A halo-independent lower bound on the DM capture rate in the Sun from a DD signal

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Motivation
Motivation: derive a new HI framework for DD and capture

Uncertainties on $f(v)$, $\rho$ are crucial on interpretation of DM signals.

1 Many studies:
   - For DD [McCabe, Frandsen, Drees, Savage...]. Talk by N. Bozorgnia.
   - For $C_{\text{Sun}}$ [Bruch, Choi...].

2 Also halo-independent (HI) methods:
   - For DD: for fixed $m_\chi$, compare $\tilde{\eta}(v_m, t)$ (detector independent) in same $v_m$ range [Fox, Del Nobile, Bozorgnia, Feldstein, JHG...]. Talk by P. Gondolo.
   - HI upper bounds from DD and $\Gamma_{\text{Sun}}$ [Ferrer...]. Talks by S. Wild.
   - New HI framework to compare DD with $\rho_\chi$, LHC, $\Omega_\chi$, and indirect detection [Blennow, JHG, Ferrer]. Talk by S. Vogl.

Here new HI framework for comparing a positive DD signal with $\Gamma_{\text{Sun}}$:

- We use that both the DD signal and $C_{\text{Sun}}$ depend on $\sigma_{\text{SI/SD}}$.
- However, the velocities probed by both are very different.
Direct detection and the capture rate in the Sun
Direct detection: there is a minimum velocity $v_m$

Goodman, Drukier, Freese...

- For elastic SI interactions the event rate can be expressed as

$$\mathcal{R}(E_R, t) = A^2 F_A^2(E_R) \tilde{\eta}(v_m, t),$$

where

$$\tilde{\eta}(v_m, t) \equiv C \int_{v_m}^{\infty} dv v \tilde{f}_{\text{det}}(v, t), \quad \text{with} \quad C \equiv \frac{\rho \chi \sigma_{\text{SI/SD}}}{2 m \chi \mu p^2}.$$

- $v > v_m$ by kinematics, with:

$$v_m = \sqrt{\frac{m_A E_R}{2 \mu_s^2 \chi_A}}.$$
Capture: there is a maximum velocity $v_{\text{cross}}$ (shown H, SD)

For the DM to be captured there is a minimal and maximal $E_R$:

$$E_{\text{min}} = \frac{m_\chi}{2} v^2, \quad E_{\text{max}} = \frac{2\mu^2}{m_A} (v^2 + u_{\text{esc}}^2(r)).$$

These define the maximum velocity to be trapped:

$$v^A_{\text{cross}}(r) = \frac{\sqrt{4m_A m_\chi}}{|m_\chi - m_A|} u_{\text{esc}}(r).$$

![Graph showing $E_R$ and $v$ vs. $m_\chi = 10\text{ GeV}$, with $E_{\text{max}}(u_{\text{esc}} = 1381 \text{ km s}^{-1})$ and $E_{\text{max}}(u_{\text{esc}} = 618 \text{ km s}^{-1})$.](image)
A halo-independent lower bound on the DM capture rate in the Sun

Overlap in velocity \((H, SD)\) and assumptions of the bound

Gould, Edsjo, Kavanagh, Blennow...

**Overlap in** \(v_{\text{thr}} < v < v_{\text{cross}}^A(r)\):
- DD is sensitive to \(v > v_{\text{thr}}\).
- Capture for \(v < v_{\text{cross}}^A(r)\).

We can derive a lower bound on the capture, assuming:

1. We use that \(v_e \approx 29\ \text{km/s} \ll v_m\) so that:

\[
\tilde{f}_{\text{det}}(v) = \tilde{f}_{\text{Sun}}(v + v_e) \approx \tilde{f}_{\text{Sun}}(v) \equiv \tilde{f}(v).
\]

2. \(f(v), \rho_\chi\) constant so they are equal for the capture and for DD.
A lower bound on the capture

\[ C_{\text{Sun}} = 4\pi C \sum_A A^2 \int_0^{R_S} dr r^2 \rho_A(r) \int_{v_{\text{cross}}}^{v_A} dv \tilde{f}(v) v \mathcal{F}_A(v, r) \]

\[ \geq 4\pi C \sum_A A^2 \int_0^{R_S} dr r^2 \rho_A(r) \int_{v_{\text{thr}}}^{v_{\text{cross}}} dv \tilde{f}(v) v \mathcal{F}_A(v, r). \]

From a perfectly measured DD spectrum one can extract:

\[ C \tilde{f}(v) = - \frac{1}{vA^2} \frac{d}{dv} \left( \frac{\mathcal{R}(E_R)}{F_A^2(E_R)} \right) \]

- The bound on \( C_{\text{Sun}} \) can be expressed in terms of DD quantities.
- It is independent of \( f(v), v_{\text{esc}}, \sigma_{\text{SI/SD}} \) and \( \rho_X \).
Equilibrium between capture and annihilations in the Sun

- DM obeys ($A_{\text{Sun}}$ annihilation rate, no evaporation for $m_\chi \gtrsim 3.5$ GeV):
  \[ \frac{dN}{dt} = C_{\text{Sun}} - A_{\text{Sun}} N^2. \]

- Equilibrium ($dN/dt = 0$) occurs for $t_{\text{eq}} \ll t_{\text{Sun}} \sim 4.5$ Gyr, where:
  \[ t_{\text{eq}} = \frac{1}{\sqrt{C_{\text{Sun}} A_{\text{Sun}}}} \approx \approx 0.5 \text{ Gyr} \left( \frac{10^{21} \text{ s}^{-1}}{C_{\text{Sun}}} \right)^{1/2} \left( \frac{3 \cdot 10^{-26} \text{ cm}^3 \text{ s}^{-1}}{\langle \sigma v \rangle_{\text{ann}}} \right)^{1/2} \left( \frac{100 \text{ GeV}}{m_\chi} \right)^{3/4}, \]
  
  where $\langle \sigma v \rangle_{\text{ann}}$ is the thermally-averaged annihilation cross section.

- For $C_{\text{Sun}} \lesssim 10^{21}$ s$^{-1}$, no equilibrium reached if $\langle \sigma v \rangle_{\text{ann}} < \langle \sigma v \rangle_{\text{fo}}$.

- If it is reached $\Gamma_{\text{Sun}}$ given solely in terms of $C_{\text{Sun}}$:
  \[ \Gamma_{\text{Sun}}^{\text{eq}} = \frac{N^2}{2} A_{\text{Sun}} = \frac{C_{\text{Sun}}}{2}. \]
Numerical analysis
Comparison with the SHM

Our bounds are strongest:

- For Xe in $20 \lesssim m_\chi \lesssim 1000$ GeV (SD) and $m_\chi \gtrsim 50$ GeV (SI).
\( C_{\text{Sun}} \) and upper bounds on BR for Xe mock signal

**Results:**

- For xenon, for SD, annihilations into \( \nu \) would be constrained to BR at the few \% level. \( \tau\tau, WW \) at the 10\% level.
- Strong dependence on \( m_\chi \). Stronger bounds at the wrong \( m_\chi \).
DAMA (already strongly disfavored HI by DD)

Using HI bounds on modulation: \( A_\eta(v_m) \leq -v_e s_\alpha \frac{d\tilde{\eta}}{dv_m} \) [Schwetz, Zupan, JHG].

DAMA in strong tension:

- Na: for SI annihilation into \( \nu\nu, \tau\tau \) (\( bb \)) are strongly constrained for \( m_\chi \gtrsim 5 \) (15) GeV, while SD is excluded for all channels.
- I: for SI, strong bounds for \( m_\chi \gtrsim 10, 35 \) GeV for \( \nu\nu, \tau\tau \). For SD \( m_\chi \gtrsim 40 \) (80) GeV excluded for \( \nu\nu, \tau\tau \) (\( bb \)).
Unknown couplings to $n$ and $p$, and uncertainties in SD FF

The extracted $f(v)$ depends on couplings to $n$ and $p$, and FF:

$$C\tilde{f}_{\text{extr}}(v) = C\tilde{f}(v) \frac{A^2_{\text{true}} F^2_{\text{true}}(E_R)}{A^2_{\text{wrong}} F^2_{\text{wrong}}(E_R)} - \frac{\tilde{\eta}(v)}{v} \frac{d}{dv} \left( \frac{A^2_{\text{true}} F^2_{\text{true}}(E_R)}{A^2_{\text{wrong}} F^2_{\text{wrong}}(E_R)} \right).$$

Isospin violation (IV), $\kappa \equiv f_n/f_p$: SD Xe (n) & F (p), $\kappa \equiv a_n/a_p$:
Preliminary: inelastic \((m_\chi^* - m_\chi = \delta)\), self-interactions


- **Left**) \(C_{\text{Sun}}\) for inelastic scattering (solid) and bounds (dashed)
  \(\delta\) (keV) black: 0, red: 25, blue: 50.

- **Right**) \(C_{\text{Sun}}\) for inelastic self-interactions (solid) [no bounds possible]
  \(m_\chi\) (GeV) red: 10, green: 50, blue: 100, yellow: 500, black: 1000.
Summary and conclusions
We derived a lower bound on $C_{\text{Sun}}$ in terms of a positive DD signal that is independent of $f(v), v_{\text{esc}}, \sigma_{\text{SI/SD}}$ and $\rho_\chi$.

We assumed $f(v), \rho_\chi$ constant on $t_{\text{eq}}$ and equal in DD and $C_{\text{Sun}}$.

If equilibrium between capture-annihilations is reached (otherwise need $\sigma_{\text{ann}}$) one can derive upper bounds on BR from no $\nu$ signal.

It is strong for SD and channels to $\nu\nu, \tau\tau$ and $m_\chi \gtrsim 100$ GeV.

Extension to inelastic scattering, isospin-violation and self-interactions in preparation [Blennow, Clementz, JHG].
## Concluding remarks: complementarity of the signals

<table>
<thead>
<tr>
<th>DD</th>
<th>$\Gamma_{\text{Sun}}$</th>
<th>Lesson</th>
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<tbody>
<tr>
<td>No</td>
<td>No</td>
<td>Keep trying... Axions? Eventually, does DM interact non-gravitationally?</td>
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| No  | Yes             | There is NO halo-independent lower bound on $R$ from a $\nu$ signal Dark disk? [Bruch, Choi...]
|     |                 | Self-interactions? [Zentner, JHG (in preparation) ...]
|     |                 | IV? Inelastic? [Nussinov, Menon, Shu, JHG (in preparation)...] |
| Yes | No              | $\rightarrow$ HI upper bounds on BRs [this work]. SD dominated by neutrons? Asymmetric DM with suppressed $\Gamma$? [Kaplan, Nussinov...]
|     |                 | p-wave suppressed $\sigma_{\text{ann}}$? [Kappl...]
|     |                 | Channels without $\nu$ (or not energetic enough)? IV? Inelastic? [Nussinov, Menon, Shu, JHG (in preparation)...] |
| Yes | Yes             | Check if the lower bounds here derived are fulfilled. If so, extract DM properties by a fit [Arina, Serpico, Kavanagh...]. |
Back-up slides
The DM velocity inside the gravitational potential of the Sun:
\[ w^2 = v^2 + u_{\text{esc}}^2(r), \]
with \( u_{\text{esc}}(r) \) the escape velocity from the Sun.

The capture rate is given by (notice that \( wdw = vdv \))

\[
C_{\text{Sun}} = 4\pi \rho_X \frac{m_X}{m_\chi} \sum_A \int_0^{R_S} dr \, r^2 \int_0^\infty dv \, \tilde{f}(v) \, v \, w \, \Omega_A(w, r),
\]

with

\[
\Omega_A(w, r) = w \frac{\rho_A}{m_A} \int_{E_{\text{min}}(w)}^{E_{\text{max}}(w)} dE_R \frac{d\sigma_A}{dE_R}(w),
\]

where \( E_R \) is the nuclear recoil energy.

If equilibrium between capture and annihilation is reached:

\[
\Gamma_{\text{Sun}} = \frac{1}{2} C_{\text{Sun}}
\]
A lower bound on the capture

\[ C_{\text{Sun}} \geq 4\pi \sum_A A^2 \int_0^{R_S} drr^2 \rho_A(r) \int_{v_{\text{thr}}}^{v_{\text{cross}}} dv \left( -\frac{d\tilde{\eta}(v)}{dv} \right) F_A(v, r) \]

\[ = 4\pi \sum_A A^2 \int_0^{R_S} drr^2 \rho_A(r) \left[ \tilde{\eta}_{\text{thr}} F_A(v_{\text{thr}}, r) + \int_{v_{\text{thr}}}^{v_{\text{cross}}} dv \tilde{\eta}(v) F'_A(v, r) \right], \]

where in the last line we integrated by parts, with \( F_A(v_{\text{cross}}, r) = 0 \).

**Features:**

- Either the derivative or the function \( \tilde{\eta}(v) \), including its value at the threshold, have to be determined from DD.
- The bound is independent of the DM velocity distribution, the galactic escape velocity, the scattering cross section and the local DM density.
We simulate mock data motivated by future experiments:

- **Xenon**, with $\sigma_{\text{SI}} = 10^{-45} \text{ cm}^2$ and $\sigma_{\text{SD}} = 2 \cdot 10^{-40} \text{ cm}^2$. Assuming $m_\chi = 100 \text{ GeV}$, for an exposure of 1 ton yr, about 154 (267) events in the range 5 – 45 keV for SI (SD) are predicted.

- **Germanium**, with $E_{\text{thr}} = 1 \text{ keV}$, focusing on low DM masses. Assuming $m_\chi = 6 \text{ GeV}$ and $\sigma_{\text{SI}} = 5 \cdot 10^{-42} \text{ cm}^2$ and $\sigma_{\text{SD}} = 2 \cdot 10^{-40} \text{ cm}^2$, $1.5 \times 10^4 \ (2–3)$ events for SI (SD) predicted in the range 1–10 keV for an exposure of 100 kg yr with energy resolution of 30%.
Results:

- For Ge, for both SD and SI direct annihilations into $\nu$ would be constrained to BR at the few % level. $\tau\tau$ are at wrong $m_\chi$.
- Strong dependence on $m_\chi$. Stronger bounds at the wrong $m_\chi$. 
The annual modulation $A_{\eta}(v_m)$ can be constrained in terms of the constant rate $\bar{\eta}(v_m)$ (almost) halo-independently [JHG, Schwetz, Zupan], by expanding $\eta(v, t)$ in $v_e/v \ll 1$, with $v_e \simeq 30$ km/s.

If there is a preferred direction in the DM velocity:

$$A_{\eta}(v_m) \leq -v_e \sin \alpha_{\text{halo}} \frac{d\bar{\eta}}{dv_m}.$$

Therefore there is also a lower bound on the capture for $A_{\eta}(v_m)$:

$$C_{\text{Sun}} \geq 4\pi \sum_A A^2 \int_0^{R_{\text{Sun}}} dr r^2 \rho_A(r) \int_{v_{\text{thr}}}^{v_{\text{cross}}} dv \frac{\tilde{A}_{\eta}(v)}{\sin \alpha_{\text{halo}} v_e} F_A(v).$$
Expansion of $\eta(v_m, t)$ in $v_e/v$ 

$v_{\text{esc}} \gg \langle v \rangle > v_m \gg v_e$, so we can expand $\eta(v_m, t)$ to first order in $v_e$:

$$
\eta(v_m, t) = \int_{v_m} d^3v \frac{f_{\text{det}}(\vec{v})}{v} = \int_{v_m} d^3v \frac{f_{\text{Sun}}(\vec{v} + \vec{v}_e(t))}{v} = 
$$

$$
= \int_{v_m} d^3v \frac{f_{\text{Sun}}(\vec{v})}{v} + \int d^3v f_{\text{Sun}}(\vec{v}) \frac{\vec{v} \cdot \vec{v}_e(t)}{v^3} [\Theta(v - v_m) - \delta(v - v_m) v_m] 
\equiv 
$$

$$
\equiv \bar{\eta}(v_m) + A_\eta(v_m) \cos 2\pi(t - t_0).
$$

- $\bar{\eta}(v_m)$ is constant, $A_\eta$ is modulated, with observed rates:

$$
\bar{R} \equiv CF^2(E_r)\bar{\eta}(v_m) \quad \text{and} \quad A_R \equiv CF^2(E_r)A_\eta
$$
The general bound on the annual modulation

1. Halo “smooth” on $\lesssim v_e \sim 30$ km/s.
2. Only time dependence in $v_e(t)$, not in $f_{Sun}$ (no change on months).

$$\int_{v_{m1}}^{v_{m2}} dv_m A_\eta(v_m) \leq v_e \left[ \bar{\eta}(v_{m1}) + \int_{v_{m1}} dv \frac{\bar{\eta}(v)}{v} \right]$$

3. If there is a constant $\hat{v}_{HALO}$ governing the modulation:

$$\int_{v_{m1}}^{v_{m2}} dv_m A_\eta(v_m) \leq \sin \alpha v_e \bar{\eta}(v_{m1})$$

where:
- in general $\sin \alpha$ can be set to 1.
- $\sin \alpha \simeq 0.5$ when $\hat{v}_{HALO} \propto \hat{v}_{SUN}$ (isotropic, SHM, DD...).
  And phase $t_0 = June 2nd$. 
DAMA results

- Na dominates for DM masses $m_\chi \lesssim 20$ GeV.
- I is relevant for larger DM masses.
- Small overlap for I for H (SD).

![Graphs showing DAMA results for Sodium and Iodine](image_url)
Evidence for dark matter

Rotation curves of spiral galaxies:

Bullet cluster ($X$-rays + grav. lensing):

- CMB spectrum alone: $\Omega_{DE} \approx 0.69$, $\Omega_B \approx 0.05$, $\Omega_{DM} \approx 0.26$
- Combination of data from CMB, SN Ia, BBN and clusters.
- $M/L$ ratio in galaxy clusters (virial theorem to gas).
- Growth of structure (N-body simulations).
- Globular clusters in galaxies...
Properties of a DM particle (or particles)

1. Interacts gravitationally.
2. With the observed density (long-lived/ stable).
4. Cold (or warm), otherwise small scales would have been erased.
5. Collisionless: it does not dissipate, it forms haloes.

We assume DM is a Weakly Interacting Massive Particle (WIMP):

- weak-scale cross-section with the SM.
- mass $m_\chi \sim \mathcal{O}(1 - 10^3)$ GeV.

which yield right relic abundance (WIMP miracle) and make DD possible.