Phenomenological studies in dark matter direct detection: from the impact of the escape speed estimates to tests of halo models

Stefano Magni

Laboratoire Univers et Particules de Montpellier
(Université de Montpellier, France)

Mainly based on
and ongoing work with J. Lavalle, P. Mollitor and E. Nezri
Impact of the recent estimates of the Galactic escape speed on dark matter direct detection


- Focus on recent **estimates of the escape speed** from the RAVE collaboration (Piffl et al. '14), important for **low WIMP masses**

- Investigate the **implications of these results** for direct detection (assuming isotropic velocity distribution for the dark matter)

- Bringing cosmological simulations into the picture
Astrophysics in direct detection: the standard halo model

**Standard Halo Model**

- Truncated Maxwell-Boltzmann speed distribution
- $\rho^{\text{shm}} = 0.3 \text{ GeV/cm}^3$
- $v^{\text{shm}} = 544 \text{ km/s}$
- $v^{\text{c}} = 220 \text{ km/s}$
- $\sigma_v \propto v_c$
- $v_{\text{min}}(E_r) = \sqrt{\frac{E_r m_A}{2m_{\text{red}}^2}}$

**Differential event rate:**

$$\frac{dR}{dE_r}(E_r) = \frac{A^2 \sigma_{p,SI} F^2(E_r)}{2\mu^2 m_A} \int_{|\vec{v}| > v_{\text{min}}} \frac{d^3 \vec{v} f_\odot(\vec{v})}{v} \rho_\odot$$

- Particle + hadronic + nuclear physics
- Astrophysics

**Effects at work:**

- Experimental treshold
- $v_{\text{esc}} + v_c$
- $\rho_\odot$
Qualitative impact of astrophysical parameters on exclusion curves

- $\nu_{\text{esc}}$ decreases
- $\rho$ increases
- $\nu_0$ increases
- $\nu_{\text{c}}$ decreases

$LUX$ (MGM, 0 events, 2013)
Updated escape speed from the RAVE survey (Piffl et al. '14)

- Previous estimate: \( v_{\text{esc}} = 544^{+64}_{-46} \text{ km/s} \) (90% CL) (Smith et al. '07)

- Based on a sample of \(~100\) high-velocity, non-corotating stars, to test the non local gravitational potential

- Requires assumptions!
  
  ✓ Power law assumption for the high velocity tail of the stellar distribution:

  \[
  n_\ast (v) \propto (v_{\text{esc}} - v)^k
  \]

  ✓ Correct line-of-sight speeds of observed stars to "relocate" them at Sun's position (8.28 kpc)

  ➔ Needs gravitational potential at position of each star

  ➔ Relies on assuming a Milky Way mass model
RAVE's assumption: **Milky Way mass model**

- **Circular speed at Sun's position**
  \[ v_c(R_\odot, 0) = R_\odot \frac{\partial \Phi}{\partial R}(R_\odot, 0) \]

- **Galactic escape speed at Sun's position**
  \[ v_{esc}(R_\odot) = \sqrt{2 |\Phi(R_\odot) - \Phi(R_{max})|} \]

- **Dark matter density at Sun's position**
  \[ \rho_\odot = \rho^{DM}(R_\odot) \]

- **dark matter halo**: NFW
  \[ \phi_{dm}(r) = -4\pi G \frac{\rho_s r_s^3}{r} \ln \left(1 + \frac{r}{r_s}\right) \]

- **disk**: Miyamoto-Nagai
  \[ \phi_d(R, |z|) = -G \frac{M_d}{\sqrt{R^2 + (R_d + \sqrt{z^2 + z_{d}^2})^2}} \]

- **bulge**: Hernquist
  \[ \phi_b(r) = -G \frac{M_b}{(r + r_b)} \]

**Fixed baryons**

- bulge: Hernquist

- dark matter halo: NFW

(taken from http://pages.uoregon.edu)

(taken from: http://universeshots.tumblr.com)
RAVE's escape speed reconstruction

• In Piffl et al. '14, two different likelihood analyses:

  1a) **fixed** $v_c = 240 \text{ km/s}$
  $$v_{esc} = 511^{+48}_{-35} \text{ km/s (90\% CL)}$$

  1b) **fixed** $v_c = 220 \text{ km/s}$
  $$v_{esc} = 533^{+54}_{-41} \text{ km/s (90\% CL)}$$
RAVE's escape speed reconstruction

- In Piffl et al. '14, two different likelihood analyses:

  1a) **fixed** $v_C = 240 \text{ km/s}$
      $v_{esc} = 511^{+48}_{-35} \text{ km/s (90\% CL)}$

  1b) **fixed** $v_C = 220 \text{ km/s}$
      $v_{esc} = 533^{+54}_{-41} \text{ km/s (90\% CL)}$

  2) **free** $v_C$
     - mostly for MW mass estimates
     $v_{esc} = 537 \text{ km/s}$
     (for best fit $v_C = 196 \text{ km/s}$)
Converting RAVE results in the $v_c$-$v_{\text{esc}}$ plane

- In Piffl et al. '14, two different likelihood analyses:

  1a) **fixed** $v_c = 240$ km/s
      $v_{\text{esc}} = 511^{+48}_{-35}$ km/s (90% CL)

  1b) **fixed** $v_c = 220$ km/s
      $v_{\text{esc}} = 533^{+54}_{-41}$ km/s (90% CL)

2) **free** $v_c$
   - mostly for MW mass estimates
     $v_{\text{esc}} = 537$ km/s
     (for best fit $v_c = 196$ km/s)
Beware! MW mass model induces correlations

- In Piffl et al. '14, two different likelihood analyses:

  1a) **fixed** \( v_c = 240 \text{ km/s} \)
  \[
  v_{esc} = 511^{+48}_{-35} \text{ km/s (90% CL)}
  \]

  1b) **fixed** \( v_c = 220 \text{ km/s} \)
  \[
  v_{esc} = 533^{+54}_{-41} \text{ km/s (90% CL)}
  \]

  2) **free** \( v_c \)
  - mostly for MW mass estimates
  \[
  v_{esc} = 537 \text{ km/s}
  \]
  (for best fit \( v_c = 196 \text{ km/s} \))

- **Dynamical correlations** between \( \rho_\odot, v_c, \) and \( v_{esc} \) must be taken into account!!!
Accounting for dynamical correlations requires self-consistent $f(v)$

- Shortcomings of Maxwell-Boltzmann:
  - Relies on isothermal assumption
  - Truncated M.-B. not solution of Jeans equation
- Use Eddington equation (Ullio & Kamionlowski '01, Vergados & Owens '03)

\[
f(\epsilon) = \frac{1}{\sqrt{8\pi^2}} \left\{ \frac{1}{\sqrt{\epsilon}} \frac{d\rho}{d\psi}\bigg|_{\psi=0} + \int_{0}^{\epsilon} \frac{d\psi}{\sqrt{\epsilon - \psi}} \frac{d^2\rho}{d\psi^2} \right\} \Psi = -\Phi_{MW}(r) \\
\epsilon = -E_{\text{tot}} \\
\rho = \rho_{NFW}(r)
\]

- From the DM phase-space distribution $f(\epsilon)$ compute the DM speed distribution $f(v, R_{\text{Sun}})$
Focus on one case

1a) fixed $v_c = 240 \text{ km/s}$
   
   $v_{esc} = 511^{+48}_{-35} \text{ km/s} \ (90\% \text{ CL})$

1b) fixed $v_c = 220 \text{ km/s}$
   
   $v_{esc} = 533^{+54}_{-41} \text{ km/s} \ (90\% \text{ CL})$

2) free $v_c$
   - originally estimate of MW Mass
   - give independent estimate of $v_{esc}$
   $v_c = 196 \text{ km/s} \quad v_{esc} = 537 \text{ km/s}$

- **Dynamical correlations** between $\rho_\odot, v_c$, and $v_{esc}$ must be taken into account!!!
Impact on the direct detection exclusion curves


- Above ~10 GeV: **RAVE-inferred limit more constraining** than SHM by 40% (larger $\rho_\odot$)
- At low masses: **RAVE-inferred limit beaten** by SHM because
  \[ v_c + v_{\text{esc}} = 751 \text{ km/s} \quad \text{VS} \quad v_c + v_{\text{esc}} = 764 \text{ km/s} \]
- The **form of the speed distribution** is relevant only when $m_\chi \ll m_A$
- Relative RAVE stat. uncertainties saturate at $\pm 10\% \ (90\% \text{ CL})$ at large masses
Different direct detection experiments

- **CRESST-II**
- **SuperCDMS**
- **LUX**

- Reduced uncertainties if more experiments are put together (same for more nuclei)
Different analyses

- LUX
- SuperCDMS
- CRESST-II

Graph showing the cross-section versus mass for different dark matter analysis with various labels and fits.
Perspective I: cross-checks with cosmological simulations

In collaboration with P. Mollitor, E. Nezri, J. Lavalle

- **Cosmological hydrodynamical simulation of Milky Way-like Galaxy**
  run by our collaborators P. Mollitor, E. Nezri,

- Using **RAMSES code** (R. Teyssier 2002)

- Including **baryons**, important role in central part of galaxy

- **Balance** between star formation and super-Novae feedback

- Presence of a dark disk in this simulation?
  Dark matter component in disk, due to a subhalo, not yet a conclusive answer.
Perspective II:
The escape speed in a simulated galaxy

In collaboration with P. Mollitor, E. Nezri, J. Lavalle

- **Goal**: use simulated galaxy to study how the employed methods behave

- Particularly interesting for this work: the escape speed in the simulation

- Different ways of computing the escape speed:
  - from fitted speed distributions
  - from the last bin
  - from the gravitational potential, which can be computed exactly

In any case: consider 1) spherical shell or 2) ring

![Graph showing the escape speed](image1)

DM particles in a spherical shell at Sun's position

![Graph showing speeds](image2)

Points = max speed of DM particles (red/black for shell/ring)
Crosses = escape speed computed from the gravitational potential (green/blue for shell/ring averaging)
Perspective III: Validity of Eddington equation?

In collaboration with P. Mollitor, E. Nezri, J. Lavalle

- Phase-space distribution from Eddington equation

\[ f(\varepsilon) = \frac{1}{\sqrt{8\pi^2}} \left\{ \frac{1}{\varepsilon \psi} \left| \psi = 0 \right. + \int_0^\varepsilon \frac{d\psi}{\sqrt{\varepsilon - \psi}} \frac{d^2 \rho}{d\psi^2} \right\} = \frac{1}{\sqrt{8\pi^2}} \frac{d}{d\varepsilon} \int_0^\varepsilon \frac{d\phi}{d\psi} \frac{d\Psi}{\sqrt{\varepsilon - \Psi}} \geq 0 \]

- Issue: **Eddington not applicable to certain couples of** \( \rho(r), \psi(r) \).

- Sufficient condition for the applicability of Eddington equation:

\[ \frac{d^2 \rho(\Psi)}{d\Psi^2} \geq 0 \]

- Eddington not applicable to mass models fit to this simulation:

![Graphs showing phase-space distribution and related derivatives](image)

- Same kind of issue also with generalizations of Eddington procedure to anisotropic velocity distributions
Perspective IV: Considering anisotropic dark matter velocity distributions

(see e.g. N. Bozorgnia et al. JCAP 1312 (2013) arXiv:1310.0468 [astro-ph.CO])

- **Anisotropy parameter**: $\beta (r) \doteq 1 - \frac{v_T^2 (r)}{2v_r^2 (r)}$
  
  Tangential velocity dispersion
  
  Radial velocity dispersion

  Assume: $\begin{cases} v_\beta^2 = \overline{v_\phi^2} \\ \overline{v_T^2} (r) = \overline{v_\phi^2} (r) + \overline{v_\phi^2} (r) \end{cases}$

- **Different models** to obtain anisotropic velocity distributions from Eddington-like procedures

  1) **Osipkov-Merrit models**

  \[
f (\mathcal{E}, L) = f (Q) \quad \text{with} \quad Q = \mathcal{E} - \frac{L^2}{2r_a^2} \quad \Longrightarrow \quad \beta (r) = \frac{r^2}{r^2 + r_a^2}
  \]

  2) **Constant anisotropy parameter**

  \[
f (\mathcal{E}, L) = L^{-2\beta} G (\mathcal{E}) \quad \Longrightarrow \quad \beta (r) = \beta
  \]

  3) **Linear combination**, useful to reproduce simulations

  \[
f (\mathcal{E}, L) = w f_{OM} (Q) + (1 - w) f_\beta (\mathcal{E}, L)
  \]

- **In simulations, different behavior of anisotropy parameter** between spherical shell and ring
Conclusions

• RAVE's estimates of escape speed cannot be used blindly because rely on assumptions

• A Milky Way mass model must be assumed, which induces correlations among the astrophysical parameters

• These correlations propagate to limits computed in direct detection

• Direct detection limits on the plane $\sigma_{p,SI} - m_\chi$ self-consistently inferred from RAVE's results more constraining by up to 40% w.r.t. SHM (due to larger $\rho_\odot$).

• Main question: how reliable is to use current observables to reconstruct dark matter's phase space?

Perspectives

• Cosmological simulations to cross check the employed procedures (with P. Mollitor, E. Nezri & J. Lavalle)

• Limits of validity of Eddington equation (and generalizations)

• Comparison with uncertainties on WIMP-nucleon interactions (evaluated by L. Lellouche and C. Torrero using LQCD methods)
Thank you very much for your attention!
Backup slides
Considering an independent determination of $v_c$

\[ v_c = 243 \pm 6 \text{ km/s (1\sigma)} \]
\[ v_c = 243 \pm 12 \text{ km/s (2\sigma)} \]

(Reid et al., '14)

Additional constraints (OK within 3 sigma):

\[ \frac{dv_c}{dR} = -0.2 \pm 0.4 \text{ km/s/kpc} \]
\[ r_\odot = 8.33 \pm 0.16 \text{ kpc} \]
Analysis with free $\nu_c$ versus forced correlation between $\nu_c$ and $\nu_{esc}$

- Taking into account also the $\nu_{esc}, \nu_c$ anticorrelation provides the most consistent analysis
Spin-Independent interpretation of the current experimental results

\[ m_\chi \ (\text{GeV}/c^2) \]

(Figure from J. Billard Phys.Rev. D89 (2014) 2, 023524)
Exclusion curves computed with e.g. the Maximum Gap Method

Example: XENON10 (J. Angle et al, 2007)
Reproducing the spin independent interpretation of the experimental results

\[ R_i(t) = \int_0^\infty dE_r \epsilon(E_r) \frac{dR(E_r)}{dE_r} \int_{E_i^{\text{min}}}^{E_i^{\text{max}}} dE_i' G(E_r, E_i') \]

- Total event rate
- Experimental efficiency (acceptance)
- Energy resolution
- Time average
- Isotopic composition of targets

\[ \sigma_{\text{SI}} \text{ (cm}^2\text{)} \]

\[ m_\chi \text{ (GeV/c}^2\text{)} \]
The cosmological simulations


- **Cosmological hydrodynamical simulation of Milky-Way like Galaxy** run by our collaborators P. Mollitor and E. Nezri, (Halo B)

- **Using RAMSES code** (R. Teyssier 2002)
  - a grid-based hydrodynamical solver with adaptive mesh refinement
  - dark matter only simulations + hydrodinamical simulations of baryons
  - "Zoom-in" simulations (highly resolved structures within a cosmological context)

- **Including baryons**, the physics of which is very important for galaxies

- **Star formation**: conversion of gas into star particles (Modeled by a Schmidt law)

- **Super-Novae feedback**:
  - Explosion 10 Myr after the star (particle) creation
  - 20 % of the star mass is re-injected into the gas

- **Balance between star formation rate and super-novae feedback**
  - Star formation contracts the dark matter profile
  - Super-Novae feedback originates a **core** in the dark matter profile
The cosmological simulations

Characterizing astrophysical properties of dark matter in simulation

Goals:
➢ understanding the **physical processes**
➢ project the impact on **direct detection**
➢ test analytical methods (Eddington-like)

• Dark matter velocity distribution
  ➢ Fundamental for direct detection
  ➢ Fundamental to test on Eddington

• Dark matter spatial distribution
  ➢ Cored dark matter profile

• Dark disk?
  ➢ Suggested by dark matter density
  ➢ Just an effect of the triaxiality of the halo?

• Offset between the centers of dark matter and stellar distributions?
  ➢ Between 0 and 2 kpc...
  ➢ Meaningful definition of “centers”?