Spinning particles coupled with axions

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Outline

- Motivation
- Spin-axion coupling phenomenology
- Some applications
- Concluding remarks

Motivation

Spontaneous breaking of Peccei-Quinn symmetry provide axion interactions with

photons	$g_\gamma \, \phi \, F_{ij} \tilde{F}^{ij} \,,$
gluons	$g_g \, \phi \, G_{ij} \tilde{G}^{ij} \,,$
fermions	$g_f \, \partial_i \phi \; \overline{\psi} \gamma^i \gamma^5 \psi \,.$

Motivation

Photon polarization coupling with axions

Axion electrodynamics

$$\nabla_l F^{kl} = -\alpha \tilde{F}^{kl} \nabla_l \phi$$

Geometrical optics approximation $\Rightarrow$

$$A_m = a \xi_m e^{i\Psi}$$

Equations

$$\frac{Dk^j}{D\tau} = 0$$

$$\frac{D\xi^j}{D\tau} = \frac{\alpha}{2} \epsilon^{jlmn} \nabla_l \phi \; \xi_m \; k_n$$

massless photons $\rightarrow$ spinning particles

$$k^i \rightarrow p^i$$

$$\xi^i \rightarrow S^i$$

$\Rightarrow$

$$\frac{Dp^j}{D\tau} = 0$$

$$\frac{DS^j}{D\tau} = \frac{\beta}{2} \epsilon^{jlmn} \nabla_l \phi \; S_m \; p_n$$

Spin-axion coupling phenomenology

Equations of motion for spinning particles

$$\frac{Dp^i}{D\tau} = \mathcal{F}^i\,, \qquad \frac{DS^i}{D\tau} = \mathcal{G}^i$$

$$\begin{array}{lcl} p^i p_i = m^2 c^2 = \text{const} & \Rightarrow & p_i \mathcal{F}^i = 0 \\ S^i S_i = -S^2 = \text{const} & \Rightarrow & S_i \mathcal{G}^i = 0 \\ S^i p_i = 0 & \Rightarrow & \mathcal{F}^i S_i + \mathcal{G}_i p^i = 0 \end{array}$$

The equations are satisfied identically, when

$$\mathcal{F}^i = \omega^{ik} p_k\,, \qquad \mathcal{G}^i = \omega^{ik} S_k\,,$$

with an arbitrary anti-symmetric tensor $\omega^{ik} = -\omega^{ki}$.

Spin-axion coupling phenomenology

Ansatz: structure of ω^{ik}

The tensor ω^{ik} is up to the first order

in the Maxwell tensor F_{ik},

in the axion field ϕ or its four-gradient $\nabla_i \phi$,

in the Riemann tensor R^{ijkl} and its convolutions,

and does not include the spin four-vector S^i,

$$\omega^{ik} =$$

no axions $\rightarrow$ $$\frac{e}{2mc}\left[gF^{ik} + \frac{(g-2)}{m^2c^2}\delta^{ik}_{mn}p_j F^{jm}p^n\right]$$

axions $\rightarrow$ $$\frac{e\lambda}{2mc}\phi \left[g_A \tilde{F}^{ik} + \frac{(g_A-2)}{m^2c^2}\delta^{ik}_{mn}p_j \tilde{F}^{jm}p^n\right]$$

gradients $\rightarrow$ $$\nabla_l \phi \left(\omega_{33}\epsilon^{ikjn}p_n p^l p^s F_{js} + \omega_{34}\epsilon^{ikmn}p_n F_m{}^l + \omega_{35}\epsilon^{ikln}p_n + \cdots\right.$$

and the non-minimal terms

Models

Bargmann-Michel-Telegdi equations

$$\omega^{ij} = \frac{e}{2mc}\delta^{ik}_{mn}\left(gF^{mn} + \frac{g-2}{m^2c^2}\,p_l F^{ml} p^n\right)$$

$$\mathcal{F}^i = \frac{e}{mc}F^i{}_k\; p^k\,,$$

$$\mathcal{G}^i = \frac{e}{2mc}\left[\,gF^i{}_k\; S^k + \frac{(g-2)}{m^2c^2}p^i F_{kl}S^k p^l\right].$$

The simple model with the direct spin-axion coupling

$$\omega^{ij} = \omega_{35}\epsilon^{iklj}p_j\nabla_l\phi$$

$$\frac{Dp^i}{D\tau} = 0\,, \qquad \frac{DS^i}{D\tau} = \omega_{35}\epsilon^{iklj}S_k p_j \nabla_l \phi\,.$$

Application 1. De Sitter cosmology

Preliminaries

Metrics $$ds^2 = dt^2 - a^2(t)\left[{dx^1}^2 + {dx^2}^2 + {dx^3}^2\right]$$

de Sitter regime $a(t) \propto \exp\{H_0 t\}$

Axion field $\ddot{\phi} + 3H_0\dot{\phi} + m_{\rm a}^2\phi = 0\,.$

$$\phi(t) \propto \exp\{-\frac{3}{2}H_0 t\}\cos\Omega_{\rm a} t\,, \quad \Omega_{\rm a} \equiv \sqrt{m_{\rm a}^2 - \frac{9}{4}H_0^2}\,.$$

The gradient of axion field is timelike $\nabla_i\phi\nabla^i\phi > 0$

Particle motion $\dfrac{Dp_j}{D\tau} = 0$

$$p_1(t) = p_2(t) = 0, \quad p_3(t) = q = \text{const}, \quad p_0(t) = \sqrt{m^2 + q^2 a^{-2}(t)}$$

Application 1. De Sitter cosmology

$$\frac{DS^0}{D\tau} = 0\,, \qquad \text{no axion effect}$$

$$\frac{DS^3}{D\tau} = 0\,. \qquad S^0 = S^3 = 0$$

Application 1. De Sitter cosmology

$$\frac{d(aS^1)}{dt} = -\frac{m\omega_{35} q\dot{\phi}(t)}{\sqrt{m^2 + q^2 a^{-2}}} \cdot S^2$$

$$\frac{d(aS^2)}{dt} = \frac{m\omega_{35} q\dot{\phi}(t)}{\sqrt{m^2 + q^2 a^{-2}}} \cdot S^1$$

Application 1. De Sitter cosmology

$$\frac{d(aS^1)}{dt} = -\frac{m\omega_{35}q\dot{\phi}(t)}{\sqrt{m^2+q^2a^{-2}}} \cdot S^2$$

$$\frac{d(aS^2)}{dt} = \frac{m\omega_{35}q\dot{\phi}(t)}{\sqrt{m^2+q^2a^{-2}}} \cdot S^1$$

The solution describes a spin precession in the transverse plane:

$$S^1(t) = \frac{S}{a(t)}\cos\Psi(t), \qquad S^2 = \frac{S}{a(t)}\sin\Psi(t),$$

$$\Psi(t) = \int_{t_0}^{t} \frac{m\omega_{35}q\dot{\phi}(t)}{a(t)\sqrt{m^2+q^2a^{-2}(t)}}dt$$

Application 2. Static spherical symmetry

Preliminaries

Metric $ds^2 = B(r)dt^2 - A(r)dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$

Axion field $\phi'' + \phi'\left[\frac{1}{2}\left(\frac{B'}{B} - \frac{A'}{A}\right) + \frac{2}{r}\right] = A(r)\phi\left[m_{\rm a}^2 + \nu_{\rm a}(\phi^2 - \phi_0^2)\right]$

The gradient of axion field is spacelike $\nabla_i\phi\nabla^i\phi < 0$

The particle moves in the equatorial plane $\theta = \frac{\pi}{2}$

$$p_0 = K = \text{const}\,, \quad p_\theta = 0\,, \quad p_\varphi = -J = \text{const}\,,$$

$$A(r)\left(p^r\right)^2 = \frac{K^2}{B(r)} - \frac{J^2}{r^2} - E^2\,, \quad E = \text{const}\,.$$

Application 2. Static spherical symmetry

Radial motion

$$\frac{dS^0}{d\tau} + S^0 \left\{ \frac{B'(r)}{2mABp^r} \left[2\frac{K^2}{B(r)} - m^2 \right] \right\} = 0\,,$$

$$\frac{dS^r}{d\tau} + S^r \left\{ \frac{p^r}{2m} \left(\frac{A'}{A} + \frac{B'}{B} \right) \right\} = 0\,.$$

no axion effect

$$S^0 = S^r = 0$$

Application 2. Static spherical symmetry

Radial motion

$$\frac{dS^{\theta}}{d\tau} + S^{\theta} \left(\frac{p^r}{mr}\right) = -\frac{\omega_{35}\phi'}{\sqrt{AB}r^2} p_0 S_{\varphi} \, ,$$

$$\frac{dS^{\varphi}}{d\tau} + S^{\varphi} \left(\frac{p^r}{mr}\right) = \frac{\omega_{35}\phi'}{\sqrt{AB}r^2} p_0 S_{\theta} \, .$$

The solution describes a spin precession in the transverse plane:

$$S^{\theta} = \frac{S}{r} \cos \Psi(r) \, , \qquad S^{\varphi} = \frac{S}{r} \sin \Psi(r)$$

$$\Psi(r) = \Psi(\infty) + \int_{\infty}^{r} \frac{mK\omega_{35}\phi'}{\sqrt{K^2 - E^2 B}} dr$$

Application 2. Static spherical symmetry

Circular motion

$r = $const, $\quad p^r = 0$

$$S^0 = S^\varphi \frac{J}{K}$$
$$\frac{dS^r}{d\tau} = S^\varphi \cdot H^r(r)$$
$$\frac{dS^\theta}{d\tau} = S^\varphi \cdot H^\theta(r)$$
$$\frac{dS^\varphi}{d\tau} + \frac{S^r}{r} = -S^\theta \cdot H^\varphi(r)\,,$$

Application 2. Static spherical symmetry

Circular motion

r =const, $\quad p^r = 0$

$$\frac{d^2 S^\varphi}{d\tau^2} + \Omega^2(r)\ S^\varphi = 0$$

Application 2. Static spherical symmetry

Circular motion

$r = $const, $\quad p^r = 0$

$$\frac{d^2 S^\varphi}{d\tau^2} + \Omega^2(r)\ S^\varphi = 0$$

The solution describes a geodesic-axion precession with the frequency

$$\Omega^2(r) = \Omega^2_{\text{geod}}(r) + \Omega^2_{\text{axion}}(r)$$

$$\Omega_{\text{geod}}(r) = \frac{EJ}{mr^2 K}\sqrt{\frac{B(r)}{A(r)}}\,, \qquad \Omega_{\text{axion}}(r) = \frac{\omega_{35} E \phi'(r)}{\sqrt{A(r)}}\,.$$

Application 3. Gravitational wave

Preliminaries

Metrics $$ds^2 = 2dudv - L^2(u)\left(e^{2\beta(u)}(dx^2)^2 + e^{-2\beta(u)}(dx^3)^2\right)$$

Axion field is non-minimally coupled with gravitation
(A. B. Balakin and W.-T. Ni, CQG 27 : 055003, 2010)

The gradient of axion field is null $\nabla_i\phi\nabla^i\phi = 0$

Particle motion $$\frac{Dp_j}{D\tau} = 0$$

$$p^u = p_v = C_0 = \text{const}\,,$$
$$p^2L^2(u)e^{2\beta(u)} = -p_2 = C_2 = \text{const}\,,$$
$$p^3L^2(u)e^{-2\beta(u)} = -p_3 = C_3 = \text{const}$$
$$p_ip^i = m^2 \quad \Rightarrow \quad p_u = \ldots$$

Application 3. Gravitational wave

Equations

$$\frac{d}{du}S^u = 0 \qquad \Rightarrow \qquad S^u = S_v = \text{const}$$

$$\frac{e^{-\beta}}{L}\frac{d}{du}\left(Le^{\beta}S^2\right) + \frac{C_2}{2C_v}S_v\left(L^{-2}e^{-2\beta}\right)' = \frac{\omega_{25}\phi'}{L^2}\left(S_v\frac{C_3}{C_v} - S_3\right),$$

$$\frac{e^{\beta}}{L}\frac{d}{du}\left(Le^{-\beta}S^3\right) + \frac{C_3}{2C_v}S_v\left(L^{-2}e^{2\beta}\right)' = -\frac{\omega_{25}\phi'}{L^2}\left(S_v\frac{C_2}{C_v} - S_2\right).$$

Exact solutions

$$Le^{\beta}S^2 = \mathcal{A}\cos\Psi(u) - S_v\frac{C_3}{C_v}\left(\frac{e^{\beta}}{L}\right)$$

$$Le^{-\beta}S^3 = \mathcal{A}\sin\Psi(u) - S_v\frac{C_2}{C_v}\left(\frac{e^{-\beta}}{L}\right)$$

$$\Psi(u) = \Psi(0) + \omega_{25}[\phi(u) - \phi(0)]$$

$$S_i p^i = 0 \quad \Rightarrow \quad S_u = \ldots$$

Application 3. Gravitational wave

Particle starts to move in the GW plane

$p_1(0) = p_3(0) = 0, \quad p_2(0) \neq 0$

All the spin components are involved in the precession
regardless of initial states

The solution with the initial state $S^0(0) = S^1(0) = S^2(0) = 0, \; S^3(0) \neq 0$ is

$$S^0(u) = S^1(u) = -\frac{C_2}{\sqrt{2}C_v} \, \mathcal{S} \left(\frac{e^{-\beta}}{L} \right) \sin \Psi(u) \,,$$

$$S^2(u) = \mathcal{S} \left(\frac{e^{-\beta}}{L} \right) \sin \Psi(u) \,,$$

$$S^3(u) = \mathcal{S} \left(\frac{e^{\beta}}{L} \right) \cos \Psi(u)$$

Application 3. Gravitational wave

Particle moves along the GW direction

$p_1 \neq 0, \quad p_2 = p_3 = 0$

The axionically induced spin precession is in the GW plane.

The simplest example is

$$S^0(u) = S^1(u) = 0\,,$$

$$S^2(u) = \mathcal{S}\left(\frac{e^{-\beta}}{L}\right)\sin\Psi(u)\,, \qquad S^3(u) = \mathcal{S}\left(\frac{e^{\beta}}{L}\right)\cos\Psi(u)\,.$$

Concluding remarks

- The axions makes the space-time to be chiral: the left-hand and right-hand rotations of the particle spin four-vector become non-equivalent.
- Gravity activates the spin-axion coupling: the spin precession is induced by the gradient of the axion field and the spin rotation becomes more pronounced in the strong gravity field.
- Spin rotation can be used as cumulative effect: the phase of rotation is presented by the integral with respect to time or to the radial variable or to the retarded time.
- Experiments and observations: the direct spin-axion coupling gives very elusive effects, which need very sophisticated methods.
- Further developments: spin-axion coupling mediated by the electromagnetic field, and nonminimal spin-axion interactions.

Thank you for attention!