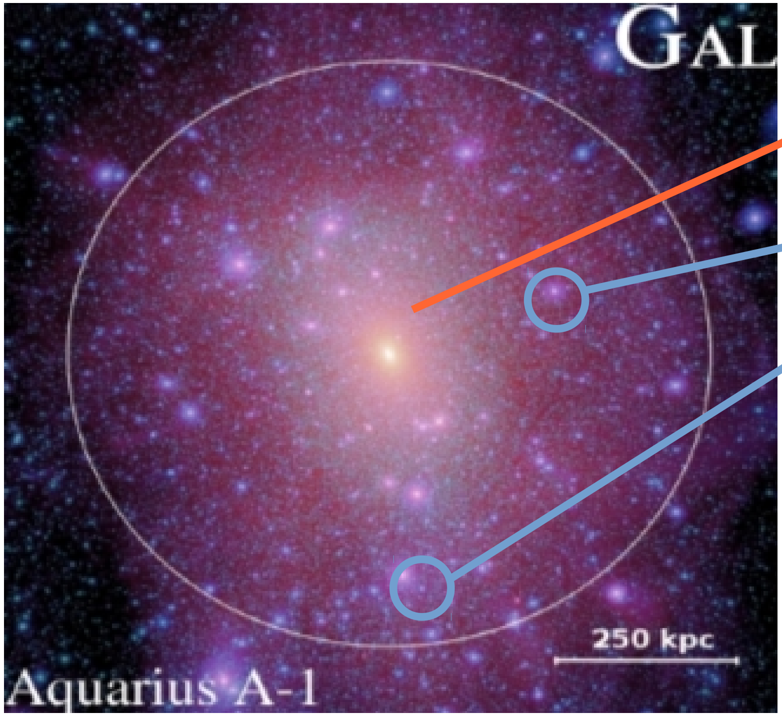
Smooth distribution

Density profile is described by NFW/Einasto functional form, independent of mass



Springel et al 2008

The inner structure of Dark Matter halos

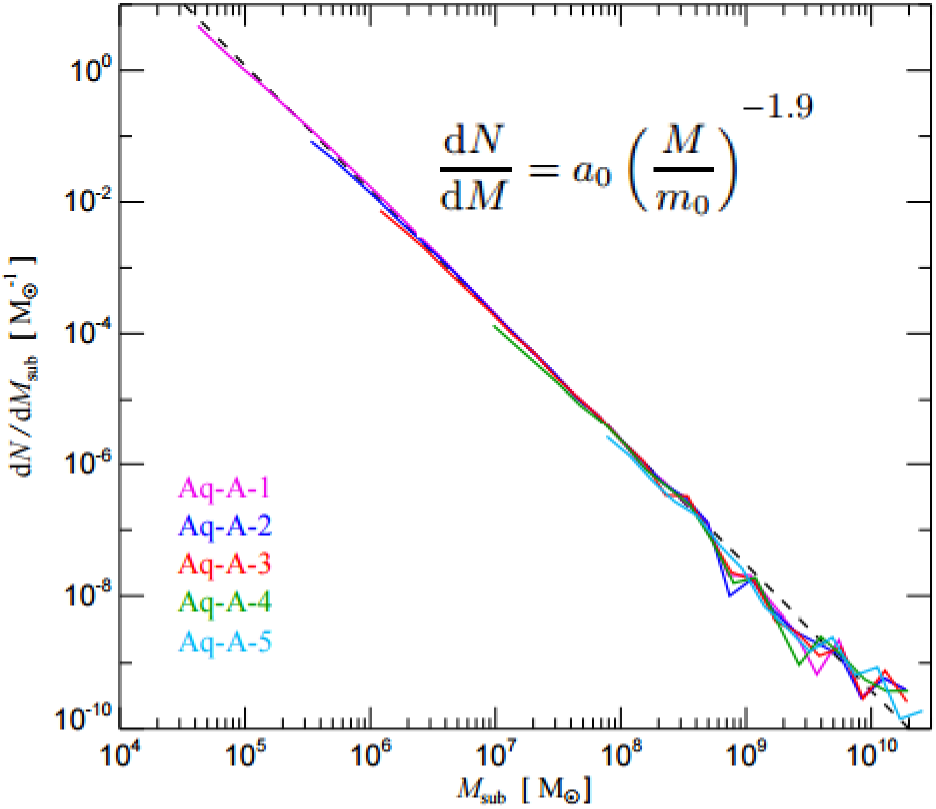


Springel et al 2008

Smooth distribution

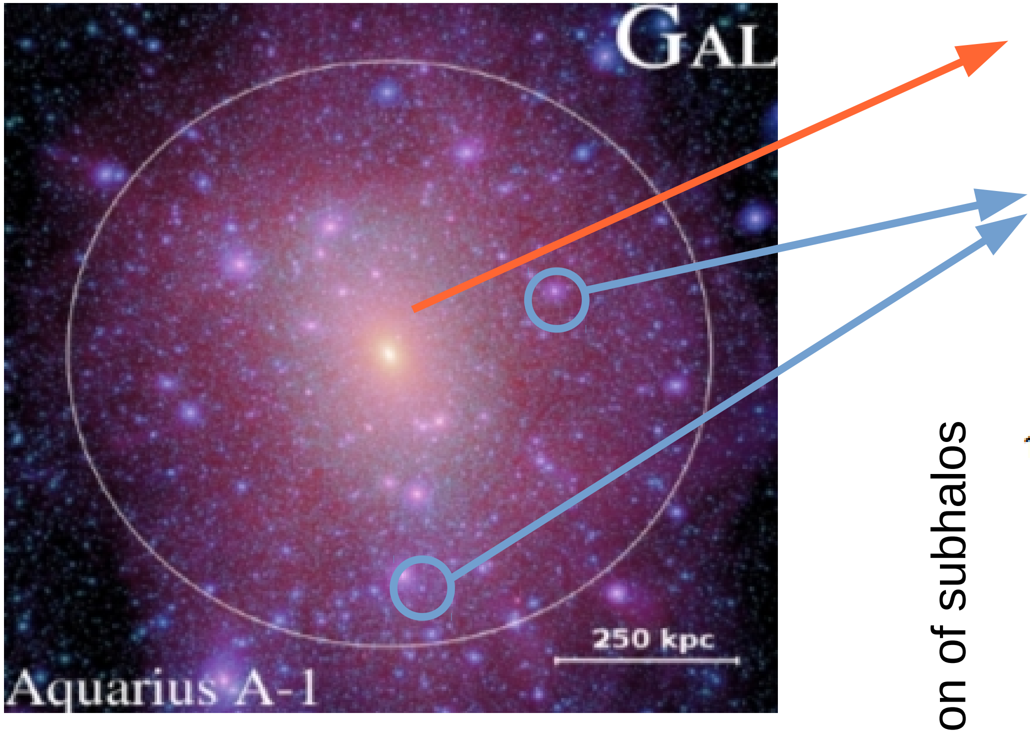
Hierarchy of substructures

→ **Abundance**



Springel et al 2008

The inner structure of Dark Matter halos

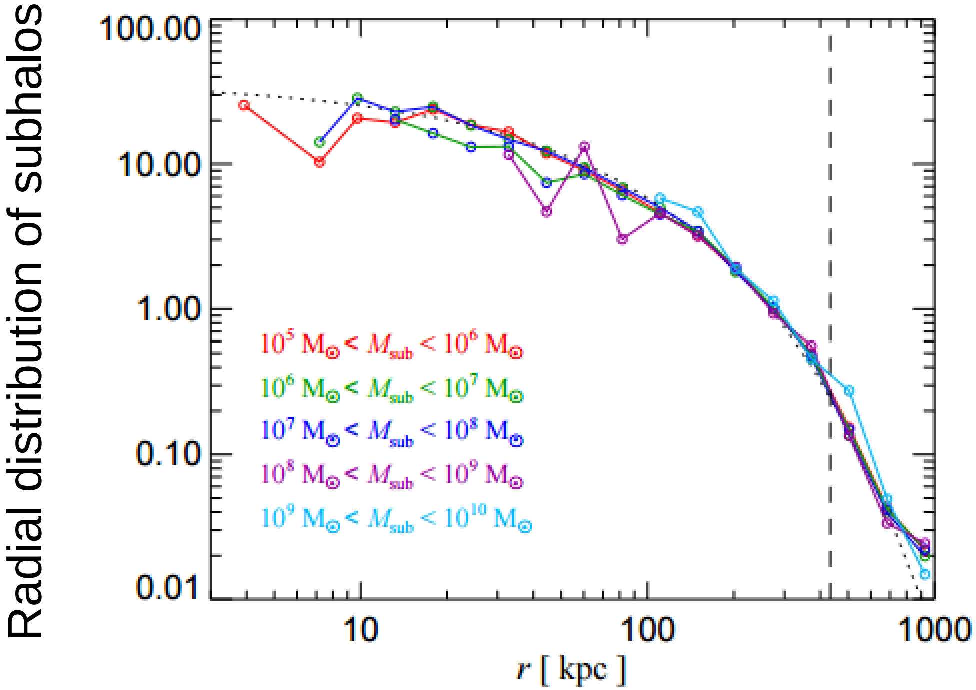


Springel et al 2008

Smooth distribution

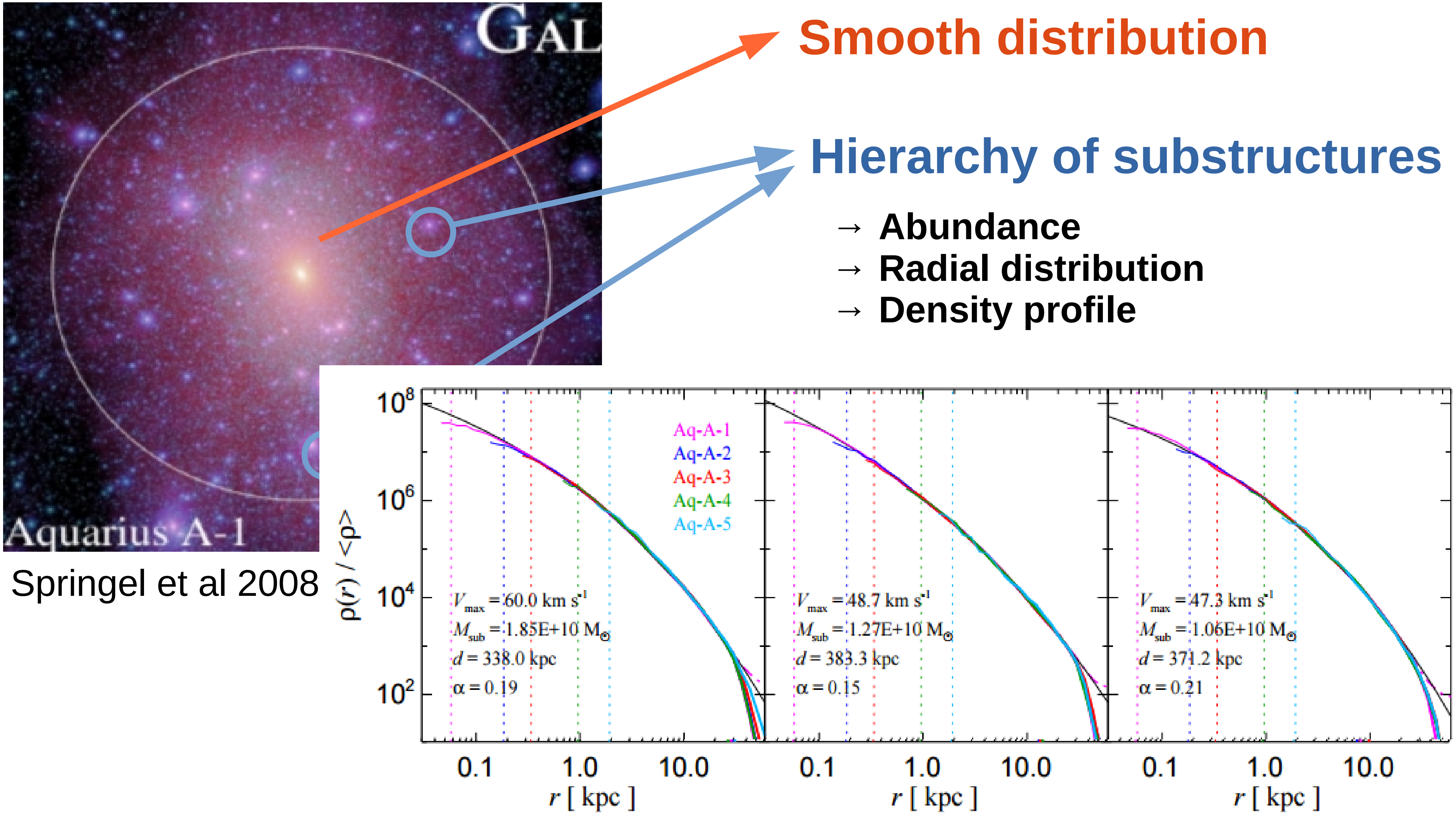
Hierarchy of substructures

→ **Abundance**
→ **Radial distribution**



Springel et al 2008

The inner structure of Dark Matter halos



Smooth distribution

Hierarchy of substructures

- → **Abundance**
- → **Radial distribution**
- → **Density profile**

Springel et al 2008

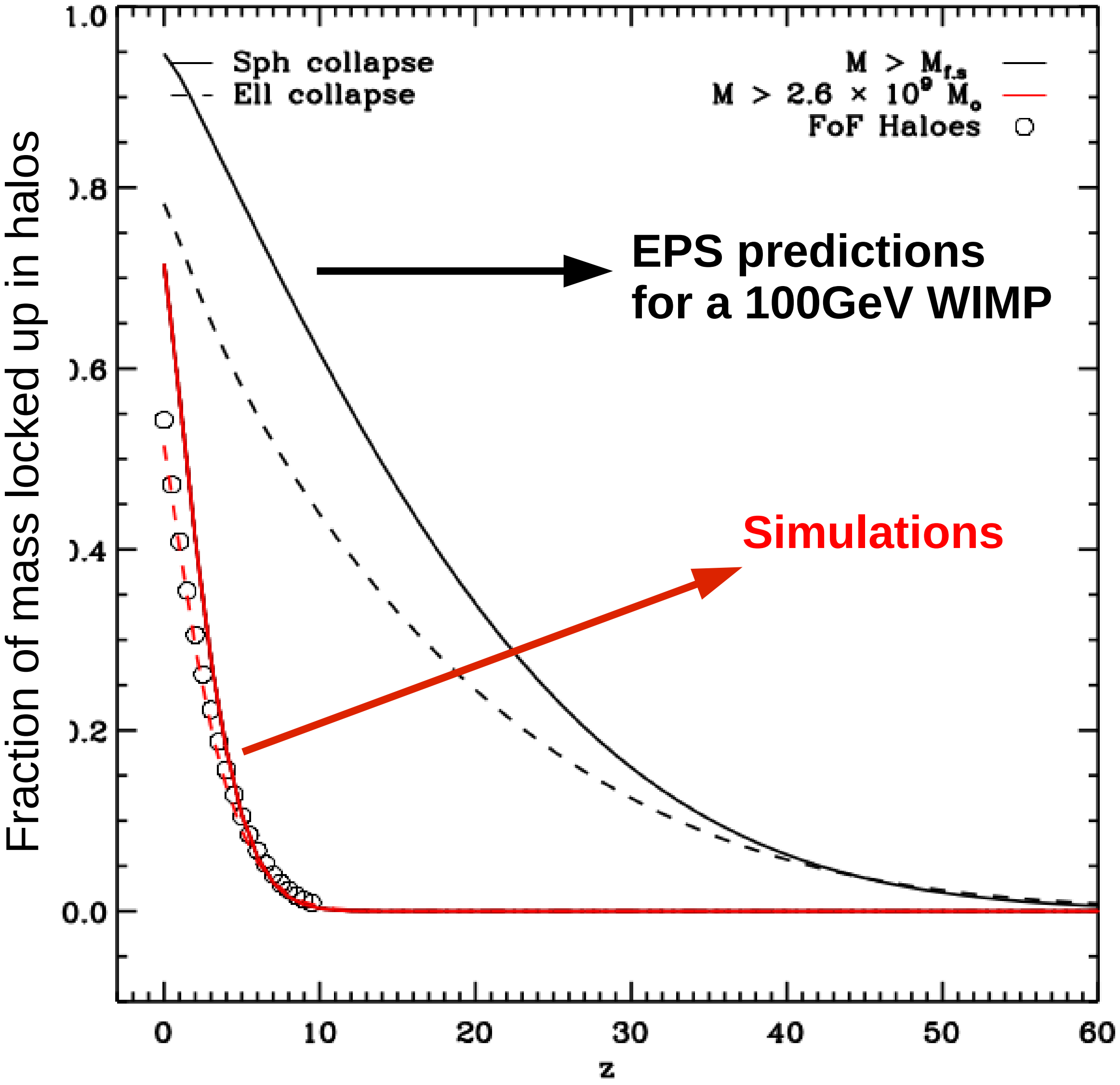
- → Dark Matter Simulations

- → Current State of the Art
  What we don't know

- → The next decade

The abundance of CDM collapsed structures

Numerical simulations only resolve a fraction of the CDM halos



EPS predictions for a 100GeV WIMP

Simulations

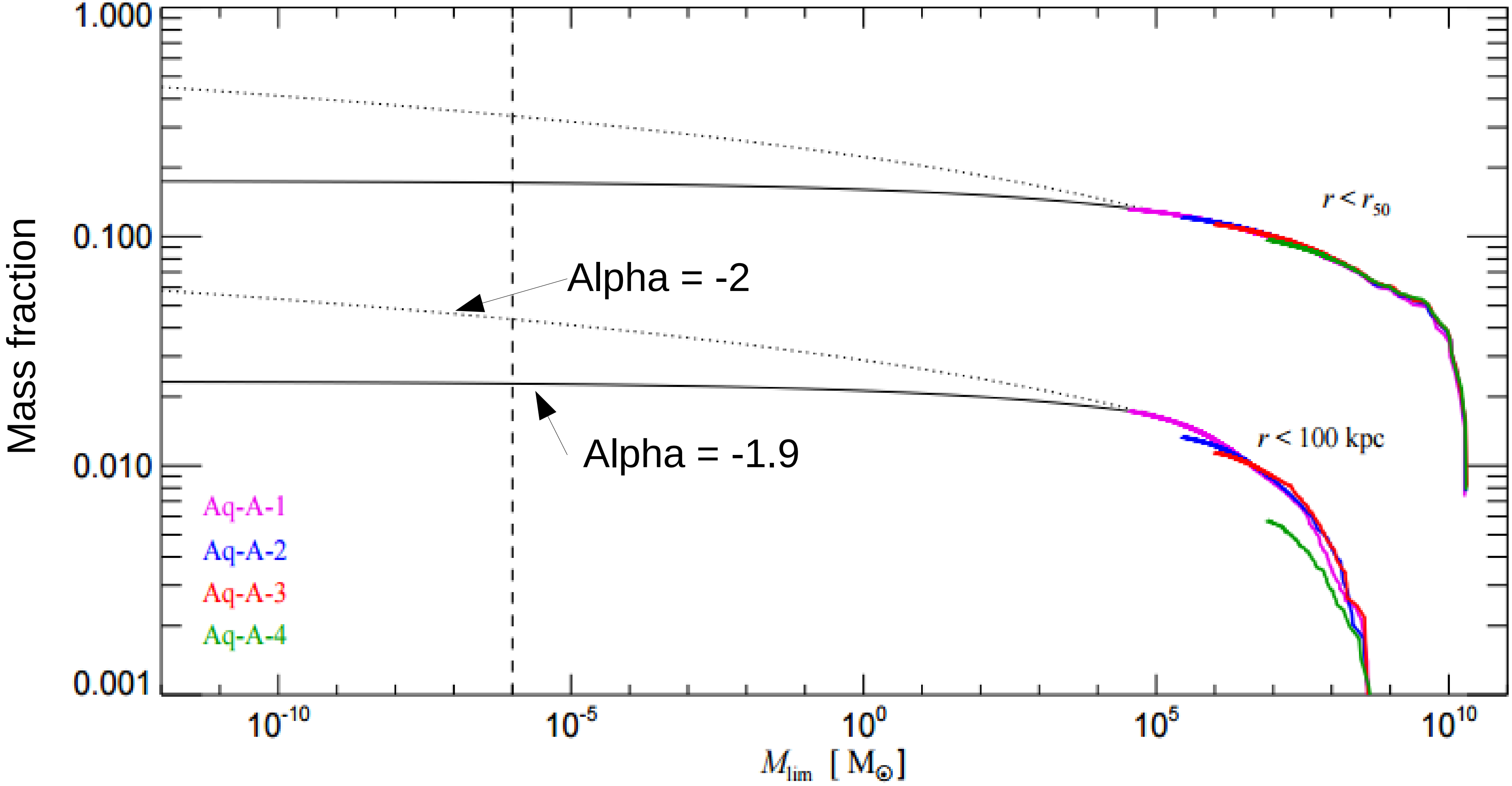
Angulo & White 2010

20% of the mass in the Universe is still diffuse today!

Most of the mass is still diffuse by redshift 10

Mass fraction in subhalos of a MW halo

Uncertainties in the extrapolation introduce a factor of 2 in the Amount of mass in substructures for a 100GeV WIMP



Alpha = -2

Alpha = -1.9

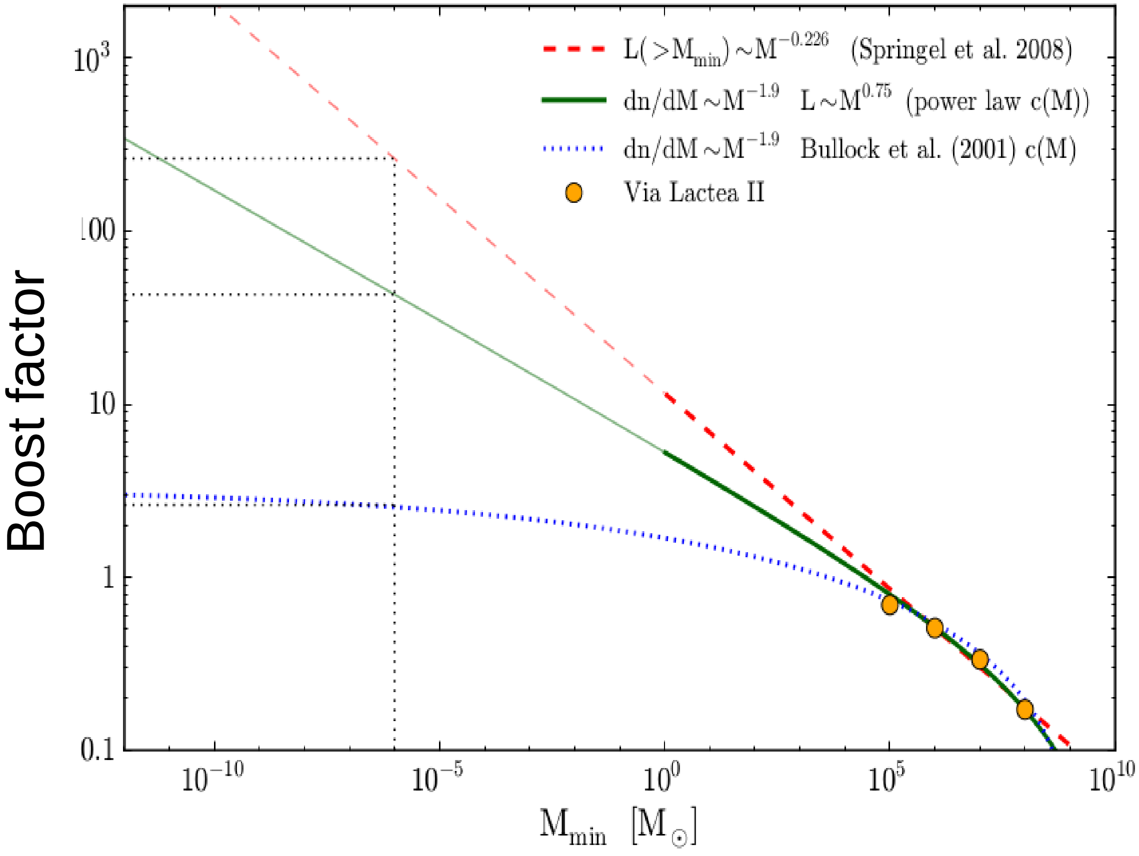
Springel et al 2008

DM annihilation boosts

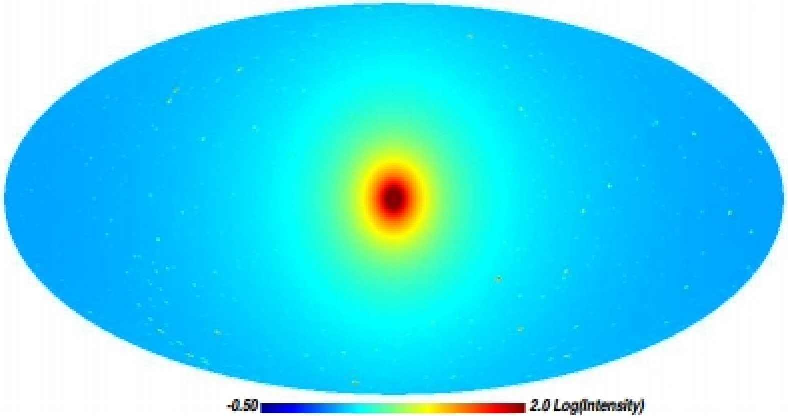
Annihilation rate scales as density squared, thus any unresolved clumpiness (e.g. subhalos) should result into an enhanced signal

$$\mathscr{L}(\mathbf{x}) = \mathscr{G}(\text{particle physics, observational setup})\,\rho^2(\mathbf{x}),$$

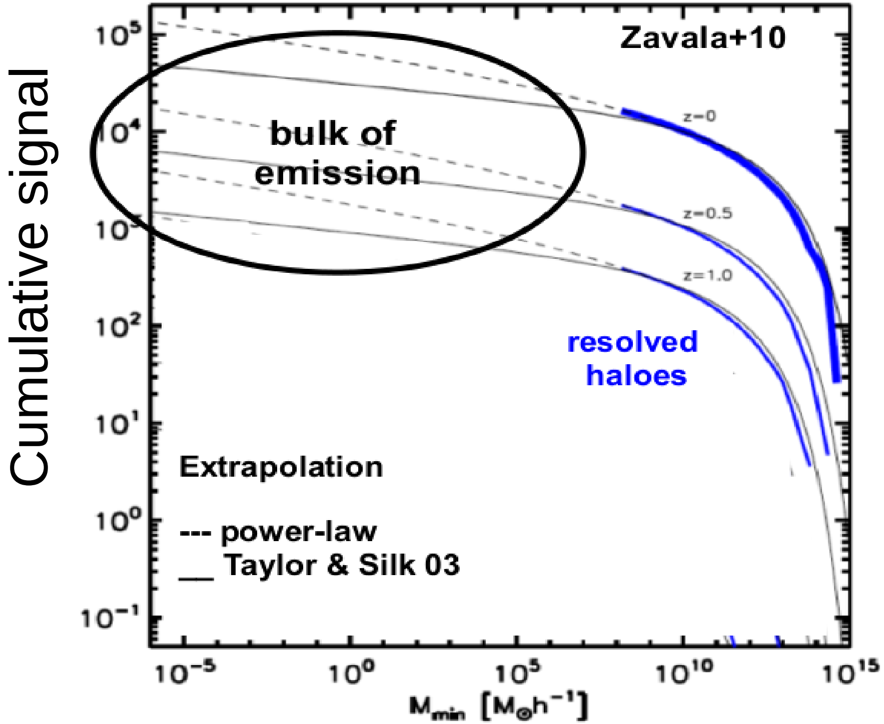
Substructure boost



Kuhlen, Vogelsberger, Angulo 2012



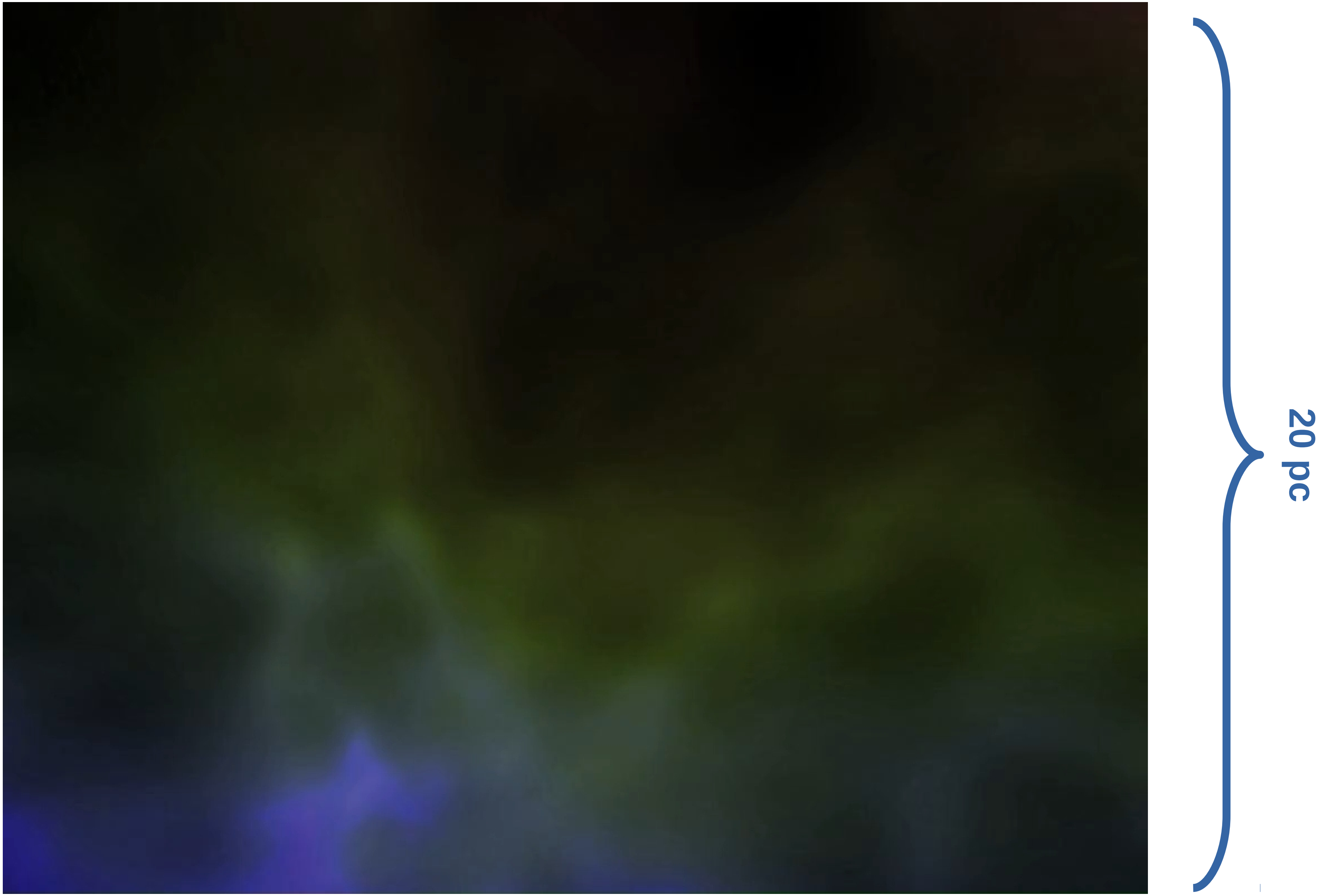
Cosmic halo boost



Structure formation at the free streaming mass

Angulo et al (in prep)

Zoom into the formation of a 0.01 Msun halo



Angulo et al (in prep)

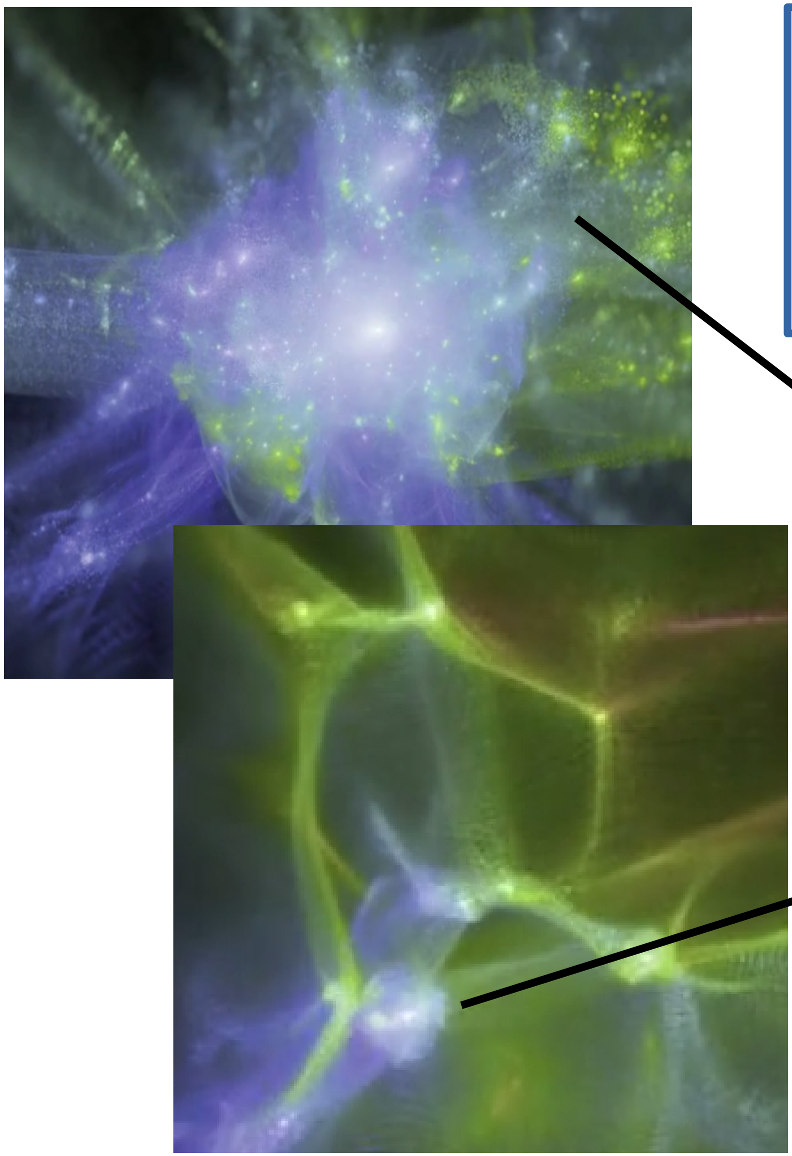
Structure formation at the free streaming mass

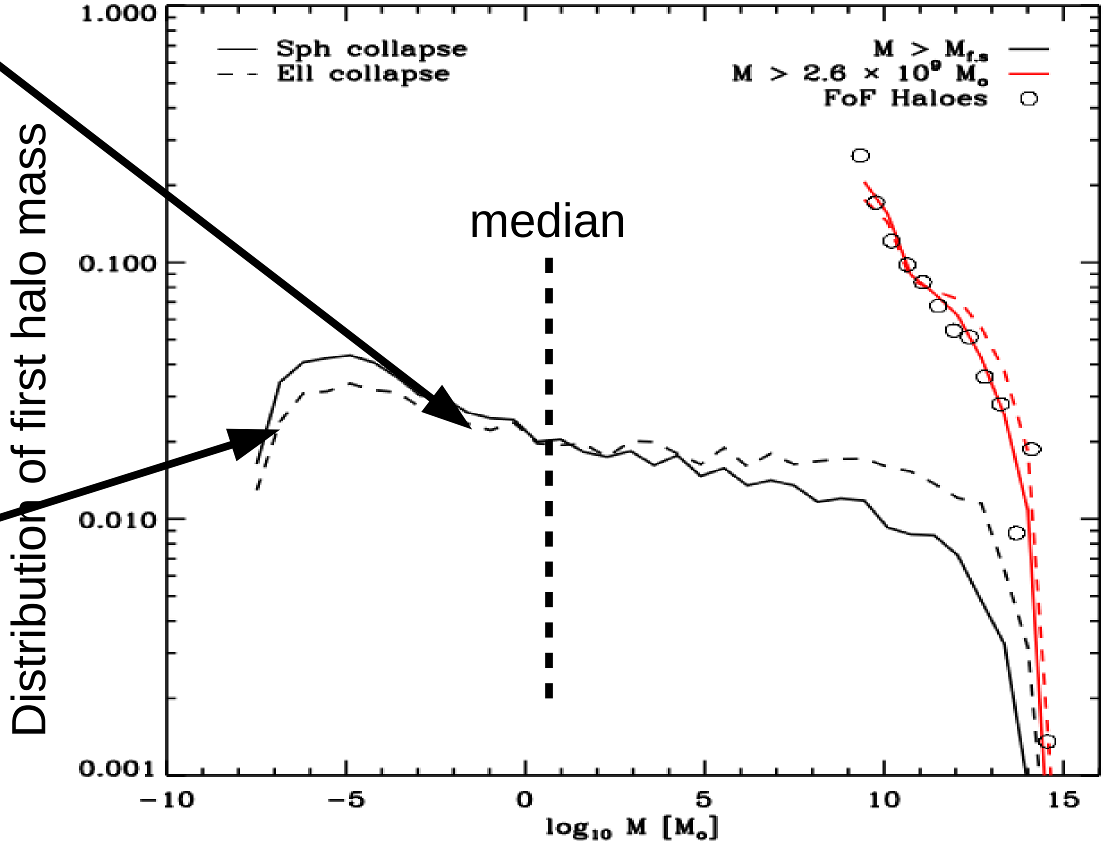
The flattening of the mass variance implies a very diverse First-generation halo mass distribution

→ First generation halos of most particles is not at high z not it happens at the free streaming mass

→ 10% of the MW halo was accreted diffused

→ Most of the mass experienced 5 accretion events





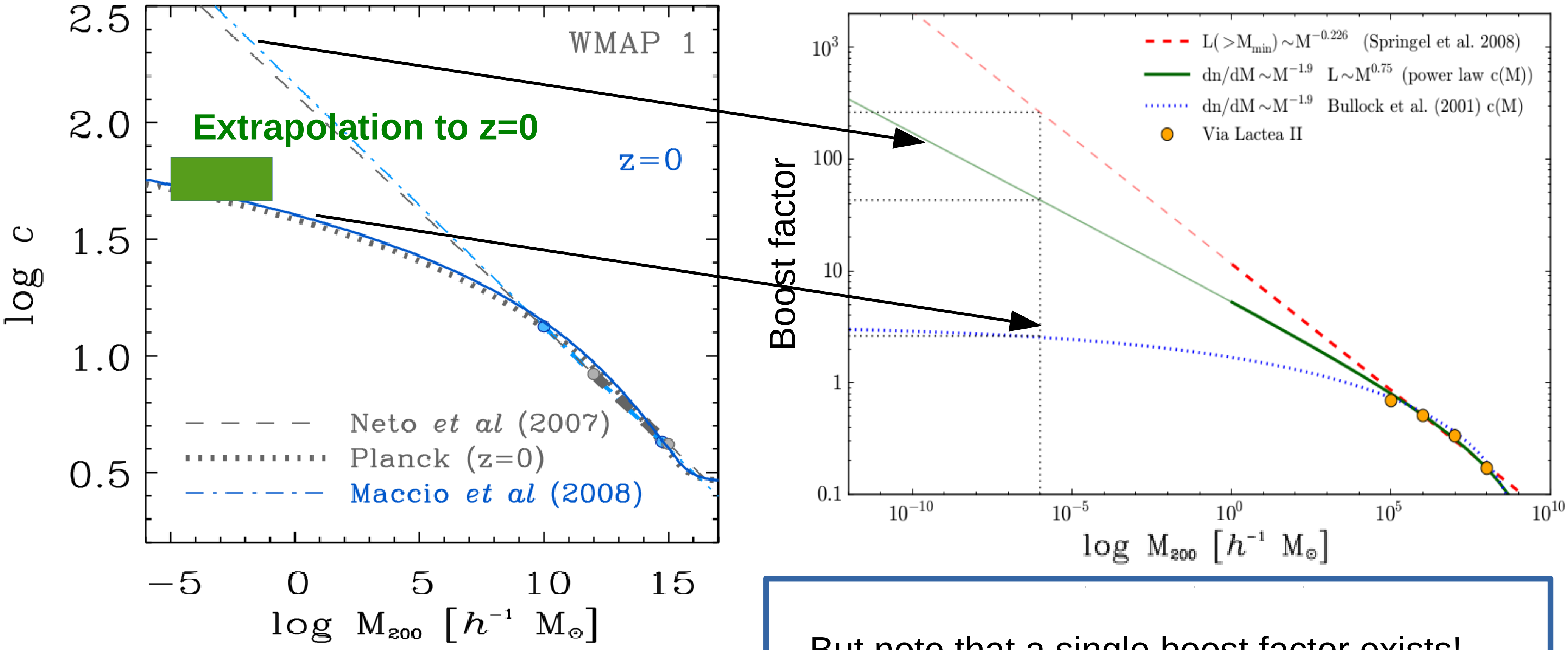
Angulo & White 2010

Structure formation at the free streaming mass

The flattening of the mass variance implies similar formation redshifts and thus concentrations

Ludlow et al 2014

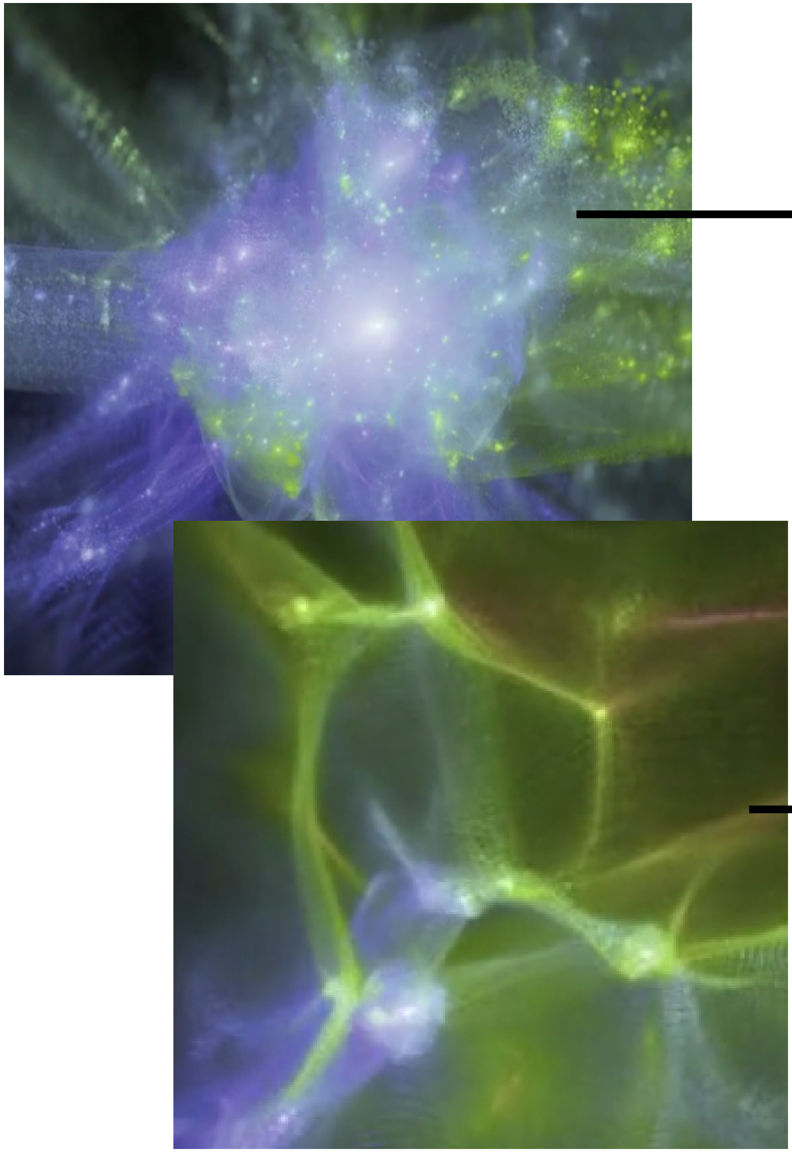
Kuhlen, Vogelsberger, Angulo 2012

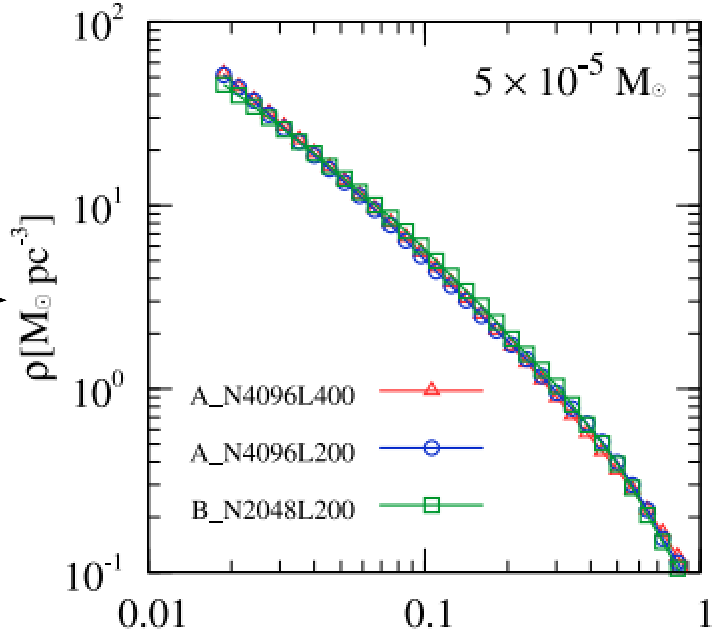


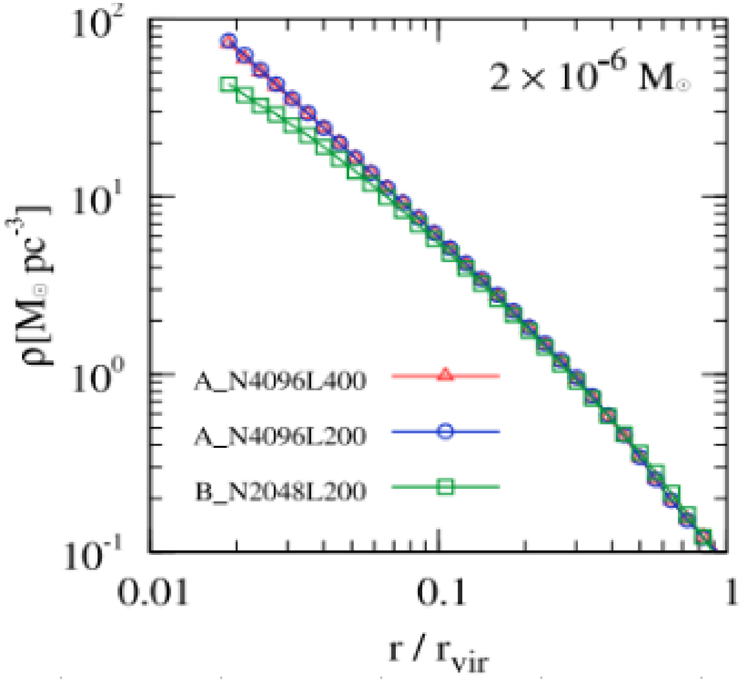
But note that a single boost factor exists!
→ it depends on the host halo mass, distance from the center, etc

Structure formation at the free streaming mass

The density profile of microhalos is steeper than a NFW







Ishiyama 2014

Structure formation at the free streaming mass

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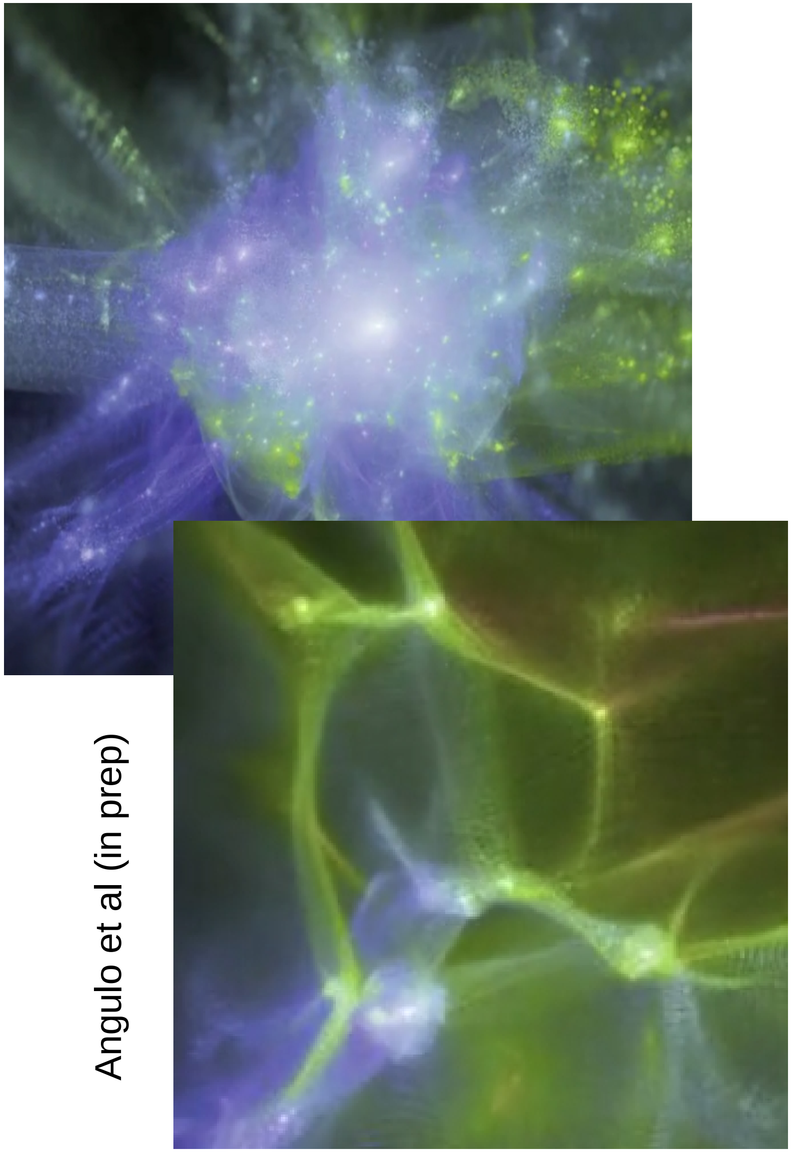
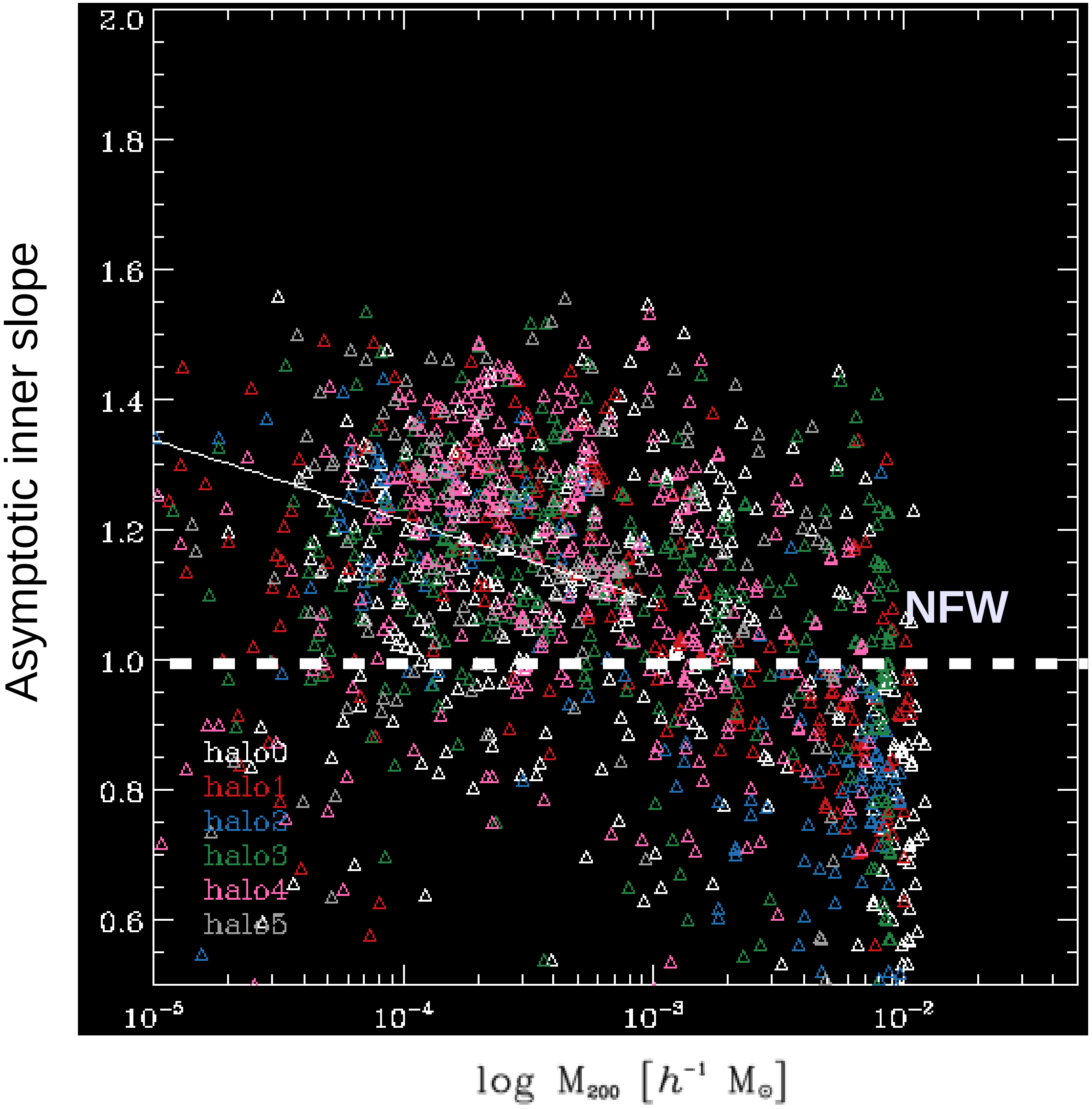


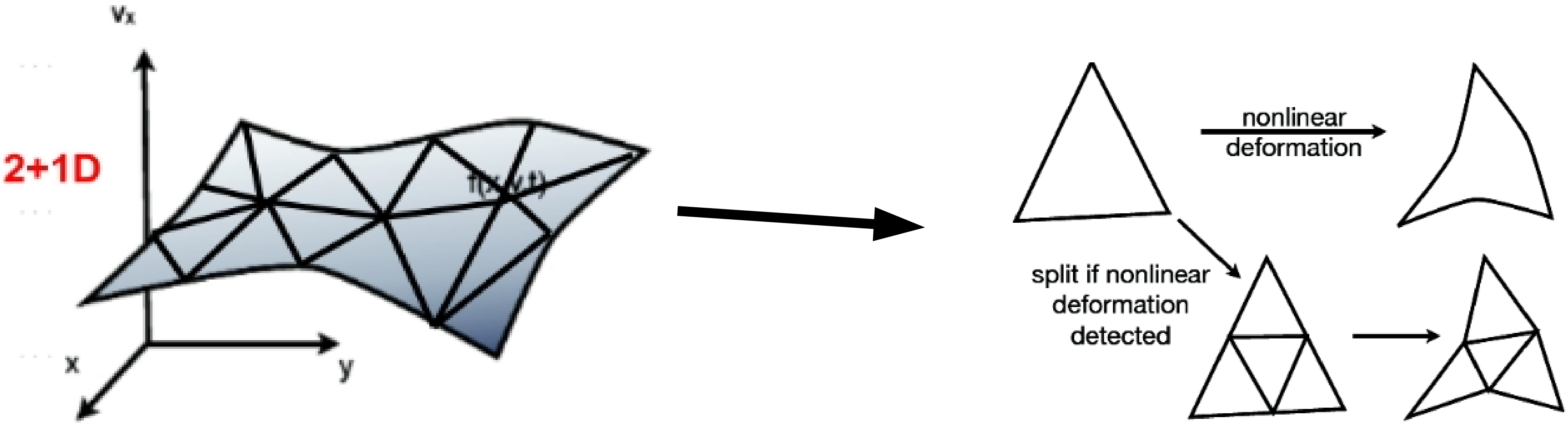
Angulo et al (in prep)



- → Dark Matter Simulations
- → Current State of the Art
- → The next decade

Structure formation in the continuous limit

Discretization of the DM fluid using adaptively refined phase-space lagrangian elements



$$\nabla^2\Phi = \frac{4\pi G}{a}\int f\,\mathrm{d}^3v$$

Advantages

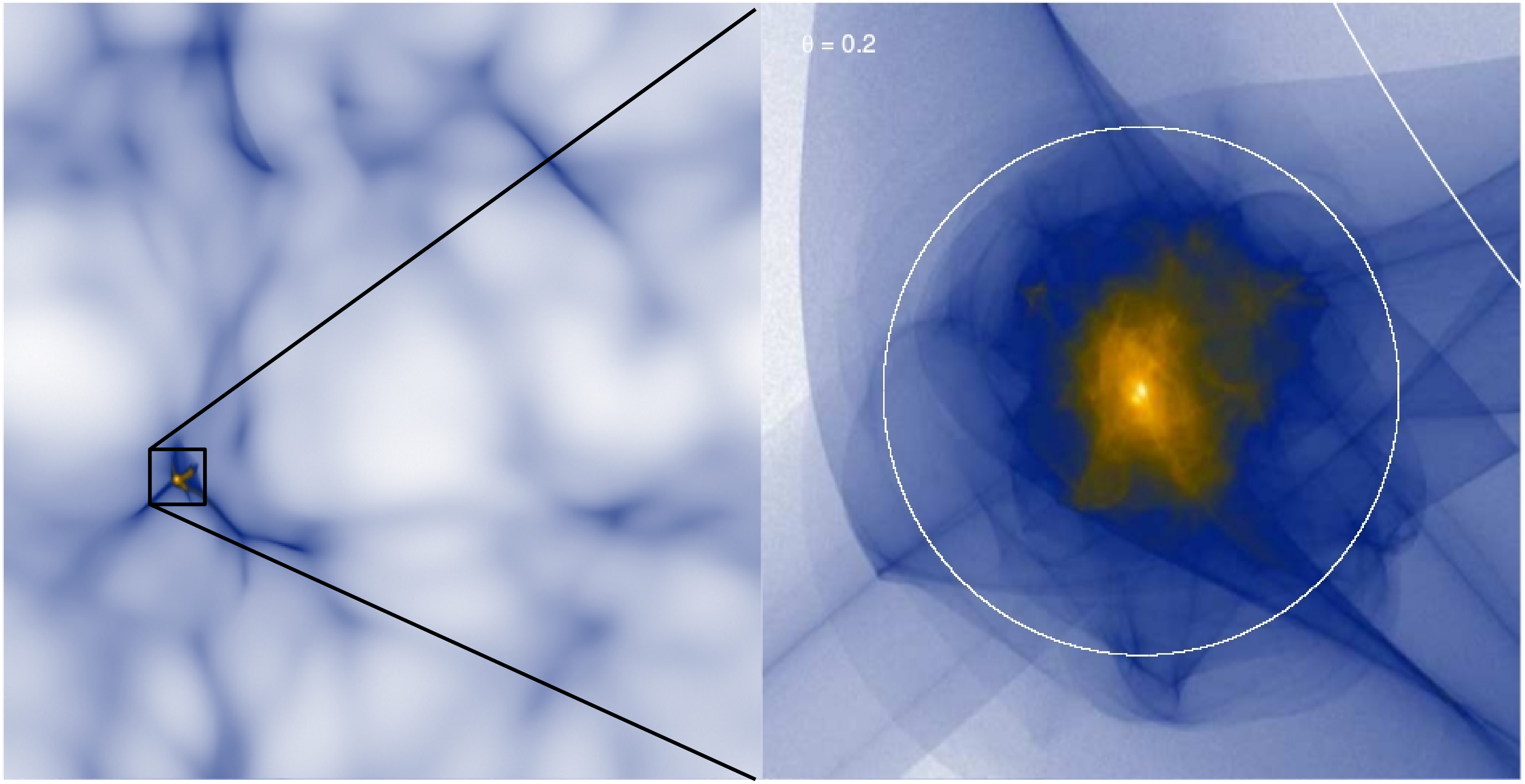
→ Full information about the phase-space structure is retained
→ No discreteness noise (2-body effects)
→ No need for a softening length

Hahn+2013; Angulo, Hahn, Abel 2013b; Hahn & Angulo 2015

The hexahalo Project

Angulo et al (in preparation)

A zoom in simulation of the formation of a MW size halo in WDM With adaptively refined hexahedra



200,000 1.5e7 Msun flow tracers
700,000,000 final (600 Msun) flow tracers
100,000,000,000 (2 Msun) mass carriers
1024^3 + 4093^3 PM force mesh


a = 0.13631347

The next decade...

Observations are way ahead of theory

→ No simulation, even DMO, can simultaneously resolve the volume and host halos of current surveys

→ We have a reasonable theory of galaxy formation and nonlinear structures, but it is computationally slow...

The next decade...

Can we resolve the full hierarchy of structures for a 100GeV WIMP?

→ **Maybe, after 2050...**

→ Resolve the kinematic of stars in the smallest dwarf galaxies
→ Understand the subhalo/cosmic boost factor
→ Improved predictions for the phase-space structure
→ Improved modelling of the microphysical properties of DM

The next decade...

Can we resolve the full hierarchy of structures for a 100GeV WIMP?

→ **Maybe, after 2050...**

→ Resolve the kinematic of stars in the smallest dwarf galaxies
→ Understand the subhalo/cosmic boost factor
→ Improved predictions for the phase-space structure
→ Improved modelling of the microphysical properties of DM

The impact of hydrodynamics

→ Under what conditions do baryons affect the central density of galaxies
→ What is the net effect for the survival of DM substructure?
→ How realistic are current implementations of stellar/AGN feedback (hydrodynamical decoupling, energy injection) of what happens in molecular clouds?
→ Better treatment of radiation/non-thermal pressure support, non resolved turbulence, etc.