Another look at collective neutrino oscillations

Evgeny Akhmedov

Max-Planck Institute für Kernphysik, Heidelberg

In collaboration with A. Mirizzi

MAX-PLANCK-INSTITUT FÜR KERNPHYSIK

ν oscillations in dense neutrino backgrounds

(Early Universe, supernovae) – interesting collective oscillation effects, absent in the case of the usual MSW oscillations:

Synchronized neutrino oscillations, bi-polar oscillations, spectral splits & swaps, multiple spectral splits

Our goal: to study late-time decoherence effects on collective neutrino oscillations in the simplest possible system: uniform and isotropic neutrino gas

Decoherence: in momentum space – due to de-phasing of different momentum modes at late times

In coordinate space: due to spatial separation of wave packets corresponding to different propagation eigenstates

The two descriptions are equivalent

Decoherence by wave packet separation

For SN neutrinos: $\sigma_{xP} \sim 10^{-11}$ cm (Kersten 2012) $\Rightarrow$ estimated $L_{\rm coh} \sim 10$ km. By the time ν's reach the region of coll. oscillations ($r \sim 100$ km) they should have already lost their coherence.

Decoherence by wave packet separation

For SN neutrinos: $\sigma_{xP} \sim 10^{-11}$ cm (Kersten 2012) $\Rightarrow$ estimated $L_{\rm coh} \sim 10$ km. By the time ν's reach the region of coll. oscillations ($r \sim 100$ km) they should have already lost their coherence.

Decoherence: leads to averaging out of neutrino oscillations with time (or with distance from the source).

Decoherence by wave packet separation

For SN neutrinos: $\sigma_{xP} \sim 10^{-11}$ cm (Kersten 2012) $\Rightarrow$ estimated $L_{\rm coh} \sim 10$ km. By the time ν's reach the region of coll. oscillations ($r \sim 100$ km) they should have already lost their coherence.

Decoherence: leads to averaging out of neutrino oscillations with time (or with distance from the source).

E.g. for 2-flavor $\nu_e \to \nu_x$ oscillations in vacuum and Gaussian WPs:

$$\boxed{P_{ee} = c^4 + s^4 + 2c^2 s^2 e^{-t^2/L_{\rm coh}^2} \cos\phi}\,, \qquad \boxed{P_{ex} = 2c^2 s^2 (1 - e^{-t^2/L_{\rm coh}^2} \cos\phi)}$$

$$\phi \equiv (\Delta m^2/2E)t\,.$$

Decoherence by wave packet separation

For SN neutrinos: $\sigma_{xP} \sim 10^{-11}$ cm (Kersten 2012) $\Rightarrow$ estimated $L_{\rm coh} \sim 10$ km. By the time ν's reach the region of coll. oscillations ($r \sim 100$ km) they should have already lost their coherence.

Decoherence: leads to averaging out of neutrino oscillations with time (or with distance from the source).

E.g. for 2-flavor $\nu_e \to \nu_x$ oscillations in vacuum and Gaussian WPs:

$$P_{ee} = c^4 + s^4 + 2c^2 s^2 e^{-t^2/L_{\rm coh}^2} \cos\phi \,, \qquad P_{ex} = 2c^2 s^2 (1 - e^{-t^2/L_{\rm coh}^2} \cos\phi)$$

$$\phi \equiv (\Delta m^2/2E)t \,.$$

♢ At the same time: synchronized oscillations in a dense neutrino gas (i.e. with sufficiently large μ) show no trace of averaging out!

Decoherence by wave packet separation

For SN neutrinos: $\sigma_{xP} \sim 10^{-11}$ cm (Kersten 2012) $\Rightarrow$ estimated $L_{\rm coh} \sim 10$ km. By the time ν's reach the region of coll. oscillations ($r \sim 100$ km) they should have already lost their coherence.

Decoherence: leads to averaging out of neutrino oscillations with time (or with distance from the source).

E.g. for 2-flavor $\nu_e \to \nu_x$ oscillations in vacuum and Gaussian WPs:

$$P_{ee} = c^4 + s^4 + 2c^2 s^2 e^{-t^2/L_{\rm coh}^2} \cos\phi\,, \qquad P_{ex} = 2c^2 s^2 (1 - e^{-t^2/L_{\rm coh}^2} \cos\phi)$$

$$\phi \equiv (\Delta m^2/2E)t\,.$$

♢ At the same time: synchronized oscillations in a dense neutrino gas (i.e. with sufficiently large μ) show no trace of averaging out!

Why does this happen? Why is WP separation inoperative in this case?

Flavour spin formalism

EoMs for individual ω-modes:

$$\diamondsuit \qquad \boxed{\dot{\vec{P}}_\omega = \vec{H}_\omega \times \vec{P}_\omega}$$

Precession of “flavour spin” around the “magn. field” $\vec{H}_\omega$.

$$\boxed{\vec{H}_\omega = \omega\vec{B} + \mu\vec{P}}\,,$$

$$\vec{B} = (s_{20},\, 0,\, -c_{20})\,, \qquad s_{20} \equiv \sin 2\theta_0\,, \quad c_{20} \equiv \cos 2\theta_0\,,$$

$$\mu = \sqrt{2}G_F n_\nu\,, \qquad \vec{P} = \int \vec{P}_\omega\, d\omega\,, \qquad \omega = \frac{\Delta m^2}{2p}.$$

$|\vec{P}_\omega| \equiv P_0 g_\omega = const.$, g_ω – normalized neutrino spectrum in ω.

Initial conditions:

$$\vec{P}_\omega(0) = P_0 g_\omega \vec{n}_z$$

EoM for flavour spin

For WPs: g_ω – spectrum of an individual WP. Assumed to have a peak at $\omega \simeq \omega_0$ and effective width σ_ω. E.g. for the Gaussian spectrum

$$g_\omega = \frac{1}{\sqrt{2\pi}\sigma_\omega} e^{-\frac{(\omega-\omega_0)^2}{2\sigma_\omega^2}}$$

EoM for flavour spin

For WPs: g_ω – spectrum of an individual WP. Assumed to have a peak at $\omega \simeq \omega_0$ and effective width σ_ω. E.g. for the Gaussian spectrum

$$g_\omega = \frac{1}{\sqrt{2\pi}\sigma_\omega} e^{-\frac{(\omega-\omega_0)^2}{2\sigma_\omega^2}}$$

Integrating over ω:

$$\dot{\vec{P}} = \int d\omega[(\omega\vec{B} + \mu\vec{P}) \times \vec{P}_\omega] = \vec{B} \times \vec{S}\,,$$

$$\vec{S} \equiv \int d\omega\omega\vec{P}_\omega\,. \qquad \text{Initial condition :} \qquad \vec{P}(0) = P_0\hat{\vec{n}}_z\,.$$

EoM for flavour spin

For WPs: g_ω – spectrum of an individual WP. Assumed to have a peak at $\omega \simeq \omega_0$ and effective width σ_ω. E.g. for the Gaussian spectrum

$$g_\omega = \frac{1}{\sqrt{2\pi}\sigma_\omega} e^{-\frac{(\omega-\omega_0)^2}{2\sigma_\omega^2}}$$

Integrating over ω:

$$\dot{\vec{P}} = \int d\omega[(\omega\vec{B} + \mu\vec{P}) \times \vec{P}_\omega] = \vec{B} \times \vec{S}\,,$$

$$\vec{S} \equiv \int d\omega\omega\vec{P}_\omega\,. \qquad \text{Initial condition :} \qquad \vec{P}(0) = P_0\hat{\vec{n}}_z\,.$$

From EoM:

$$\vec{B}\cdot\dot{\vec{P}} = 0 \qquad \Rightarrow \qquad \vec{P}\cdot\vec{B} = const. = -c_{20}P_0\,.$$

But: $|\vec{P}| \neq const.$!

EoM for flavour spin

For WPs: g_ω – spectrum of an individual WP. Assumed to have a peak at $\omega \simeq \omega_0$ and effective width σ_ω. E.g. for the Gaussian spectrum

$$g_\omega = \frac{1}{\sqrt{2\pi}\sigma_\omega} e^{-\frac{(\omega-\omega_0)^2}{2\sigma_\omega^2}}$$

Integrating over ω:

$$\dot{\vec{P}} = \int d\omega [(\omega \vec{B} + \mu \vec{P}) \times \vec{P}_\omega] = \vec{B} \times \vec{S}\,,$$

$$\vec{S} \equiv \int d\omega \omega \vec{P}_\omega\,. \qquad \text{Initial condition :} \qquad \vec{P}(0) = P_0 \hat{\vec{n}}_z\,.$$

From EoM:

$$\vec{B} \cdot \dot{\vec{P}} = 0 \qquad \Rightarrow \qquad \vec{P} \cdot \vec{B} = const. = -c_{20} P_0\,.$$

But: $|\vec{P}| \neq const.$! From conservation of $|\vec{P}_\omega|$ and $\vec{P} \cdot \vec{B}$:

♢ $$\boxed{c_{20} P_0 \leq |\vec{P}(t)| \leq P_0}$$

EoM for flavour spin

From the EoMs:

$$E_{tot} \equiv \frac{\mu P^2}{2} + \vec{B}\cdot\vec{S} = const.$$

Can be interpreted as conservation of the ‘total energy’ of the system of flavour spins (Duan, Fuller & Qian, 2006).

Oscillations in uniform isotropic neutrino gas

Studied in many papers (Samuel 1993; Kostelecky & Samuel, 1993, 1994, 1995; Pastor, Raffelt & Semikoz, 2001; Duan, Fuller & Qian 2006, 2008; ...; Raffelt & Tamborra, 2010).

Oscillations in uniform isotropic neutrino gas

Studied in many papers (Samuel 1993; Kostelecky & Samuel, 1993, 1994, 1995; Pastor, Raffelt & Semikoz, 2001; Duan, Fuller & Qian 2006, 2008; ...; Raffelt & Tamborra, 2010).

Three main regimes:

Oscillations in uniform isotropic neutrino gas

Studied in many papers (Samuel 1993; Kostelecky & Samuel, 1993, 1994, 1995; Pastor, Raffelt & Semikoz, 2001; Duan, Fuller & Qian 2006, 2008; ...; Raffelt & Tamborra, 2010).

Three main regimes:

- Large μ regime – "perfect synchronization". All $\vec{P}_\omega$ evolve in a synchronized way despite differences in ω. $\vec{P}$ satisfies

$$\dot{\vec{P}} = \omega_0 \vec{B} \times \vec{P}$$

– precesses around $\vec{B}$ with frequency ω_0; $P = P_0$.

Oscillations in uniform isotropic neutrino gas

Studied in many papers (Samuel 1993; Kostelecky & Samuel, 1993, 1994, 1995; Pastor, Raffelt & Semikoz, 2001; Duan, Fuller & Qian 2006, 2008; ...; Raffelt & Tamborra, 2010).

Three main regimes:

- Large μ regime – "perfect synchronization". All $\vec{P}_\omega$ evolve in a synchronized way despite differences in ω. $\vec{P}$ satisfies

$$\dot{\vec{P}} = \omega_0 \vec{B} \times \vec{P}$$

  – precesses around $\vec{B}$ with frequency ω_0; $P = P_0$.

- Small μ regime – complete de-synchronization at late times. Oscillations average out, no evolution at asymptotically large times. $|\vec{P}|$ shrinks to its minimal value $P_{min} = c_{20} P_0$.

Oscillations in uniform isotropic neutrino gas

Studied in many papers (Samuel 1993; Kostelecky & Samuel, 1993, 1994, 1995; Pastor, Raffelt & Semikoz, 2001; Duan, Fuller & Qian 2006, 2008; ...; Raffelt & Tamborra, 2010).

Three main regimes:

- Large μ regime – "perfect synchronization". All $\vec{P}_\omega$ evolve in a synchronized way despite differences in ω. $\vec{P}$ satisfies

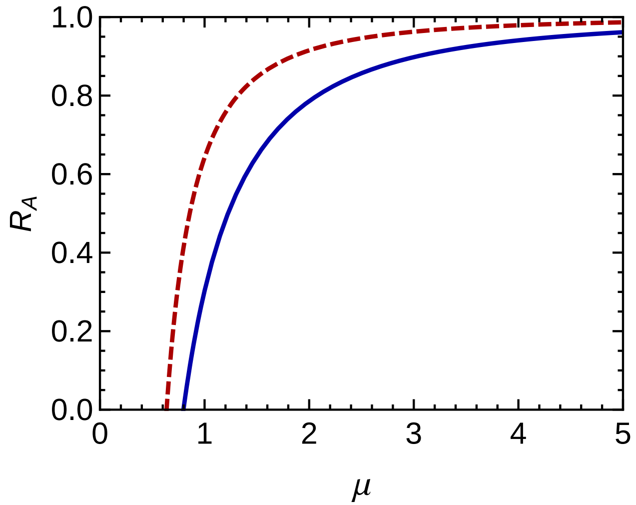
$$\dot{\vec{P}} = \omega_0 \vec{B} \times \vec{P}$$

  – precesses around $\vec{B}$ with frequency ω_0; $P = P_0$.

- Small μ regime – complete de-synchronization at late times. Oscillations average out, no evolution at asymptotically large times. $|\vec{P}|$ shrinks to its minimal value $P_{min} = c_{20} P_0$.

- Intermediate μ regime – partial de-synchronization. at late times. Precession around $\vec{B}$ with some frequency ω_s; $P_{min} < P < P_0$.

Raffelt & Tamborra results



Raffelt & Tamborra, arXiv:1006.0002. Order parameter $R_A = P_\perp / \sin 2\theta_0$ for Gaussian (solid line) and box-type (dashed line) neutrino spectra.

Approximations used in the analytic approach:

- "Sudden approximation" – $\vec{P}$ replaced by its asymptotic (late time) value starting immediately at $t = 0$.
- The angles between $\vec{P}_\omega$ and $\vec{B}$ at the onset of asymptotic regime are taken to be those corresponding to $t = 0$ (i.e. all equal $2\theta_0$).
- In the co-rotating frame $\vec{P}_\omega$ are replaced by their asymptotic averages assumed to be given by their projections on $\vec{H}'_\omega$: $\vec{P}_\omega \to \frac{\vec{P}_\omega \cdot \vec{H}'_\omega}{H'^2_\omega} \vec{H}'_\omega$.

Very good agreement with numerical results.

Spectral moments formalism

I. Exact relations.

$$\dot{\vec{P}} = \int d\omega[(\omega\vec{B} + \mu\vec{P}) \times \vec{P}_\omega] = \vec{B} \times \vec{S}\,,$$

Contains 2 global flavour spin vectors (spectral moments):

$$\vec{P} \equiv \int d\omega \vec{P}_\omega\,, \qquad \vec{S} \equiv \int d\omega\omega\vec{P}_\omega\,.$$

Introduce

$$\boxed{\vec{K}_n(t) = \int d\omega\, \omega^n \vec{P}_\omega(t)}\,, \qquad n \geq 0\,.$$

Well defined if the neutrino spectrum g_ω goes to zero fast enough for $|\omega| \to \infty$ (satisfied e.g. for Gaussian and box-type neutrino spectra).

$$\vec{P}(t) = \vec{K}_0(t)\,, \qquad \vec{S}(t) = \vec{K}_1(t)\,.$$

Multiplying $\dot{\vec{P}}_\omega = (\omega\vec{B} + \mu\vec{P}) \times \vec{P}_\omega$ by ω^n and integrating over ω: $\Rightarrow$

EoMs for $\vec{K}_n(t)$

$$\dot{\vec{K}}_n = \vec{B} \times \vec{K}_{n+1} + \mu \vec{P} \times \vec{K}_n$$

Combining equations for $\vec{K}_n$ and $\vec{K}_{n+1}$:

$$\vec{B} \cdot \dot{\vec{K}}_{n+1} + \mu \vec{P} \cdot \dot{\vec{K}}_n = 0$$

(Alternatively, can be derived using $\vec{H}\omega \cdot \dot{\vec{P}}_\omega = 0$ and taking $\int d\omega \omega^n$).

For $n = 0$ $\Rightarrow$ $\vec{B} \cdot \dot{\vec{S}} + \mu \vec{P} \cdot \dot{\vec{P}}$. Can be integrated: $\frac{\mu P^2}{2} + \vec{B} \cdot \vec{S} = const. = E_{tot}$
– already known.

For $n = 1$:

$$\vec{B} \cdot \dot{\vec{K}}_2 + \mu \vec{P} \cdot \dot{\vec{S}} = 0.$$

Can also be integrated! From $\dot{\vec{P}} = \vec{B} \times \vec{S}$: $\dot{\vec{P}} \cdot \vec{S} = 0$. $\Rightarrow$

$\vec{P} \cdot \dot{\vec{S}} = \vec{P} \cdot \dot{\vec{S}} + \dot{\vec{P}} \cdot \vec{S} = (d/dt)(\vec{P} \cdot \vec{S})$. $\Rightarrow$

EoMs for $\vec{K}_n(t)$

$$\boxed{\vec{B}\cdot\vec{K}_2 + \mu\vec{P}\cdot\vec{S} = const.}$$

– New conservation law! Valid also for systems of ν's and $\bar{\nu}$'s.

(Can also be derived from the results of Pehlivan et al., 2011)

Above formulas: exact and satisfied for all t. Now consider the regime of asymptotically large times, at which synchronized oscillations set in.

A simplified analytic approach

Assume that at late t evolution of the system conserves the length of $\vec{P}$:

$$\vec{P}\cdot\dot{\vec{P}} = \vec{P}\cdot(\vec{B}\times\vec{S}) = 0\,.$$

A simplified analytic approach

Assume that at late t evolution of the system conserves the length of $\vec{P}$:

$$\vec{P}\cdot\dot{\vec{P}} = \vec{P}\cdot(\vec{B}\times\vec{S}) = 0\,.$$

Can be satisfied if one (or more) of the following conditions is fulfilled:
(a) $\vec{P}=0$; (b) $\vec{P}\parallel\vec{B}$; (c) $\vec{S}=0$; (d) $\vec{S}\parallel\vec{B}$; (e) $\vec{S}\parallel\vec{P}$. Non-trivial realization – case (e), $\vec{S}\parallel\vec{P}$.

A simplified analytic approach

Assume that at late t evolution of the system conserves the length of $\vec{P}$:

$$\vec{P}\cdot\dot{\vec{P}} = \vec{P}\cdot(\vec{B}\times\vec{S}) = 0\,.$$

Can be satisfied if one (or more) of the following conditions is fulfilled:
(a) $\vec{P}=0$; (b) $\vec{P}\parallel\vec{B}$; (c) $\vec{S}=0$; (d) $\vec{S}\parallel\vec{B}$; (e) $\vec{S}\parallel\vec{P}$. Non-trivial realization – case (e), $\vec{S}\parallel\vec{P}$. More precisely: $\vec{S}_\perp\parallel\vec{P}_\perp$ [def.: $\vec{P}_\perp \equiv \vec{P}-(\vec{P}\cdot\vec{B})\vec{B}$].

A simplified analytic approach

Assume that at late t evolution of the system conserves the length of $\vec{P}$:

$$\vec{P}\cdot\dot{\vec{P}} = \vec{P}\cdot(\vec{B}\times\vec{S}) = 0\,.$$

Can be satisfied if one (or more) of the following conditions is fulfilled:
(a) $\vec{P}=0$; (b) $\vec{P}\parallel\vec{B}$; (c) $\vec{S}=0$; (d) $\vec{S}\parallel\vec{B}$; (e) $\vec{S}\parallel\vec{P}$. Non-trivial realization – case (e), $\vec{S}\parallel\vec{P}$. More precisely: $\vec{S}_\perp\parallel\vec{P}_\perp$ [def.: $\vec{P}_\perp\equiv\vec{P}-(\vec{P}\cdot\vec{B})\vec{B}$].

For now we assume:

$$\boxed{\vec{S}(t)=\omega_s\vec{P}(t)}\,.$$

(Additional assumption for the longitudinal components. Satisfied well if μ is not too close to the threshold μ_0).

A simplified analytic approach

Assume that at late t evolution of the system conserves the length of $\vec{P}$:

$$\vec{P}\cdot\dot{\vec{P}} = \vec{P}\cdot(\vec{B}\times\vec{S}) = 0\,.$$

Can be satisfied if one (or more) of the following conditions is fulfilled:
(a) $\vec{P}=0$; (b) $\vec{P}\parallel\vec{B}$; (c) $\vec{S}=0$; (d) $\vec{S}\parallel\vec{B}$; (e) $\vec{S}\parallel\vec{P}$. Non-trivial realization – case (e), $\vec{S}\parallel\vec{P}$. More precisely: $\vec{S}_\perp \parallel \vec{P}_\perp$ [def.: $\vec{P}_\perp \equiv \vec{P} - (\vec{P}\cdot\vec{B})\vec{B}$].

For now we assume:

$$\boxed{\vec{S}(t) = \omega_s \vec{P}(t)}\,.$$

(Additional assumption for the longitudinal components. Satisfied well if μ is not too close to the threshold μ_0).

From $\dot{\vec{P}} = \vec{B}\times\vec{S} \quad \Rightarrow$

$$\boxed{\dot{\vec{P}} = \omega_s \vec{B}\times\vec{P}}$$

$\vec{P}$ precesses around $\vec{B}$ with the constant frequency ω_s.

A simplified analytic approach – contd.

In principle, ω_s could be time dependent (but we shall see that it is constant).

$$\vec{S} = \omega_s \vec{P} \qquad \Rightarrow \qquad \omega_s = \frac{\int d\omega \omega \vec{P}_\omega}{\int d\omega \vec{P}_\omega}$$

(in N and D components of $\vec{P}_\omega$ along a fixed direction should be taken).

Substitute $\vec{S} = \omega_s \vec{P}$ into E_{tot}:

$$\boxed{\frac{\mu}{2}[P_0^2 - P^2] = c_{20} P_0 (\omega_0 - \omega_s)}$$

- All quantities except possibly ω_s are constant $\Rightarrow$ ω_s must be constant.
- Since l.h.s. is non-negative, so must be r.h.s. $\Rightarrow$ $\omega_s \le \omega_0$.
- Because ω_0 and ω_s are certain averages of ω over the spectrum g_ω of effective width σ_ω, $\omega_0 - \omega_s \lesssim \sigma_\omega$. $\Rightarrow$
- In the limit $\mu \to \infty$: asympt.-lly $P \to P_0$ (no shrinkage of $\vec{P}$ for large μ).

A simplified analytic approach – contd.

Rewrite equation for E_{tot}:

$$\frac{P_0^2 - P}{P_0^2} = 2c_{20}\frac{\omega_0 - \omega_s}{\mu P_0} \lesssim 2c_{20}\,\frac{\sigma_\omega}{\mu P_0}\,,$$

'Perfect synchronization' ($P = P_0$) achieved for $\mu \gg \text{ some } \mu_0$; $\Rightarrow$

$$\boxed{\mu_0 \sim \sigma_\omega}\,,$$

while it is irrelevant whether or not μ is large compared to $\omega_0,\, \omega_s$.

A simplified analytic approach – contd.

From EoM $\dot{\vec{P}} = \omega_s \vec{B} \times \vec{P}$ and $\vec{S} = \omega_s \vec{P}$:

$$\dot{\vec{S}} = \omega_s \vec{B} \times \vec{S}$$

– at asymptotic times $\vec{S}$ satisfies EoM similar to that of $\vec{P}$.
$\vec{S} \cdot \dot{\vec{S}} = 0$; on the other hand, from EoM for $\vec{K}_n$ with $n = 1$:

$$\dot{\vec{S}} = \vec{B} \times \vec{K}_2 + \mu \vec{P} \times \vec{S}\,.$$

$\vec{S} \cdot \dot{\vec{S}} = 0 \quad \Rightarrow \qquad \vec{S} \cdot (\vec{B} \times \vec{K}_2) = 0\,.$

Non-trivially realized only if at asymptotically large times $\vec{K}_2$ is parallel or antiparallel to $\vec{S}$ (and therefore also to $\vec{P}$), that is $\vec{K}_2(t) = \omega_1 \vec{S}(t)$.

A simplified analytic approach – contd.

Comparing EoMs $\dot{\vec{S}} = \omega_s \vec{B} \times \vec{S}$ and $\dot{\vec{S}} = \vec{B} \times \vec{K}_2 + \mu \vec{P} \times \vec{S}$:
$\omega_1 = \omega_s$, that is, asymptotically

$$\vec{K}_2(t) = \omega_s \vec{S}(t) = \omega_s^2 \vec{P}(t) \,.$$

Substituting this into the conservation law $\vec{B}\cdot\vec{K}_2 + \mu\vec{P}\cdot\vec{S} = const.$:

$$-\omega_s^2 c_{20} P_0 + \omega_s \mu P^2 = P_0[\mu P_0 \omega_0 - c_{20}(\omega_0^2 + \sigma_\omega^2)] \,.$$

Combining this with $E_{tot} = const.$ $\Rightarrow$ quadratic equation for $\omega_0 - \omega_s$:

$$\boxed{(\omega_0 - \omega_s)^2 - \frac{\mu P_0}{c_{20}}(\omega_0 - \omega_s) + \sigma_\omega^2 = 0}\,.$$

A simplified analytic approach – contd.

The solution:

$$\boxed{\omega_0 - \omega_s = \frac{1}{2c_{20}}\left[\mu P_0 - \sqrt{\mu^2 P_0^2 - 4c_{20}^2\sigma_\omega^2}\,\right]}\,.$$

Exhibits a threshold behaviour: real solution only exists (and synchronized oscillations are only take possible) for $\mu > \mu_0$ where

$$\boxed{\mu_0 P_0 = 2c_{20}\sigma_\omega}\,.$$

For $\mu P_0 \gg 2c_{20}\sigma_\omega$ (far above the threshold):

$$\omega_0 - \omega_s \simeq c_{20}\frac{\sigma_\omega}{\mu P_0}\sigma_\omega \ll \sigma_\omega \qquad (\omega_s \to \omega_0).$$

$\omega_0 - \omega_s$ reaches its maximum at the threshold $\mu = \mu_0$: $\omega_0 - \omega_s(\mu_0) = \sigma_\omega$.
The asymptotic value of P vs μ:

$$P = P_0\left[1 - \frac{\mu_0^2}{\mu^2}\right]^{1/4}.$$

A simplified analytic approach – contd.

Results for the asymptotic regime are quite reasonable in a number of respects:

- demonstrate the existence of the threshold μ_0
- give a good estimate for its value (typically within a factor of 2 of the true value)
- lead to a reasonable behaviour of P far above the threshold.

<u>But:</u> they lead to a wrong result for the value of P at the threshold: $P = 0$.
(Due to conservation of $\vec{P}\cdot\vec{B}$ the length of $\vec{P}$ cannot be smaller than $c_{20}P_0$).

This problem is related to the additional assumption about the relation between the longitudinal components of $\vec{S}$ and $\vec{P}$ that is not valid close to the threshold

II. More accurate consideration

The goal: to get a better description close to the threshold μ_0.

$\Rightarrow$ Remove an additional assumption regarding the relation between the longitudinal components of $\vec{P}$ and $\vec{S}$, $\vec{P}_\parallel \equiv (\vec{P}\cdot\vec{B})\vec{B}$ and $\vec{S}_\parallel \equiv (\vec{S}\cdot\vec{B})\vec{B}$.

Non-trivial realization of the condition $\vec{P}\cdot(\vec{B}\times\vec{S}) = 0$:

$$\boxed{\vec{S}_\perp(t) = \omega_s \vec{P}_\perp(t)}$$

In previous analysis: $\omega_s = const.$ (followed from $E_{tot} = const.$ and late-time relation $P = const$).

Now: ω_s relates only the transverse components of $\vec{P}$ and $\vec{S}$, whereas $E_{tot} = const.$ constrains only the longitudinal component of $\vec{S}$. $\Rightarrow$ cannot prove $\omega_s = const.$.

In general, no longer possible to directly use the conservation laws to find μ_0 and the asymptotic value of P. $\Rightarrow$ Follow a different strategy.

Different strategy

Assume that the asymptotic steady-state evolution of $\vec{P}$ is a simple precession with constant angular velocity, that is $\omega_s = const$.
Differentiating $\vec{S}_\perp = \omega_s \vec{P}$ and using EoM for $\vec{P}$:

$$\dot{\vec{S}}_\perp = \omega_s \vec{B} \times \vec{S}_\perp .$$

From conservation of E_{tot} and $P^2 = const.$: $S_\| = \vec{B}\cdot\vec{S}$ is also constant. $\Rightarrow$

$$\dot{\vec{S}} = \omega_s \vec{B} \times \vec{S} .$$

From $\vec{S}\cdot\dot{\vec{S}} = 0$ and $\dot{\vec{S}} = \vec{B} \times \vec{K}_2 + \mu \vec{P} \times \vec{S}$:

$$\vec{S}\cdot(\vec{B} \times \vec{K}_2) = 0 .$$

Non-trivial realization:

$$\vec{K}_{2\perp}(t) = \omega_1 \vec{S}_\perp(t)$$

with constant ω_1 (steady-state evolution assumption). But: $\omega_1 \neq \omega_s$! (cannot be immediately found).

Different strategy – contd.

Just as for $\vec{S}$: $\vec{B}\cdot\vec{K}_2$ is conserved, $\dot{\vec{K}}_2 = \omega_s \vec{B}\times\vec{K}_2$, $\vec{K}_{2\perp} = \omega_1\omega_s\vec{P}_\perp$.
Can be shown by induction: if for some n

$$\dot{\vec{K}}_n = \omega_s \vec{B}\times\vec{K}_n\,, \qquad\qquad \vec{K}_{n\perp} = \alpha_n \vec{P}_\perp$$

with constant α_n (steady-state evolution assumption), the the same relations hold for $\vec{K}_{n+1}$. $\Rightarrow$ In particular

$$\vec{B}\cdot\dot{\vec{K}}_n(t) \equiv \int d\omega\,\omega^n\,\vec{B}\cdot\dot{\vec{P}}_\omega(t) = 0\,.$$

Has to be satisfied for all n, which is only possible if

$$\vec{B}\cdot\dot{\vec{P}}_\omega = \vec{B}\cdot(\mu\vec{P}\times\vec{P}_\omega) = 0\,,$$

Non-trivial realization:

$$\vec{P}_{\omega\perp}(t) = a_\omega \vec{P}_\perp(t)\,, \qquad a_\omega = const.$$

Different strategy – contd.

$$P_{\omega\perp} = P_\omega \sin\theta_\omega = P_0 g_\omega \frac{\mu P_\perp}{\sqrt{(\omega-\omega_r)^2 + (\mu P_\perp)^2}}\,,$$

$$P_{\omega\parallel} = P_\omega \cos\theta_\omega = P_0 g_\omega \frac{(\omega-\omega_r)}{\sqrt{(\omega-\omega_r)^2 + (\mu P_\perp)^2}}\,,$$

Longitudinal and transverse components of all $\vec{K}_n$ can be reconstructed as integrals of $P_{\omega\parallel}$ and $P_{\omega\perp}$ with proper ω-dependent factors.
E.g., integrating $P_{\omega\perp}$ and $P_{\omega\parallel}$ over ω gives $P_\perp$ and $P_\parallel$ $\Rightarrow$

$$\diamondsuit \quad 1 = \mu P_0 \int d\omega g_\omega \frac{1}{\sqrt{(\omega-\omega_r)^2 + (\mu P_\perp)^2}}\,,$$

$$\diamondsuit \quad -c_{20} = \int d\omega g_\omega \frac{(\omega-\omega_r)}{\sqrt{(\omega-\omega_r)^2 + (\mu P_\perp)^2}}\,.$$

Coincide with results of Raffelt and Smirnov (arXiv:0705.1830, 0709.4641) and also of Duan, Fuller & Qian (arXiv:0706.4293) obtained under completely different assumptions!

Different strategy – contd.

$\omega_s P_\perp = S_\perp = \int d\omega \omega P_{\omega\perp} \quad \Rightarrow$

$$\omega_s(\mu) = \mu P_0 \int d\omega g_\omega \frac{\omega}{\sqrt{(\omega-\omega_r)^2 + (\mu P_\perp)^2}}\,.$$

Defines $\omega_s(\mu)$ implicitly (r.h.s. depends on ω_s through ω_r).

Qualitative results:

- In the limit $\mu \gg \mu_0$:

$$1 = \frac{P_0}{P}\int d\omega g_\omega$$

  Gives $P = P_0$. Then

$$\omega_s = \int d\omega g_\omega \omega = \omega_0\,.$$

Different strategy – contd.

- Assume $\mu P_0 \lesssim \sigma_\omega$. Because of the factor g_ω, $(\omega - \omega_r)^2$ in the integrands is $\sim \sigma_\omega^2$. $\Rightarrow$

$$1 = \frac{\mu P_0}{\sigma_\omega} C_1\,, \qquad C_1 \lesssim 1\,.$$

Can only be satisfied for $\mu P_0 \gtrsim \sigma_\omega \quad \Rightarrow$ there should exist a minimum value μ_0 such that $\mu_0 P_0 \sim \sigma_\omega$.

For $\mu < \mu_0$ only trivial solution possible $P_\perp = 0 \quad \Rightarrow$ complete decoherence.

Different strategy – contd.

- Assume $\mu P_0 \lesssim \sigma_\omega$. Because of the factor g_ω, $(\omega - \omega_r)^2$ in the integrands is $\sim \sigma_\omega^2$. $\Rightarrow$

$$1 = \frac{\mu P_0}{\sigma_\omega} C_1\,, \qquad C_1 \lesssim 1\,.$$

Can only be satisfied for $\mu P_0 \gtrsim \sigma_\omega$ $\Rightarrow$ there should exist a minimum value μ_0 such that $\mu_0 P_0 \sim \sigma_\omega$.

For $\mu < \mu_0$ only trivial solution possible $P_\perp = 0$ $\Rightarrow$ complete decoherence.

Solutions for box-type spectrum using $\int d\omega P_{\omega\|} = -c_{20} P_0$ and $E_{tot} = const.$:

$$\mu_0 P_0 = \sigma \qquad (\text{N.B.: } \sigma = \sqrt{3}\sigma_\omega)$$

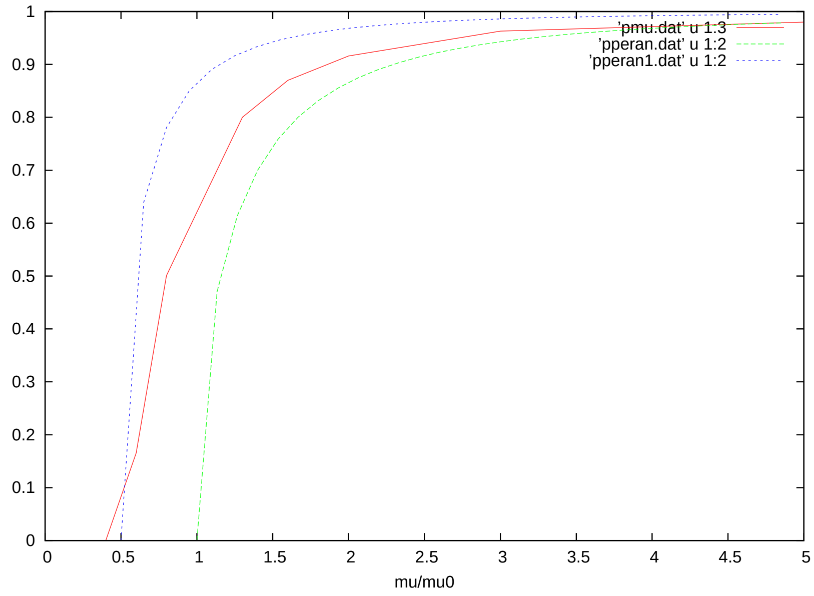
$$\boxed{P_\perp = s_{20} P_0 \sqrt{1 - \frac{\mu_0^2}{\mu^2}}}\,.$$

Different strategy – contd.

Consistency check: substitute into the second conservation law:

$$\vec{B}\cdot\vec{K}_2 + \mu\vec{P}\cdot\vec{S} = P_0\Big\{\mu P_0\omega_0 - c_{20}\Big(\omega_0^2 + \frac{1}{3}\sigma^2\Big) - \frac{1}{6}s_{20}^2 c_{20}\sigma^2\Big\}\,.$$

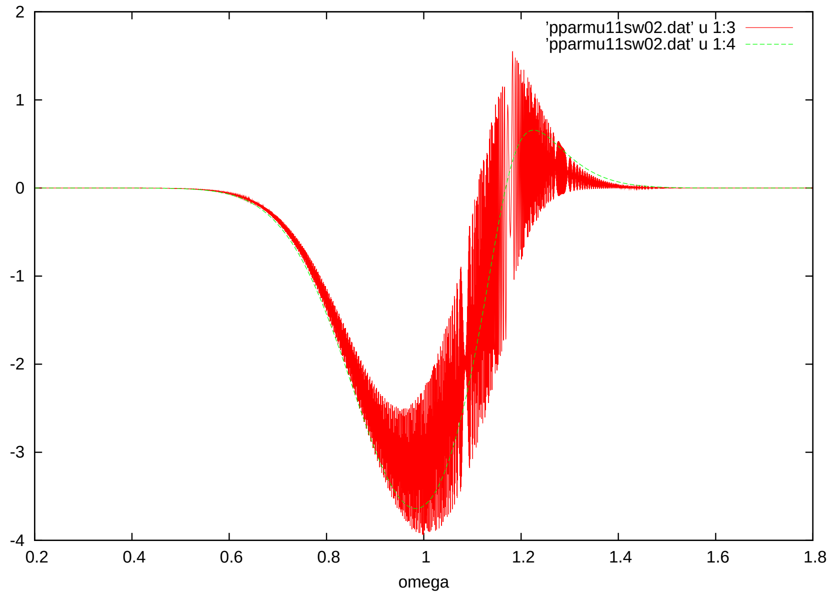
Last term $(\sim s_{20}^2\sigma^2/(\mu P_0\omega_0),\ s_{20}^2\sigma^2/\omega_0^2)$ violates the cons. law. Violation can be sizeable near threshold ($\mu \sim \mu_0$).



$P_\perp/s_{20}$ vs. μ/μ_0 for box-type spectrum of width 2σ. $\theta_0 = 0.5$.

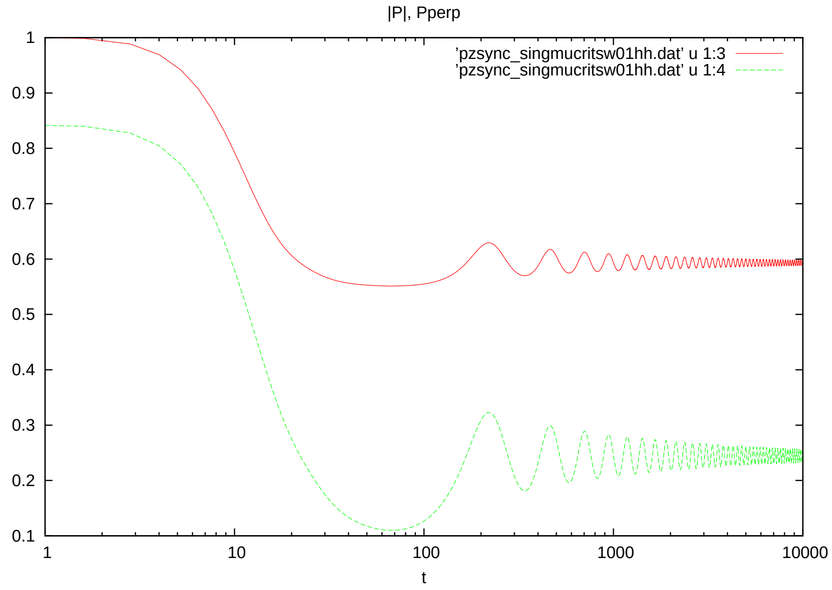
Red curve – numerical, green curve – analytical with $\mu_0 P_0 = \sigma$, blue curve – analytical with $\mu_0 P_0 = 0.5\sigma$.

Why?



Gaussian spectrum, $\theta_0 = 0.5$, $\sigma_\omega = 0.2$, $\mu = 1.1\mu_0^{theo}$.

Red: $P_{\omega\|}^{num}$, green: $P_{\omega\|}^{theo}$.



Gaussian spectrum, $\theta_0 = 0.5$, $\omega_0 = 1,\ \sigma_\omega = 0.2,\ \ \mu = \mu_0^{theo}$.

Red: $|\vec{P}|$, green: $P_\perp$.

Note:

$t_{max} = 10000 \gg L_{\rm coh}^{naive} \sim \sigma_\omega^{-1}$!

Interpretation

Synchronization/de-synchronization and decoherence

by WP separation

Decoh. by WP separation and flavour spin

Propagation eigenstate basis: the basis which diagonalizes $\mathcal{H}$.

(Evolution in prop. eigenstate basis considered by Galais, Kneller & Volpe, 2011)

Decoh. by WP separation and flavour spin

Propagation eigenstate basis: the basis which diagonalizes $\mathcal{H}$.

(Evolution in prop. eigenstate basis considered by Galais, Kneller & Volpe, 2011)

WP separation: In the propagation eigenstate basis off-diagonal elements of the density matrix $\rho_{ik} = \langle \nu_i \nu_k^* \rangle$ get suppressed

Decoh. by WP separation and flavour spin

Propagation eigenstate basis: the basis which diagonalizes $\mathcal{H}$.

(Evolution in prop. eigenstate basis considered by Galais, Kneller & Volpe, 2011)

WP separation: In the propagation eigenstate basis off-diagonal elements of the density matrix $\rho_{ik} = \langle \nu_i \nu_k^* \rangle$ get suppressed $\Rightarrow$

Each $\vec{P}_\omega$ aligns (or anti-aligns) with its $\vec{H}_\omega = \omega\vec{B} + \mu\vec{P}$.

Decoh. by WP separation and flavour spin

Propagation eigenstate basis: the basis which diagonalizes $\mathcal{H}$.

(Evolution in prop. eigenstate basis considered by Galais, Kneller & Volpe, 2011)

WP separation: In the propagation eigenstate basis off-diagonal elements of the density matrix $\rho_{ik} = \langle \nu_i \nu_k^* \rangle$ get suppressed $\Rightarrow$

Each $\vec{P}_\omega$ aligns (or anti-aligns) with its $\vec{H}_\omega = \omega \vec{B} + \mu \vec{P}$.

This can happen for all ω only when $\vec{P} = \int d\omega \vec{P}_\omega$ becomes collinear with $\vec{B}$

Decoh. by WP separation and flavour spin

Propagation eigenstate basis: the basis which diagonalizes $\mathcal{H}$.

(Evolution in prop. eigenstate basis considered by Galais, Kneller & Volpe, 2011)

WP separation: In the propagation eigenstate basis off-diagonal elements of the density matrix $\rho_{ik} = \langle \nu_i \nu_k^* \rangle$ get suppressed $\Rightarrow$

Each $\vec{P}_\omega$ aligns (or anti-aligns) with its $\vec{H}_\omega = \omega\vec{B} + \mu\vec{P}$.

This can happen for all ω only when $\vec{P} = \int d\omega \vec{P}_\omega$ becomes collinear with $\vec{B}$

$\diamondsuit$ Decoherence means alignment of the flavour spin vector $\vec{P}$ with $\vec{B}$ ($P_\perp = 0$). Also: all $P_{\omega\perp} = 0$.

Decoh. by WP separation and flavour spin

Propagation eigenstate basis: the basis which diagonalizes $\mathcal{H}$.

(Evolution in prop. eigenstate basis considered by Galais, Kneller & Volpe, 2011)

WP separation: In the propagation eigenstate basis off-diagonal elements of the density matrix $\rho_{ik} = \langle \nu_i \nu_k^* \rangle$ get suppressed $\Rightarrow$

Each $\vec{P}_\omega$ aligns (or anti-aligns) with its $\vec{H}_\omega = \omega \vec{B} + \mu \vec{P}$.

This can happen for all ω only when $\vec{P} = \int d\omega \vec{P}_\omega$ becomes collinear with $\vec{B}$

$\diamondsuit$ Decoherence means alignment of the flavour spin vector $\vec{P}$ with $\vec{B}$ ($P_\perp = 0$). Also: all $P_{\omega\perp} = 0$.

(Taken as the definition of complete decoherence in Raffelt & Tamborra, arXiv:1006.0002; integration over spectrum rather than WP separation has been considered).

Propagation eigenstates and adiabaticity

Propagation eigenstates: Physically meaningful when the adiabaticity condition is satisfied – the rate of change of the Hamiltonian is small compared to the characteristic frequency of neutrino oscillations.
(Aadiabaticity violation parameter $\lambda \ll 1$)

In ordinary matter: adiabaticity $\Leftrightarrow$ matter density varies slowly along the neutrino path compared to in-matter "oscillation frequency" $2\pi/|E_1 - E_2|$.

In dense neutrino environments: adiabaticity may be violated even when $n_\nu = const.$! The Hamiltonian $\mathcal{H}$ depends not only on the density of neutrino gas, but also on its flavour compostion which changes during evolution.

Adiabaticity condition in terms of flavour spin: the rate of evolution of $\vec{H}_\omega$ (angular velicity Ω_H of its rotation and the rate of its length decrease) is small compared to the frequency $|\vec{H}_\omega|$ of precession of individual $\vec{P}_\omega$ around their $\vec{H}_\omega$ – "tracking" of moving $\vec{H}_\omega$ by $\vec{P}_\omega$.

Strong or moderate non-adiabaticity

What happens when adiabaticity is violated?

- Propagation eigenstates are not physically meaningful; WP separation does not occur (or is suppressed).
- In adiabatic regime: prop. eigenstate evolve independently without going into each other.
- If adiabaticity is strongly violated: prop. eigenstates strongly mix – fully (or almost fully) interchange on a time scale τ that is short compared to $L_{\rm coh}$. The slow state becomes the fast one and vice versa – small WP separation ($\ll \sigma_x$) over the period τ is compensated during the next period τ.
- If adiabaticity is moderately violated, the shuffling of prop. eigenstates occurs with amplitude < 1 – only partial compensation of WP separation that occured during the period τ. Over long times ($> L_{\rm coh}$) the overlap of different prop. eigenstates tends to a finite value.

Adiabaticity – contd.

I. Large μ limit ($\mu P_0 \gg \omega_0$): $\lambda \simeq s_{20}\frac{\omega_0}{\mu P_0} \ll 1$ – good adiabaticity.

II. Subcritical μ ($\mu < \mu_0$): $P_\perp \to 0 \quad \Rightarrow \quad \lambda = 0$ (perfect adiabaticity).

III. $\mu P_0 \sim \omega_0$: $\lambda \sim 1$ (or $\gg 1)$: strong violation of adiabaticity.

Adiabaticity – contd.

I. Large μ limit ($\mu P_0 \gg \omega_0$): $\lambda \simeq s_{20}\frac{\omega_0}{\mu P_0} \ll 1$ – good adiabaticity.

II. Subcritical μ ($\mu < \mu_0$): $P_\perp \to 0 \quad \Rightarrow \quad \lambda = 0$ (perfect adiabaticity).

III. $\mu P_0 \sim \omega_0$: $\lambda \sim 1$ (or $\gg 1)$: strong violation of adiabaticity.

Synchronized oscillations:

- Case $\mu P_0 \gg \omega_0$. Good adiabaticity. Propagation eigenstates physically meaningful. At neutrino production $\theta(0) \simeq \pi/2$ – produced flavour state practically coincides with one of the propagation eigenstates (mixing strongly suppressed). $\Rightarrow$ No WP separation $\Rightarrow$ no decoherence.

Adiabaticity – contd.

I. Large μ limit ($\mu P_0 \gg \omega_0$): $\lambda \simeq s_{20}\frac{\omega_0}{\mu P_0} \ll 1$ – good adiabaticity.

II. Subcritical μ ($\mu < \mu_0$): $P_\perp \to 0 \quad \Rightarrow \quad \lambda = 0$ (perfect adiabaticity).

III. $\mu P_0 \sim \omega_0$: $\lambda \sim 1$ (or $\gg 1$) : strong violation of adiabaticity.

Synchronized oscillations:

- Case $\mu P_0 \gg \omega_0$. Good adiabaticity. Propagation eigenstates physically meaningful. At neutrino production $\theta(0) \simeq \pi/2$ – produced flavour state practically coincides with one of the propagation eigenstates (mixing strongly suppressed). $\Rightarrow$ No WP separation $\Rightarrow$ no decoherence.
- Case $\sigma_\omega \ll \mu P_0 \lesssim \omega_0$. Moderate or strong adiabaticity violation. Go to the co-rotating frame: $\omega \to \omega' = (\omega - \omega_s)$. $\Rightarrow \quad \mu P_0 \gg |\omega'|$. Good adiabaticity, suppressed mixing. Reduces to the previous case.

Adiabaticity – contd.

I. Large μ limit ($\mu P_0 \gg \omega_0$): $\lambda \simeq s_{20} \frac{\omega_0}{\mu P_0} \ll 1$ – good adiabaticity.

II. Subcritical μ ($\mu < \mu_0$): $P_\perp \to 0 \quad \Rightarrow \quad \lambda = 0$ (perfect adiabaticity).

III. $\mu P_0 \sim \omega_0$: $\lambda \sim 1$ (or $\gg 1$) : strong violation of adiabaticity.

Synchronized oscillations:

- Case $\mu P_0 \gg \omega_0$. Good adiabaticity. Propagation eigenstates physically meaningful. At neutrino production $\theta(0) \simeq \pi/2$ – produced flavour state practically coincides with one of the propagation eigenstates (mixing strongly suppressed). $\Rightarrow$ No WP separation $\Rightarrow$ no decoherence.
- Case $\sigma_\omega \ll \mu P_0 \lesssim \omega_0$. Moderate or strong adiabaticity violation. Go to the co-rotating frame: $\omega \to \omega' = (\omega - \omega_s)$. $\Rightarrow \quad \mu P_0 \gg |\omega'|$. Good adiabaticity, suppressed mixing. Reduces to the previous case.
- Case $\mu P_0 < \mu_0 P_0 \sim \sigma_\omega$. Perfect adiabaticity; $\theta \simeq \theta_0$ (mixing not suppressed); for $L > L_{coh}$ WPs separate $\Rightarrow$ complete decoherence. No synchronization.

Adiabaticity & adiabaticity violation

The produced state: $\nu(t_0) = \nu_e$. In prop. eig. basis: $\nu(t_0) = \nu_e = (c_0\ s_0)^T$.
In the large t limit ($\Omega t \gg 1$) or equivalently averaging over the ω-spectrum:

$$\boxed{\rho_{12} \simeq \tilde{s}_2 s_0 c_0 \ - \ \frac{1}{2}\tilde{s}_2\tilde{c}_2(c_0^2 - s_0^2)}\,.$$

$\tilde{s}_2 = \sin 2\tilde{\theta}$, $\tilde{c}_2 = \cos 2\tilde{\theta}$, $\tilde{\theta}$ – mixing of prop. eigenstates,

$$\tilde{s}_2 = \frac{\lambda}{\sqrt{1+\lambda^2}}\,, \qquad \tilde{c}_2 = \frac{1}{\sqrt{1+\lambda^2}}\,.$$

Adiabaticity & adiabaticity violation

The produced state: $\nu(t_0) = \nu_e$. In prop. eig. basis: $\nu(t_0) = \nu_e = (c_0\ s_0)^T$.
In the large t limit ($\Omega t \gg 1$) or equivalently averaging over the ω-spectrum:

$$\boxed{\rho_{12} \simeq \tilde{s}_2 s_0 c_0 \;-\; \frac{1}{2}\tilde{s}_2\tilde{c}_2(c_0^2 - s_0^2)}\,.$$

$\tilde{s}_2 = \sin 2\tilde{\theta}$, $\tilde{c}_2 = \cos 2\tilde{\theta}$, $\tilde{\theta}$ – mixing of prop. eigenstates,

$$\tilde{s}_2 = \frac{\lambda}{\sqrt{1+\lambda^2}}\,, \qquad \tilde{c}_2 = \frac{1}{\sqrt{1+\lambda^2}}\,.$$

Mixing angle in ‘matter’ (neutrino gas) at $t = t_0$: $\theta(t_0)$ ($\neq$ vac. mix. angle θ_0!)
$s_0 \equiv \sin\theta(t_0)$, $c_0 = \cos\theta(t_0)$.

Adiabaticity & adiabaticity violation

The produced state: $\nu(t_0) = \nu_e$. In prop. eig. basis: $\nu(t_0) = \nu_e = (c_0\ s_0)^T$.
In the large t limit ($\Omega t \gg 1$) or equivalently averaging over the ω-spectrum:

$$\boxed{\rho_{12} \simeq \tilde{s}_2 s_0 c_0 \; - \; \frac{1}{2}\tilde{s}_2 \tilde{c}_2 (c_0^2 - s_0^2)}\,.$$

$\tilde{s}_2 = \sin 2\tilde{\theta}$, $\tilde{c}_2 = \cos 2\tilde{\theta}$, $\tilde{\theta}$ – mixing of prop. eigenstates,

$$\tilde{s}_2 = \frac{\lambda}{\sqrt{1+\lambda^2}}\,, \qquad \tilde{c}_2 = \frac{1}{\sqrt{1+\lambda^2}}\,.$$

Mixing angle in ‘matter’ (neutrino gas) at $t = t_0$: $\theta(t_0)$ ($\neq$ vac. mix. angle θ_0!)
$s_0 \equiv \sin\theta(t_0)$, $c_0 = \cos\theta(t_0)$.

(1) Maximal adiabaticity violation: $\tilde{s}_2 = 1$, $\tilde{c}_2 = 0$ $\;\Rightarrow\;$ $\rho_{12} = s_0 c_0$ – coincides with that at $t = t_0$. No WP separation.

Adiabaticity & adiabaticity violation

The produced state: $\nu(t_0) = \nu_e$. In prop. eig. basis: $\nu(t_0) = \nu_e = (c_0\ s_0)^T$.
In the large t limit ($\Omega t \gg 1$) or equivalently averaging over the ω-spectrum:

$$\boxed{\rho_{12} \simeq \tilde{s}_2 s_0 c_0 \ - \ \frac{1}{2}\tilde{s}_2\tilde{c}_2(c_0^2 - s_0^2)}\,.$$

$\tilde{s}_2 = \sin 2\tilde{\theta}$, $\tilde{c}_2 = \cos 2\tilde{\theta}$, $\tilde{\theta}$ – mixing of prop. eigenstates,

$$\tilde{s}_2 = \frac{\lambda}{\sqrt{1+\lambda^2}}\,, \qquad \tilde{c}_2 = \frac{1}{\sqrt{1+\lambda^2}}\,.$$

Mixing angle in 'matter' (neutrino gas) at $t = t_0$: $\theta(t_0)$ ($\ne$ vac. mix. angle θ_0!)
$s_0 \equiv \sin\theta(t_0)$, $c_0 = \cos\theta(t_0)$.

(1) Maximal adiabaticity violation: $\tilde{s}_2 = 1$, $\tilde{c}_2 = 0 \ \Rightarrow \ \rho_{12} = s_0 c_0$ – coincides with that at $t = t_0$. No WP separation.

(2) Perfect adiabaticity: $\tilde{s}_2 = 0 \ \Rightarrow \ \rho_{12} = 0$ – complete decoherence at late t.

Adiabaticity & adiabaticity violation

The produced state: $\nu(t_0) = \nu_e$. In prop. eig. basis: $\nu(t_0) = \nu_e = (c_0 \; s_0)^T$.
In the large t limit ($\Omega t \gg 1$) or equivalently averaging over the ω-spectrum:

$$\boxed{\rho_{12} \simeq \tilde{s}_2 s_0 c_0 \; - \; \frac{1}{2}\tilde{s}_2\tilde{c}_2(c_0^2 - s_0^2)}\,.$$

$\tilde{s}_2 = \sin 2\tilde{\theta}$, $\tilde{c}_2 = \cos 2\tilde{\theta}$, $\tilde{\theta}$ – mixing of prop. eigenstates,

$$\tilde{s}_2 = \frac{\lambda}{\sqrt{1+\lambda^2}}\,, \qquad \tilde{c}_2 = \frac{1}{\sqrt{1+\lambda^2}}\,.$$

Mixing angle in 'matter' (neutrino gas) at $t = t_0$: $\theta(t_0)$ ($\neq$ vac. mix. angle θ_0!)
$s_0 \equiv \sin\theta(t_0)$, $c_0 = \cos\theta(t_0)$.

(1) Maximal adiabaticity violation: $\tilde{s}_2 = 1$, $\tilde{c}_2 = 0 \;\; \Rightarrow \;\; \rho_{12} = s_0 c_0$ – coincides with that at $t = t_0$. No WP separation.

(2) Perfect adiabaticity: $\tilde{s}_2 = 0 \; \Rightarrow \; \rho_{12} = 0$ – complete decoherence at late t.

(3) Moderate non-adiabaticity: $\tilde{s}_2 = \mathcal{O}(1) \;\; \Rightarrow$ partial decoherence.

Summary

- We have studied decoherence effects on synchronized neutrino oscillations in a dense (uniform and homogeneous) neutrino gas.

  This system is the simplest model for collective neutrino oscillations that can occur in in the early Universe or core-collapse supernovae.

- We developed an exact formalism of spectral moments $\vec{K}_n$ of neutrino flavour spin and found a new conservation law for the uniform system of self-interacting neutrinos (or neutrinos and antineutrinos).

- We developed two simple analytic approaches to decoherence dense neutrino gases. They demonstrate in a straightforward way the existence of threshold μ_0 delineating the coherent and decoherent regimes and allow to estimate μ_0 within a factor of 2.

Summary – contd.

The accuracy of description of the numerical results not very good close to the threshold but is excellent away from it.

This is probably related to the fact that the condition $|\vec{P}| = const.$ is never satisfied at the threshold because the relaxation time $\to \infty$ as $\mu \to \mu_0$.

- We gave an intrepretation of synchronization and de-synchronization (i.e. coherence and decoherence) in terms of the WP separation, both in the adiabatic and non-adiabatic cases.

- Effects of decoherence by WP separation in realistic settings still remain to be studied

Backup slides

Decoherence by wave packet separation

W. packets of different mass (matter) eigenstates propagate with different group velocities

Decoherence by wave packet separation

W. packets of different mass (matter) eigenstates propagate with different group velocities

Coherence time: $\Delta v_g \cdot t_{\text{coh}} \simeq \sigma_x$.

Decoherence by wave packet separation

W. packets of different mass (matter) eigenstates propagate with different group velocities

Coherence time: $\Delta v_g \cdot t_{\text{coh}} \simeq \sigma_x$. Coherence length: $L_{\text{coh}} = v_g \cdot t_{\text{coh}} \simeq \frac{v_g}{\Delta v_g}\sigma_x$.

Decoherence by wave packet separation

W. packets of different mass (matter) eigenstates propagate with different group velocities

Coherence time: $\Delta v_g \cdot t_{\text{coh}} \simeq \sigma_x$. Coherence length: $L_{\text{coh}} = v_g \cdot t_{\text{coh}} \simeq \frac{v_g}{\Delta v_g}\sigma_x$.

In vacuum:

$$\frac{\Delta v_g}{v_g} \simeq \frac{\Delta m^2}{2p^2} \qquad \Rightarrow \qquad \boxed{L_{\text{coh}} \simeq \frac{2p^2}{\Delta m^2}\sigma_x}$$

Decoherence by wave packet separation

W. packets of different mass (matter) eigenstates propagate with different group velocities

Coherence time: $\Delta v_g \cdot t_{\text{coh}} \simeq \sigma_x$. Coherence length: $L_{\text{coh}} = v_g \cdot t_{\text{coh}} \simeq \frac{v_g}{\Delta v_g}\sigma_x$.

In vacuum:

$$\frac{\Delta v_g}{v_g} \simeq \frac{\Delta m^2}{2p^2} \qquad \Rightarrow \qquad \boxed{L_{\text{coh}} \simeq \frac{2p^2}{\Delta m^2}\sigma_x}$$

In ordinary matter:

$$\diamondsuit \qquad \frac{\Delta v_g}{v_g} \simeq \frac{\Delta m^2}{2p^2} \cdot \frac{\frac{\Delta m^2}{2p} - V_e c_2}{\sqrt{\left(\frac{\Delta m^2}{2p} c_2 - V_e\right)^2 + \left(\frac{\Delta m^2}{2p} s_2\right)^2}}$$

Typically of the same order as in vacuum (exception: close to the MSW res.).

Decoherence by wave packet separation

In dense neutrino backgrounds: the Hamiltonian $\mathcal{H}$ depends on the state of the neutrino system $\Rightarrow$ $\Delta v_g = \frac{\partial}{\partial p}\Delta E$ depends on the solution of the evolution equation.

$$\Delta E = |\vec{H}_\omega| = \sqrt{(\omega - c_{20}\mu P_0)^2 + \mu^2 P_\perp^2}\,, \qquad \omega \equiv \frac{\Delta m^2}{2p}$$

$$\Downarrow$$

$$\Delta v_g = \frac{\partial\omega}{\partial p}\left(\frac{\partial}{\partial\omega}|\vec{H}_\omega|\right) = \frac{\Delta m^2}{2p^2}\cdot\frac{\omega - c_{20}\mu P_0}{\sqrt{(\omega - c_{20}\mu P_0)^2 + \mu^2 P_\perp^2}}$$

The second factor is $\mathcal{O}(1)$ (except in vicinity of $\omega = c_{20}\mu P_0$) $\Rightarrow$

$\Delta v_g \simeq \Delta m^2/(2p^2)$.

Different strategy – contd.

⇒ $\vec{P}_\omega(t)$ satisfy the same evolution equations as all $\vec{K}_n(t)$:

$$\dot{\vec{P}}_\omega = \omega_s \vec{B} \times \vec{P}_\omega \,.$$

⇒ Asymptotic evolution of the system at late times:

All $\vec{P}_\omega$ are always in the same plane which rotates around $\vec{B}$ with a constant (but in μ-dependent) angular velocity $\omega_s = \omega_s(\mu)$. Same holds for all $\vec{K}_n$ which are various linear superpositions of $\vec{P}_\omega$.

At late times: $\boxed{\dot{\vec{P}}_\omega = \omega_s \vec{B} \times \vec{P}_\omega}$. At all times: $\boxed{\dot{\vec{P}}_\omega = (\omega \vec{B} + \mu \vec{P}) \times \vec{P}_\omega}$.
Consistency condition:

$$\omega_s(\mu) = \omega - c_{20}\mu P_0 - \frac{1}{a_\omega}\mu P_{\omega\|}$$

⇒ $\omega - \mu P_{\omega\|}/a_\omega$ must be ω-independent.

N.B: $P_{\omega\|} \equiv \vec{P}_\omega \cdot \vec{B}$ and a_ω can be of either sign.