GRAVITATIONAL WAVES FROM COALESCING COMPACT BINARIES

INTERFACING NUMERICAL AND ANALYTICAL RESULTS

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MOTIVATIONS

- Coalescing (stellar-mass) relativistic binaries: most promising sources for ground-based gravitational wave (GW) detectors: strong fields, high velocities
- Need waveform templates for detection
- Numerical modelizations (BH-BH, BH-NS,NS-NS)
- Analytical modelizations: predictions, interpretation of numerical results & construction of templates for GW detection

MATCHED FILTERING TECHNIQUE

To extract GW signal from detector's output (lost in broadband noise $S_n(f)$)

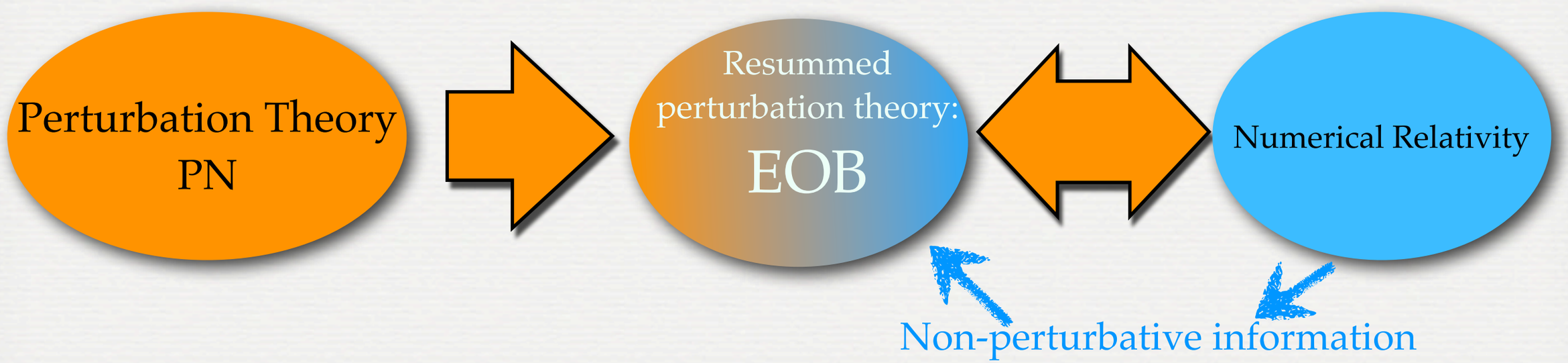
$$\langle output | h_{\text{template}} \rangle = \int \frac{df}{S_n(f)} o(f) h^*_{\text{template}}(f)$$

Detector's output

Template of expected GW signal

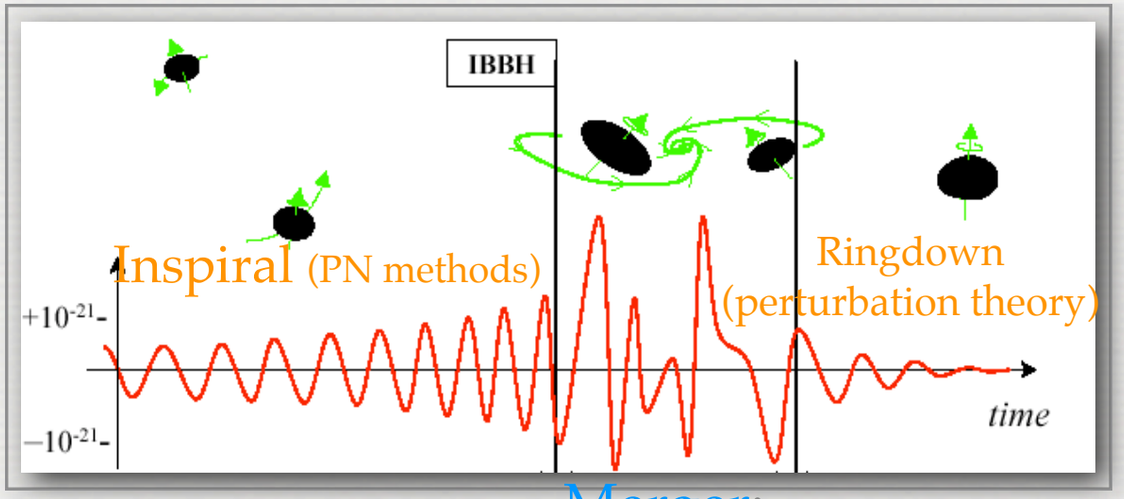
IMPORTANCE OF AN ANALYTICAL FORMALISM

- **Theoretical**: physical understanding of the coalescence process, especially in complicated situations (e.g., precessing spins).
- **Practical**: need many thousands of accurate GWs templates for detection and data analysis. Need analytical templates: $h\left(m_1, m_2, \vec{S}_1, \vec{S}_2\right)$
- **Solution**: synergy between analytical & numerical relativity

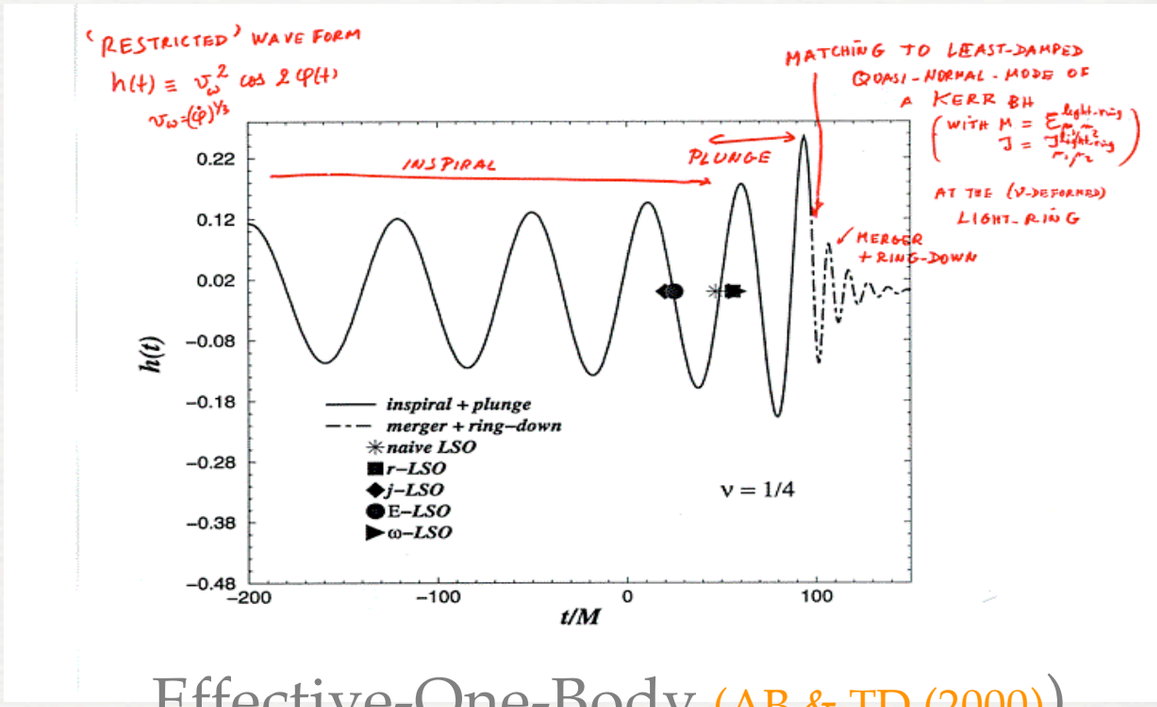


TEMPLATES FOR GWS FROM BBH COALESCENCE

Brady, Craighton & Thorne, 1998



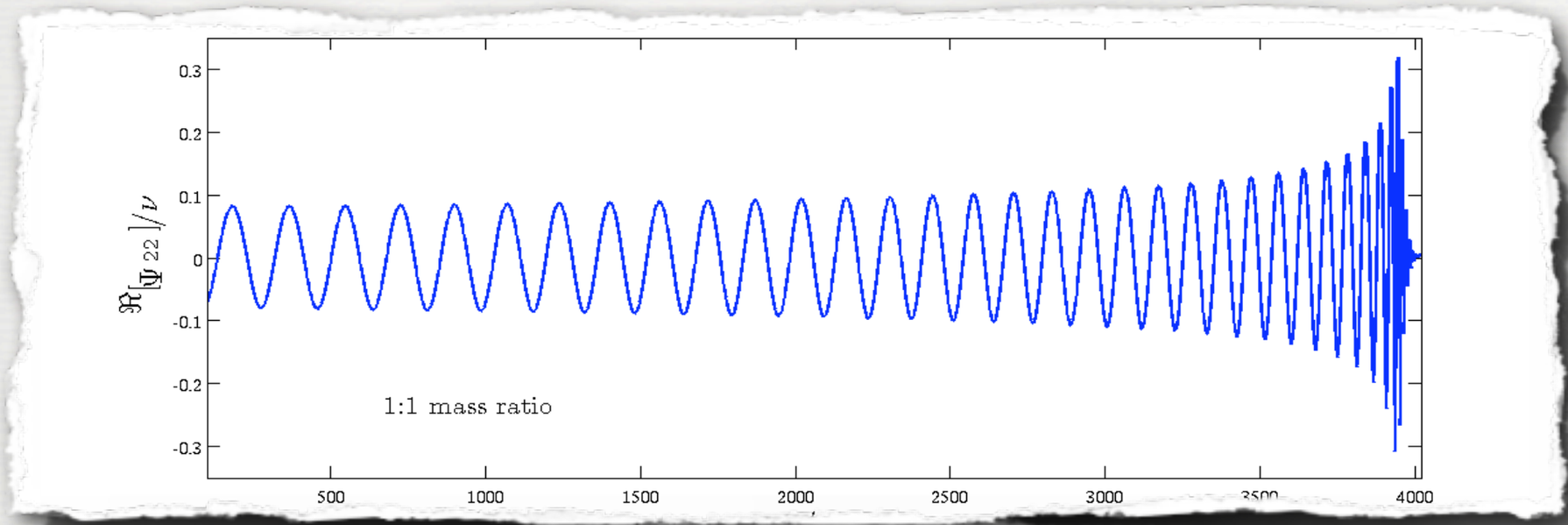
Merger:
Numerical Relativity



Effective-One-Body (AB & TD (2000))

Numerical Relativity: >= 2005 (Pretorius, Campanelli et al., Baker et al.)

Most accurate data: Caltech-Cornell spectral code (with some caveats): M. Scheel et al., 2008



Spectral code
Extrapolation (radius & resolution)

Phase error:
< 0.02 rad (inspiral)
<0.1 rad (ringdown)

A catalog of 171 high-quality binary black-hole simulations for gravitational-wave astronomy

[PRL 111 (2013) 241104]

Abdul H. Mroué,[1] Mark A. Scheel,[2] Béla Szilágyi,[2] Harald P. Pfeiffer,[1] Michael Boyle,[3] Daniel A. Hemberger,[3] Lawrence E. Kidder,[3] Geoffrey Lovelace,[4,2] Sergei Ossokine,[1,5] Nicholas W. Taylor,[2] Anıl Zenginoğlu,[2] Luisa T. Buchman,[2] Tony Chu,[1] Evan Foley,[4] Matthew Giesler,[4] Robert Owen,[6] and Saul A. Teukolsky[3]

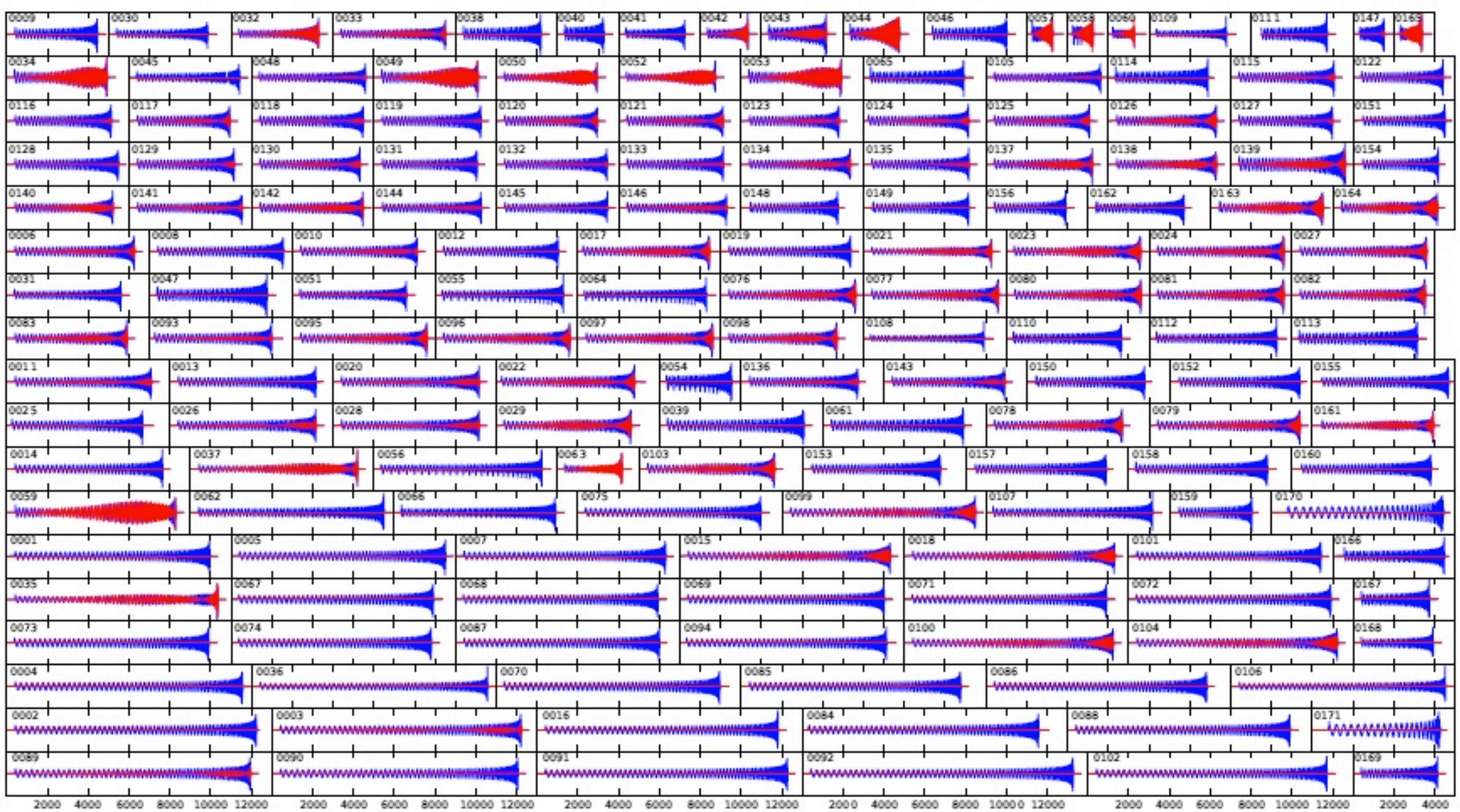


FIG. 3: Waveforms from all simulations in the catalog. Shown here are h_+ (blue) and $h_\times$ (red) in a sky direction parallel to the initial orbital plane of each simulation. All plots have the same horizontal scale, with each tick representing a time interval of $2000M$, where M is the total mass.

www.black-holes.org

SUMMARY OF BBH/BNS NR/AR RECENT RESULTS

1. NEW EOB model for coalescing [NONPRECESSING] spinning BBHs
A. Nagar, T. Damour, C. Reisswig & D. Pollney, arXiv:1506.08457
"Energetic and phasing of nonprecessing spinning black hole binaries"

2. BNS: Numerical-relativity matches EOB analytical-relativity waveforms and dynamics essentially up to merger. GW templates for LIGO/Virgo to measure EOS out of tidal effects
S. Bernuzzi, A. Nagar, T. Dietrich & T. Damour, PRL 114 (2015), 161103
"Modeling the Dynamics of Tidally Interacting Binary Neutron Stars up to Merger"
[verified/improved by Hotokezaka et al., PRD 91 (2015) 6, 064060, notably with reduced eccentricity]

3. Quasi-universality in BNS merger (binding energy, angular momentum, GW frequency vs tidal coupling constant): explained using EOB theory
S. Bernuzzi, A. Nagar, S. Balmelli, T. Dietrich & M. Ujevic, PRL 112 (2014), 201101
"Quasiuniversal properties of neutron star mergers"

4. Quasi-universality of post-merger $M f_2$ frequency vs tidal coupling constant
S. Bernuzzi, A. Nagar & T. Dietrich, arXiv:1504.01764
"Modeling the complete gravitational wave spectrum of neutron star mergers"
Unifying description of inspiral, merger and post-merger phases

EOB APPROACH IN A NUTSHELL

(Buonanno-Damour 99, 00, Damour-Jaranowski-Schäfer 00, Damour 01, Damour-Nagar 07, Damour-Iyer-Nagar 08)

key ideas:

(1) Replace two-body dynamics (m_1, m_2) by dynamics of a particle $(\mu \equiv m_1\, m_2/(m_1+m_2))$ in an effective metric $g^{\rm eff}_{\mu\nu}(u)$, with

$$u \equiv GM/c^2 R\,, \quad M \equiv m_1 + m_2$$

(2) Systematically use RESUMMATION of PN expressions (both $g^{\rm eff}_{\mu\nu}$ and $\mathcal{F}_{RR}$) based on various physical requirements

(3) Require continuous deformation w.r.t. $\nu \equiv \mu/M \equiv m_1\, m_2/(m_1+m_2)^2$ in the interval $0 \leq \nu \leq \frac{1}{4}$

STRUCTURE OF THE EOB FORMALISM

PN dynamics
(DD81,D82,DJS01,IF03,BDIF04)

PN rad losses
WW76, BDIWW95, BDEFI 05

PN waveform
BD89, B95&05,ABIQ04,

BH perturbations
RW57, Z70, Z72

Resummed (BD99)

Resummed (DIS98)

Resummed (DN07,DIN08)

QNMs spectrum
$\sigma_N = \alpha_N + i\omega_N$

EOB Hamiltonian
$H_{\rm EOB}$

EOB Rad. Reac. force
$\hat{\mathcal{F}}_\varphi$

Factorized waveform

Matching at merger time

$$h_{\ell m} = h^{(N,\epsilon)}_{\ell m}\hat{h}^{(\epsilon)}_{\ell m}$$
$$\hat{h}^{(\epsilon)}_{\ell m} = \hat{S}^{(\epsilon)}_{eff} T_{\ell m} e^{i\delta_{\ell m}} \rho^{\ell}_{\ell m}$$

$$\frac{dr}{dt} = \left(\frac{A}{B}\right)^{1/2} \frac{\partial \hat{H}_{\rm EOB}}{\partial p_{r_*}},$$
$$\frac{dp_{r_*}}{dt} = -\left(\frac{A}{B}\right)^{1/2} \frac{\partial \hat{H}_{\rm EOB}}{\partial r},$$
$$\Omega \equiv \frac{d\varphi}{dt} = \frac{\partial \hat{H}_{\rm EOB}}{\partial p_\varphi},$$
$$\frac{dp_\varphi}{dt} = \hat{\mathcal{F}}_\varphi .$$

BNS: tides
(Love numbers)

$$h^{\rm ringdown}_{\ell m}(t) = \sum_N C^+_N e^{-\sigma^+_N (t-t_m)}$$

EOB waveform

$$h^{\rm EOB}_{\ell m} = \theta(t_m - t) h^{\rm insplunge}_{\ell m}(t) + \theta(t - t_m) h^{\rm ringdown}_{\ell m}(t)$$

EXPLICIT FORM OF THE EOB HAMILTONIAN

EOB Hamiltonian

$$H_{\rm EOB} = M\sqrt{1+2\nu\left(\hat{H}_{\rm eff}-1\right)}$$

All Functions are a ν-dependent deformation of the Schwarzschild ones

$$A(r) = 1 - 2u + 2\nu u^3 + a_4 \nu u^4$$

$$a_4 = \frac{94}{3} - \frac{41}{32}\pi^2 \simeq 18.6879027$$

$$A(r)B(r) = 1 - 6\nu u^2 + 2(3\nu - 26)\nu u^3$$

$$u = GM/(c^2 R)$$

Simple effective Hamiltonian:

$$\hat{H}_{\rm eff} \equiv \sqrt{p_{r_*}^2 + A(r)\left(1 + \frac{p_\varphi^2}{r^2} + z_3 \frac{p_{r_*}^4}{r^2}\right)}$$

$$p_{r_*} = \left(\frac{A}{B}\right)^{1/2} p_r$$

Crucial EOB radial potential

Contribution at 3PN

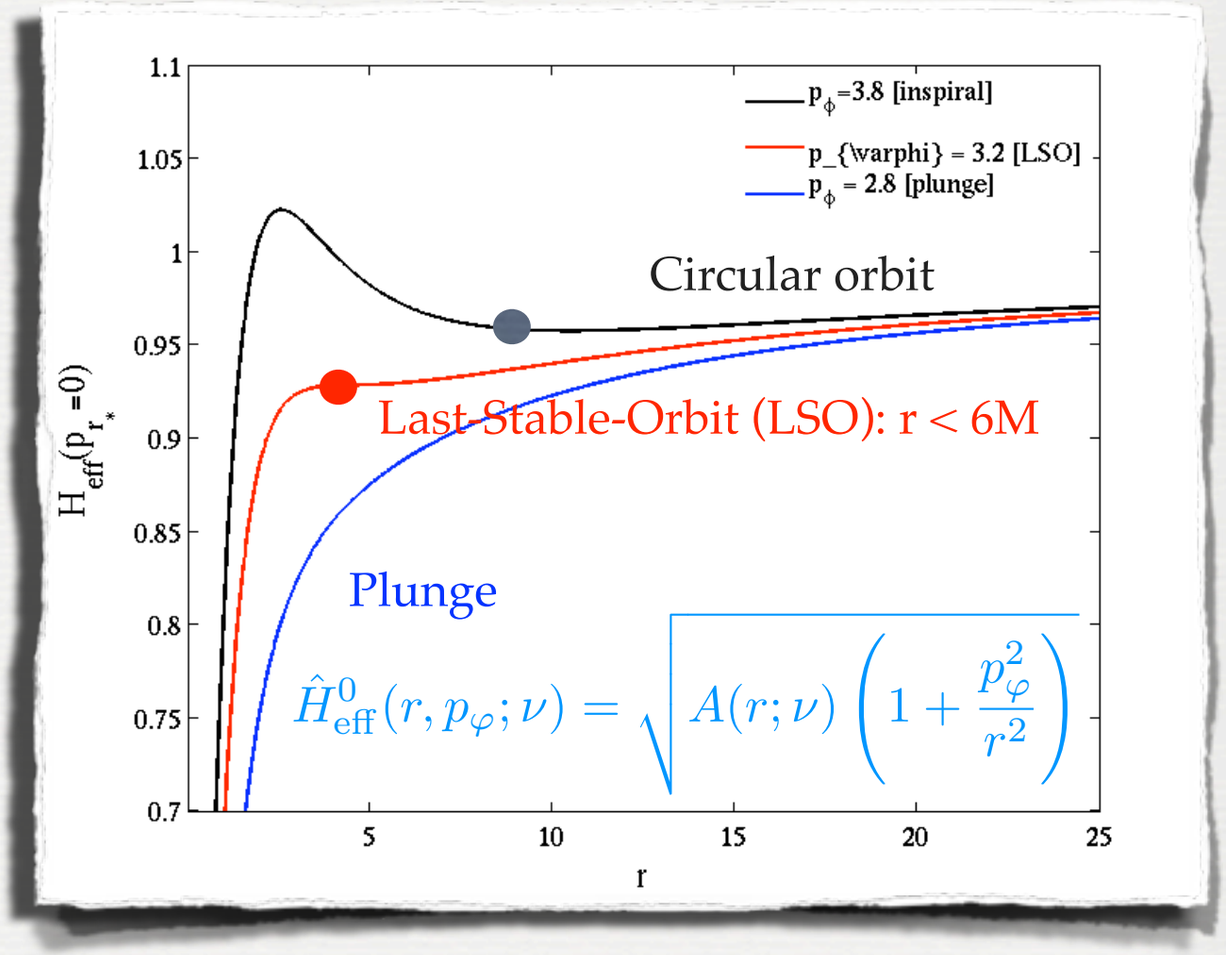
HAMILTON'S EQUATIONS & RADIATION REACTION

$$\dot{r} = \left(\frac{A}{B}\right)^{1/2} \frac{\partial \hat{H}_{\rm EOB}}{\partial p_{r_*}}$$

$$\dot{\varphi} = \frac{\partial \hat{H}_{\rm EOB}}{\partial p_\varphi} \equiv \Omega$$

$$\dot{p}_{r_*} = -\left(\frac{A}{B}\right)^{1/2} \frac{\partial \hat{H}_{\rm EOB}}{\partial r} + \hat{\mathcal{F}}_{r_*}$$

$$\dot{p}_\varphi = \hat{\mathcal{F}}_\varphi$$



- The system must radiate angular momentum
- How?Use PN-based (Taylor-expanded) radiation reaction force (ang-mom flux)
- Need flux resummation

$$\hat{\mathcal{F}}_\varphi^{\rm Taylor} = -\frac{32}{5}\nu\Omega^5 r_\Omega^4 \hat{F}^{\rm Taylor}(v_\varphi)$$

Resummation multipole by multipole
(Damour, Iyer & Nagar 2008, Damour & Nagar, 2009)

Plus horizon contribution [AN&Akcay2012]

4PN analytically complete + 5PN logarithmic term in the A(u) function:

[Damour 2009, Blanchet et al. 2010, Barack, Damour & Sago 2010, Le Tiec et al. 2011, Barausse et al. 2011,Akcay et al. 2012, Bini& Damour2013, DamourJaranowski&Schaefer 2014].

$$A_{5\mathrm{PN}}^{\mathrm{Taylor}} = 1 - 2u + 2\nu u^3 + \left(\frac{94}{3} - \frac{41}{32}\pi^2\right)\nu u^4 + \nu[a_5^c(\nu) + a_5^{\ln}\ln u]u^5 + \nu[a_6^c(\nu) + a_6^{\ln}\ln u]u^6$$

1PN 2PN 3PN 4PN 5PN

$$\left.\begin{aligned} a_5^{\log} &= \frac{64}{5} \\ a_5^c &= a_{5_0}^c + \nu a_{5_1}^c \\ a_{5_0}^c &= -\frac{4237}{60} + \frac{2275}{512}\pi^2 + \frac{256}{5}\log(2) + \frac{128}{5}\gamma \\ a_{5_1}^c &= -\frac{221}{6} + \frac{41}{32}\pi^2 \end{aligned}\right\}$$

4PN fully known ANALYTICALLY!

$$a_6^{\log} = -\frac{7004}{105} - \frac{144}{5}\nu$$

5PN logarithmic term (analytically known)

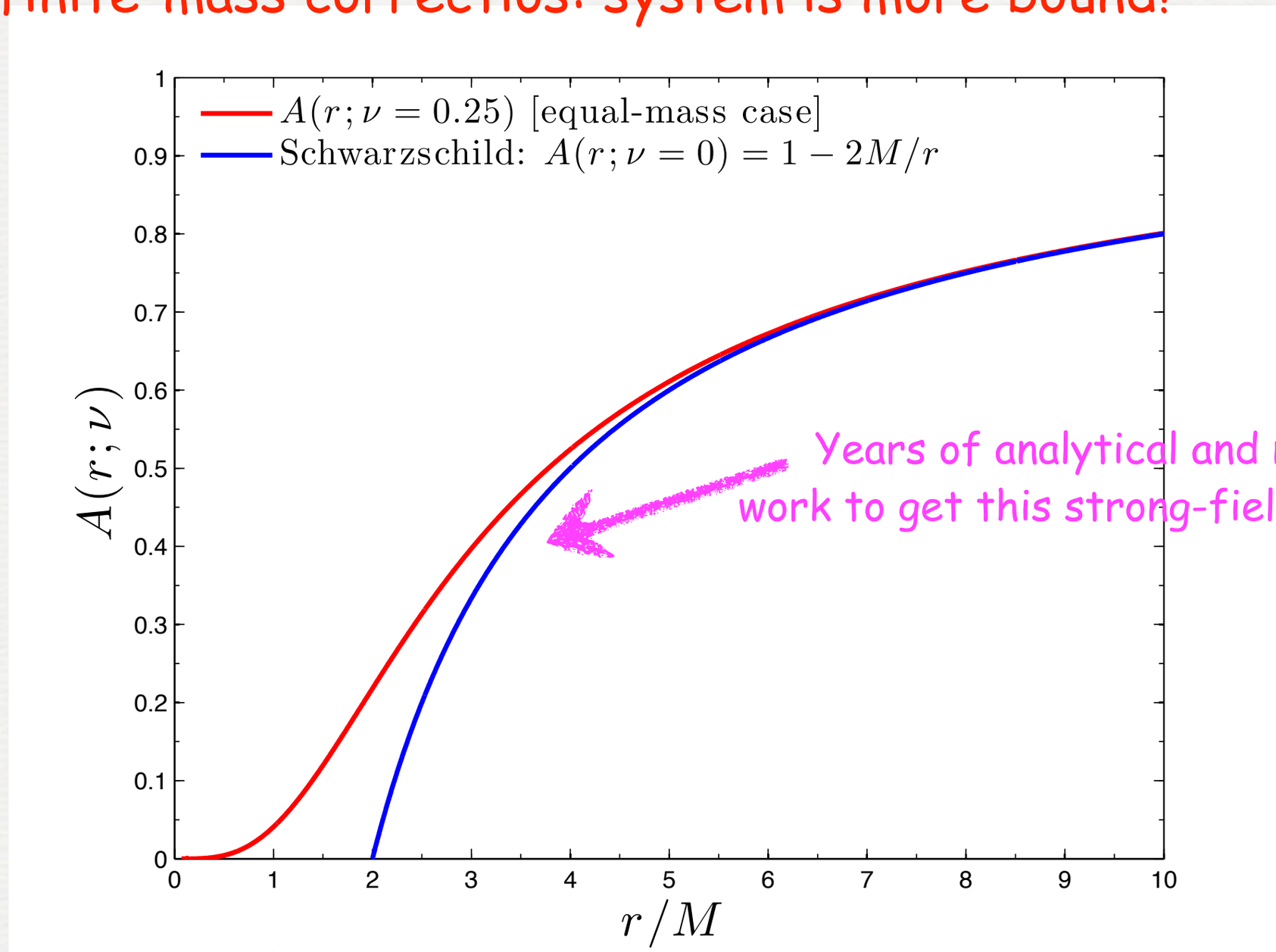
NEED ONE "effective" 5PN parameter from NR waveform data: $a_6^c(\nu)$

State-of-the-art EOB potential (5PN-resummed):

$$A(u;\nu,a_6^c) = P_5^1[A_{5\mathrm{PN}}^{\mathrm{Taylor}}(u;\nu,a_6^c)]$$

THE EOB[NR] POTENTIAL

Effect of finite-mass correctios: system is more bound!



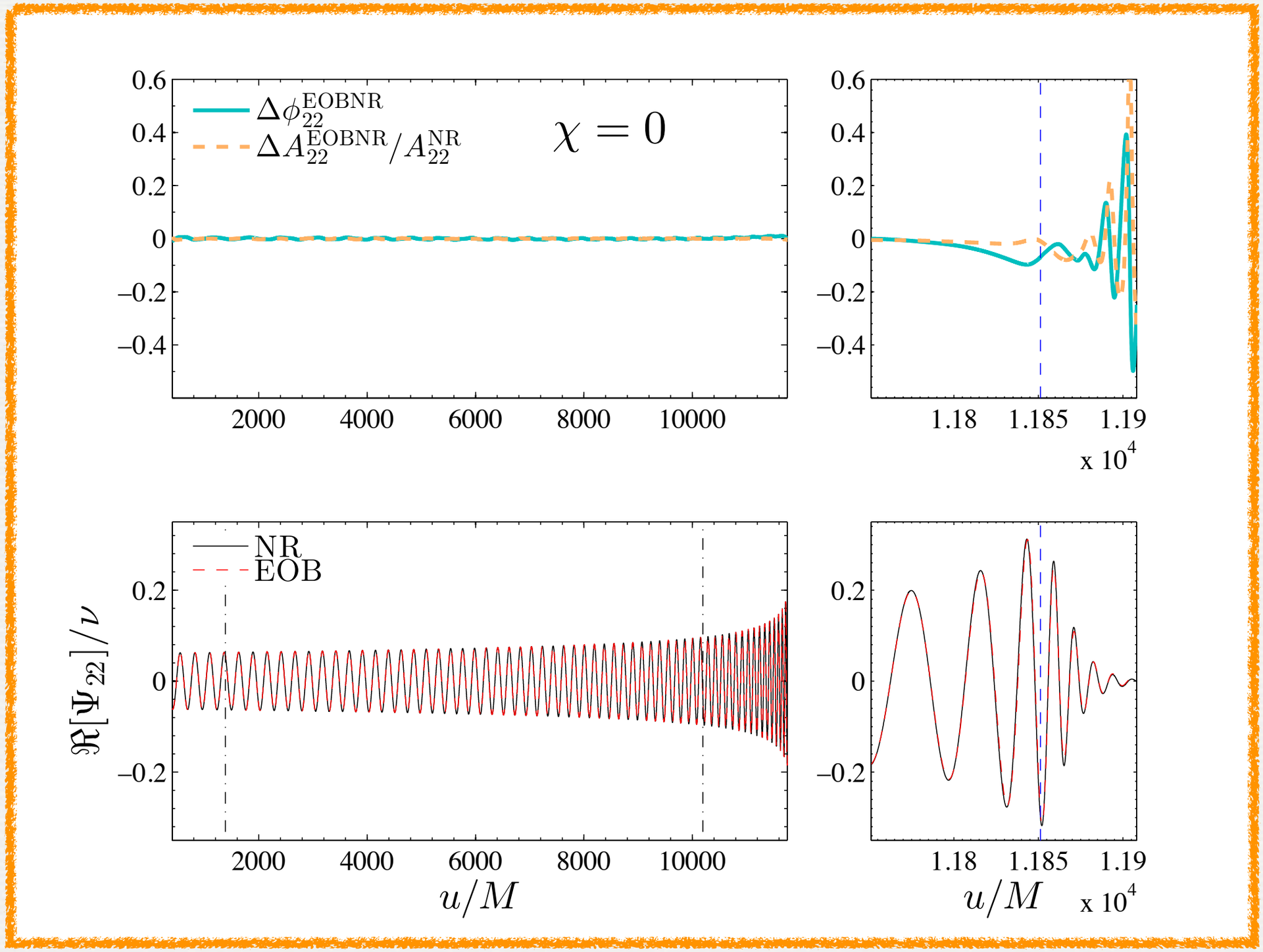
Years of analytical and numerical work to get this strong-field difference!

From (6 nonspinning) NR data sets one gets:

$$a_6^c(\nu) = 3097.3\nu^2 - 1330.6\nu + 81.3804$$

NDRP, arXiv:1506.08457

RESULTS: WAVEFORMS (NO SPIN)

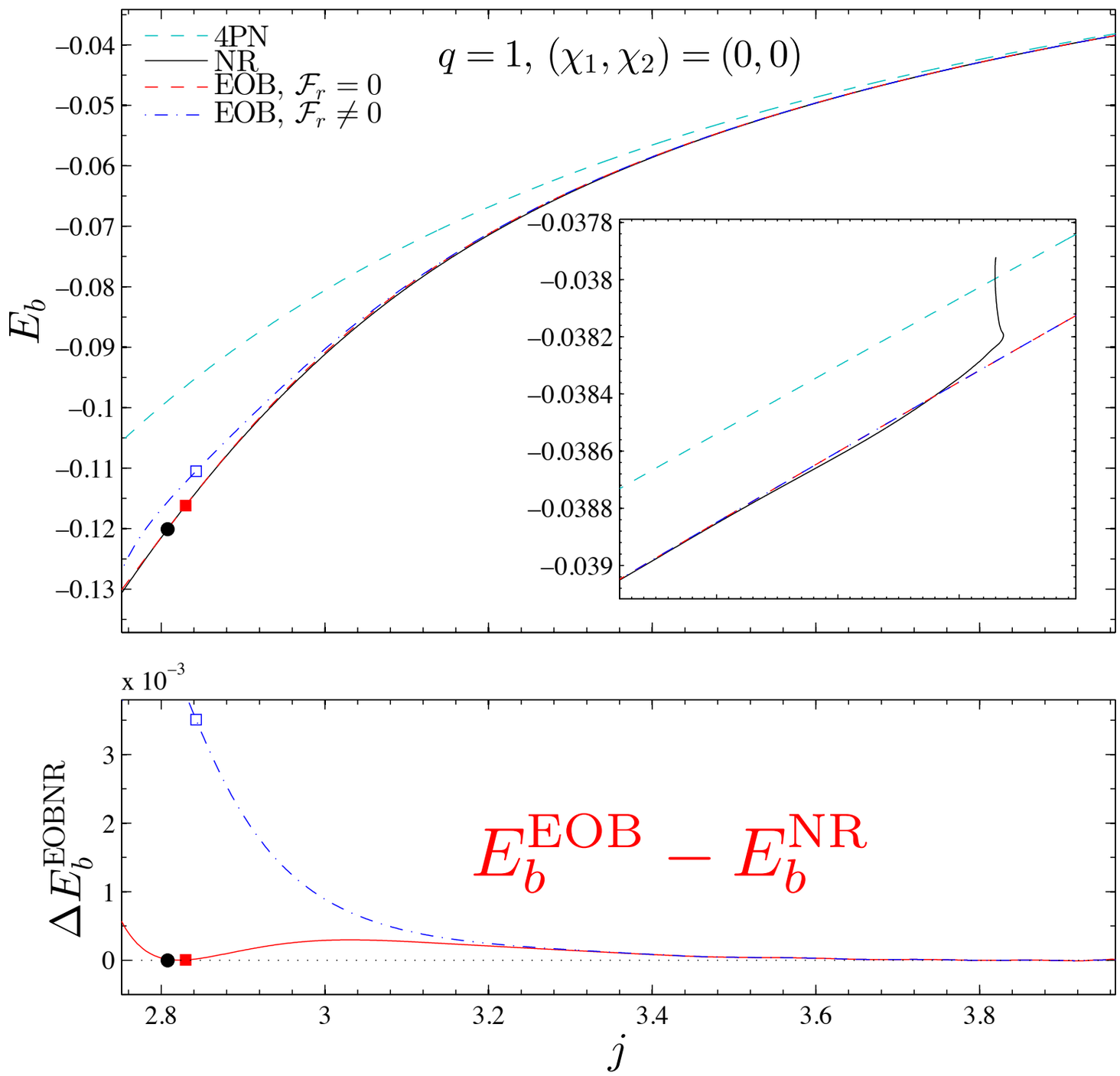


equal-mass case

NDRP, arXiv:1506.08457

ENERGETICS - NONSPINNING

Binding energy vs angular momentum (Llama NR data)



$$E_b = \frac{E - Mc^2}{\mu}$$

NDRP, arXiv:1506.08457

EOB APPROACH TO THE DYNAMICS OF TWO SPINNING BLACK HOLES

Damour01, Buonanno-Chen-Damour06, Damour-Jaranowski-Schafer08, Barausse&Buonanno10,Nagar11,Barausse&Buonanno2011,Taracchini et al. 12, Balmelli&Jetzer2013, Pan et al. 2013

Nonspinning case: EOB description = deformation of test-particle dynamics in a Schwarzschild background

Spinning case: EOB description = deformation of (spinning) test-particle dynamics in a Kerr background

Deformation parameters:

$\nu = \mu/M$ and "effective test spin" $\mathbf{S}^*$

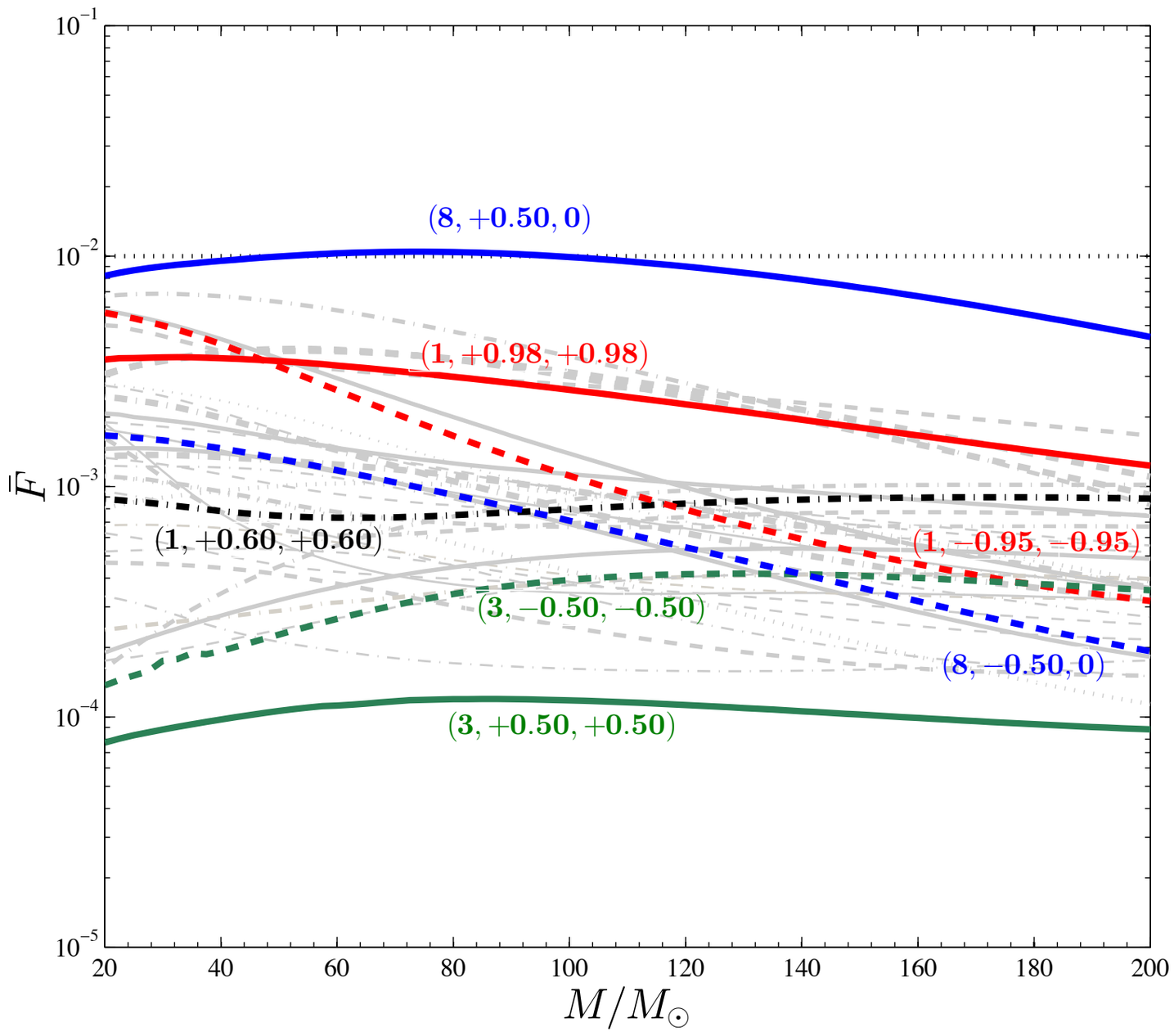
Based on Hamiltonian formulation in the center of mass frame

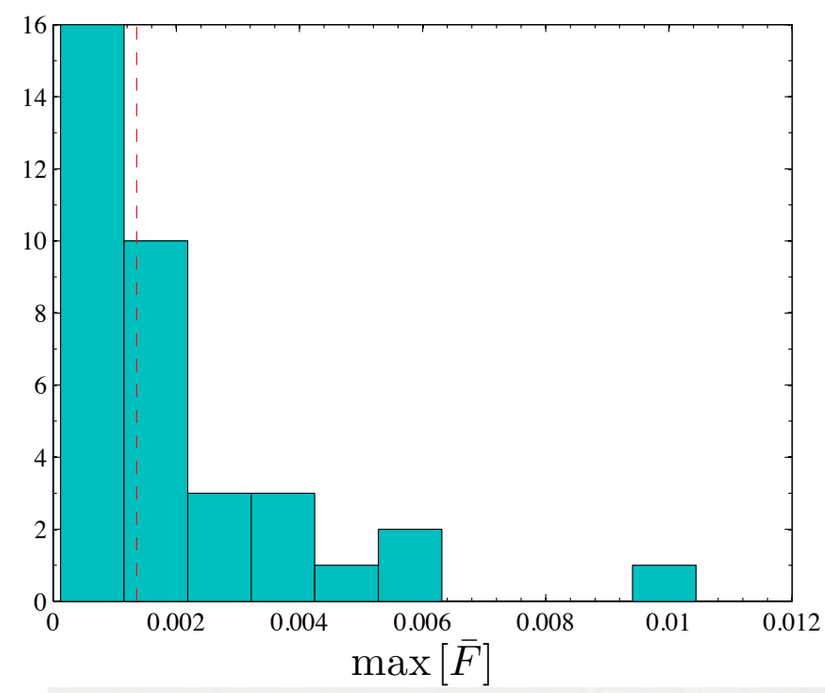
UNFAITHFULNESS

38 spin-aligned SXS NR waveforms

$$\bar{F} \equiv 1 - \max_{t_0,\phi_0} \frac{\langle h_{22}^{\rm EOB}, h_{22}^{\rm NR}\rangle}{||h_{22}^{\rm EOB}||\,||h_{22}^{\rm NR}||}$$

$$\langle h_1, h_2\rangle \equiv 4\Re \int_{f_{\rm min}}^{\infty} \tilde{h}_1(f)\tilde{h}_2^*(f)/S_n(f)\,df$$





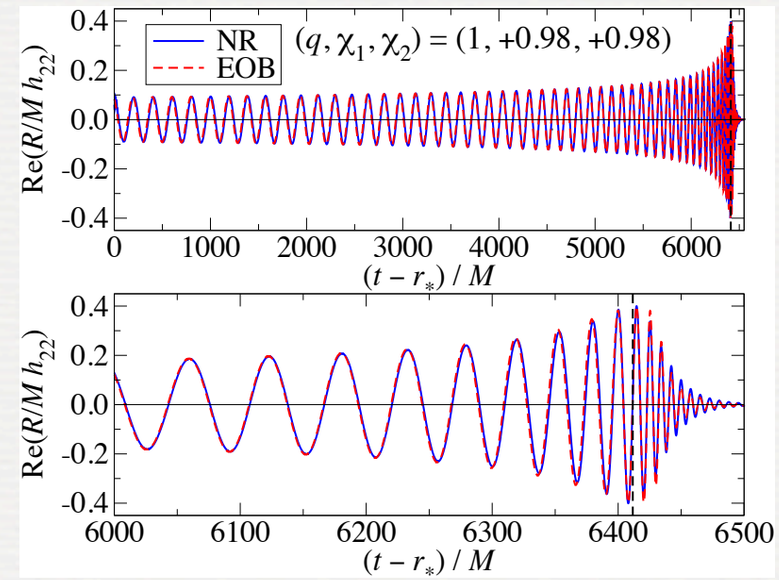
NDRP, arXiv:1506.08457

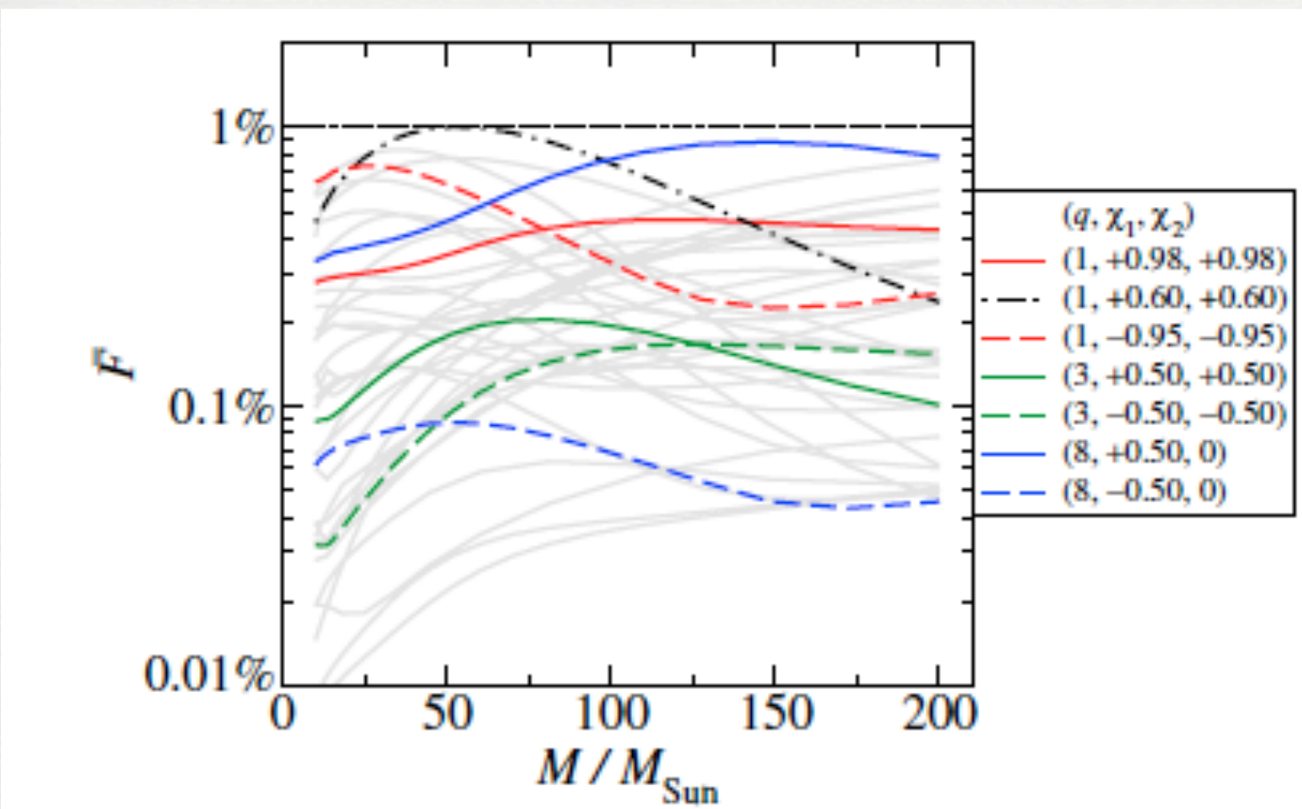
COMPARISON WITH SEOBNRV2

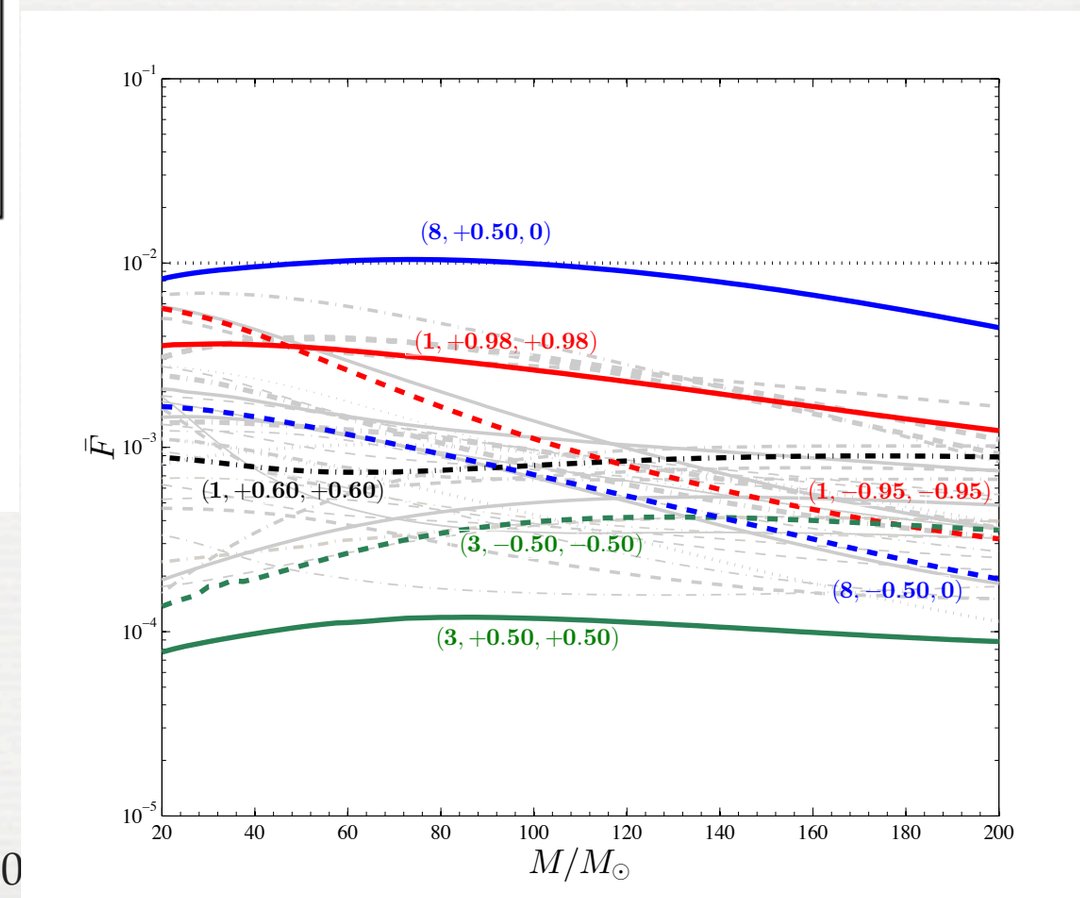
Different EOB Hamiltonian [Barausse & Buonanno11, Taracchini et al.12]

Different choices for the analytic freedom (as well as spin gauge)

Taracchini, Buonanno et al., PRD **89**, 061502 (R), 2014







BBH TAKE-AWAY

1. New way of blending finite-mass and spin effect in an EOB Hamiltonian based on the structure of the Hamiltonian of a (spinning) particle on a Kerr background

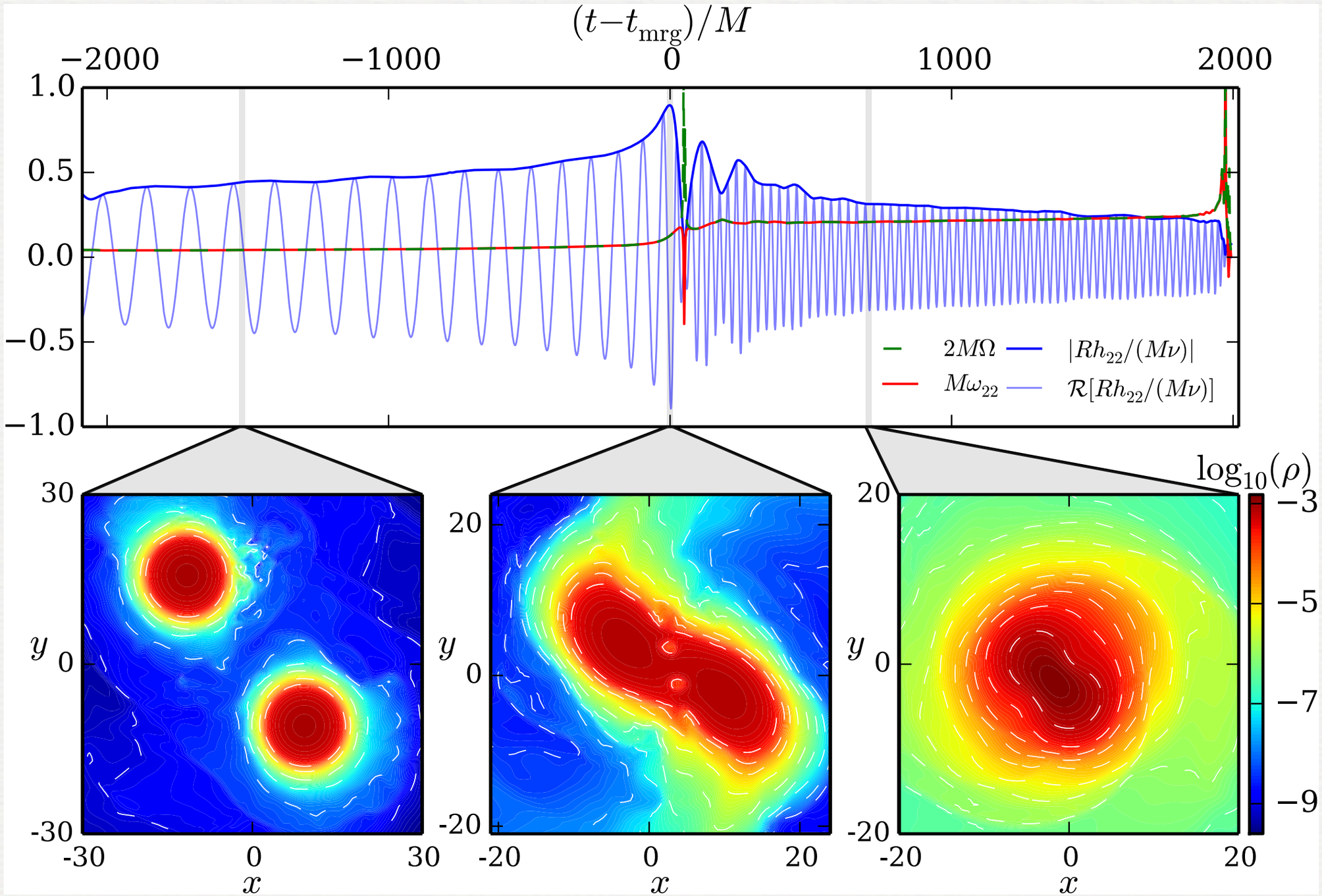
2. Analytical freedom: only two flexibility parameters that are extracted from NR data as simple (separate) functions of symmetric mass ratio and spin magnitude

3. Compatibility (within NR errors) between such EOBNR model and state-of-the art NR data over mass ratio and spin (no precession for the moment)

BINARY NEUTRON STARS

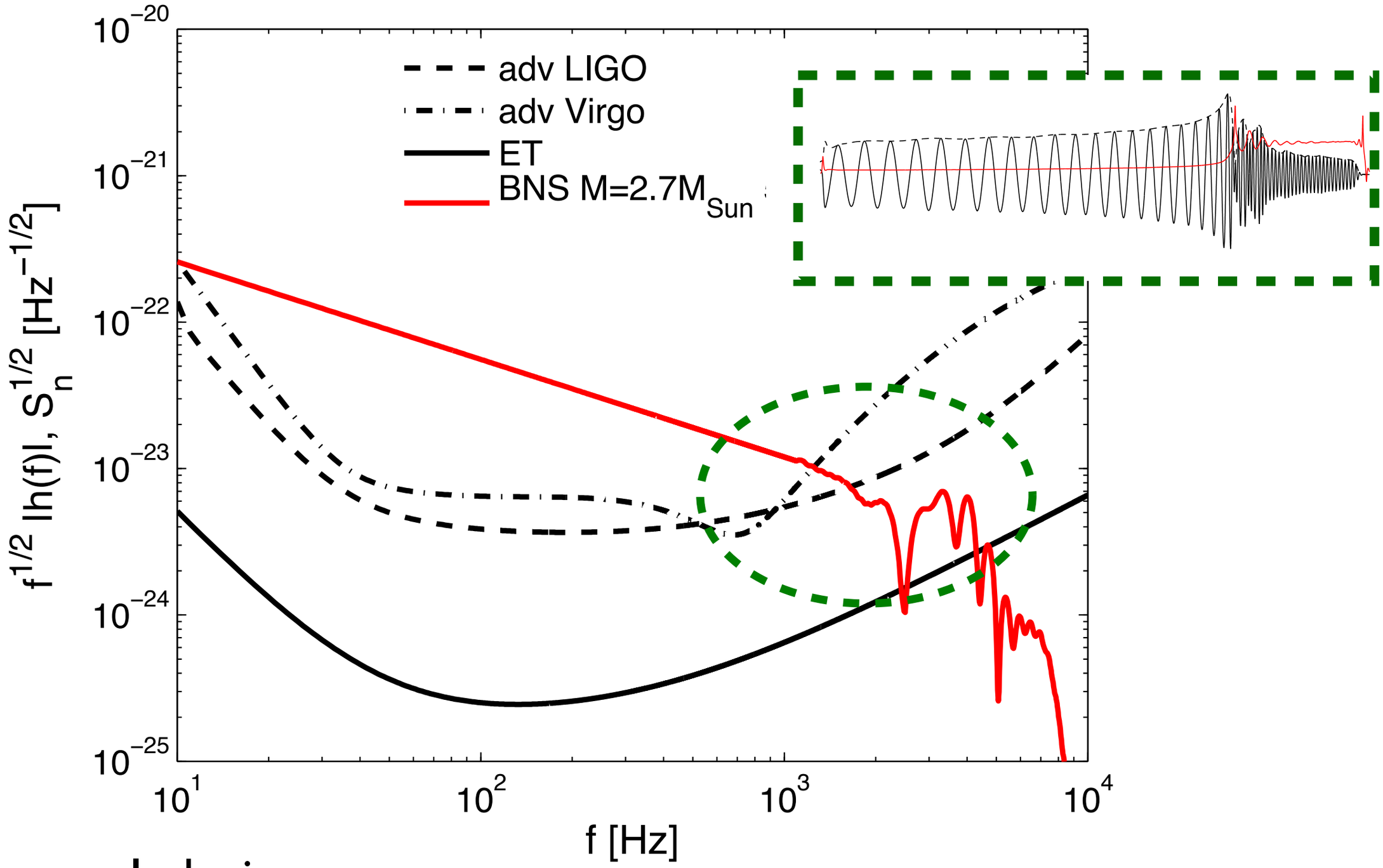
A. Nagar, TAUP - Torino 2015

BNS (NR) WAVEFORM



S. Bernuzzi, T. Dietrich & A. Nagar, PRL 115 (2015), 9, 091101

BNS: EOS detection



Induced Inducing

$$M_{ij} = \mu_2 G_{ij}$$

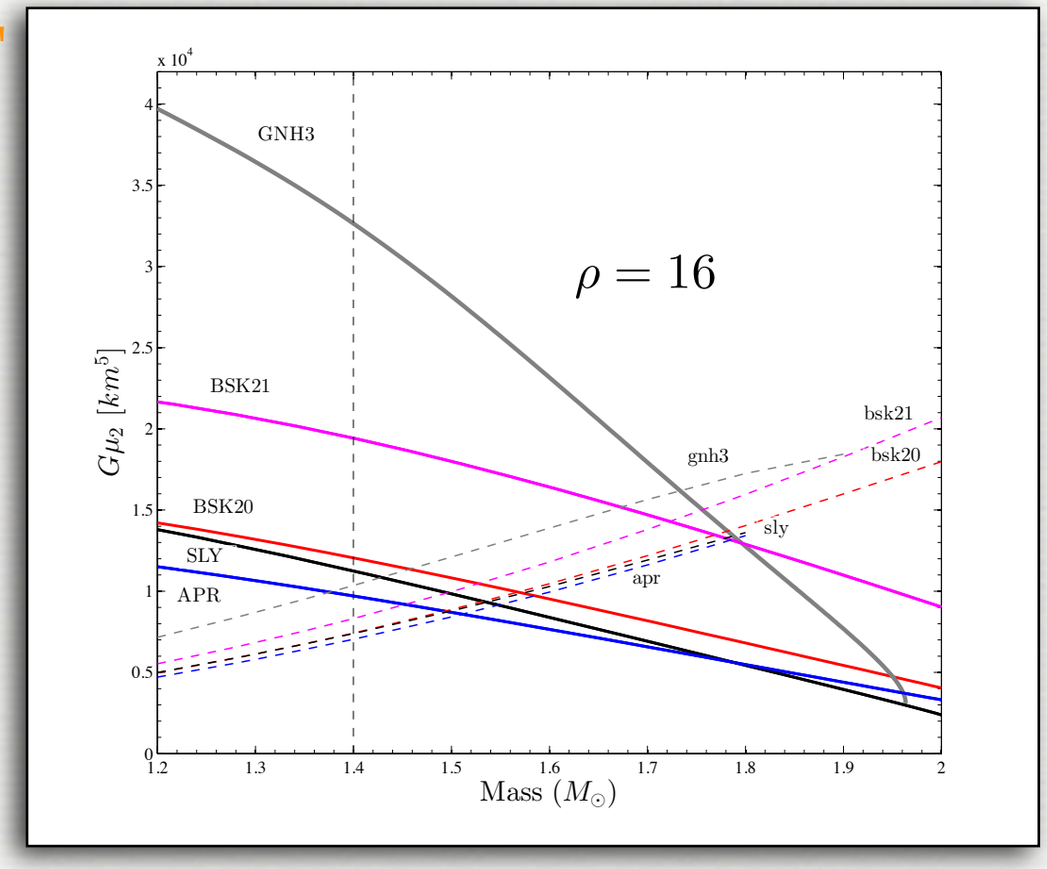
EOS detection by measuring the tidal polarizability coefficient: the Love number

MEASURING LOVE WITH GWS?

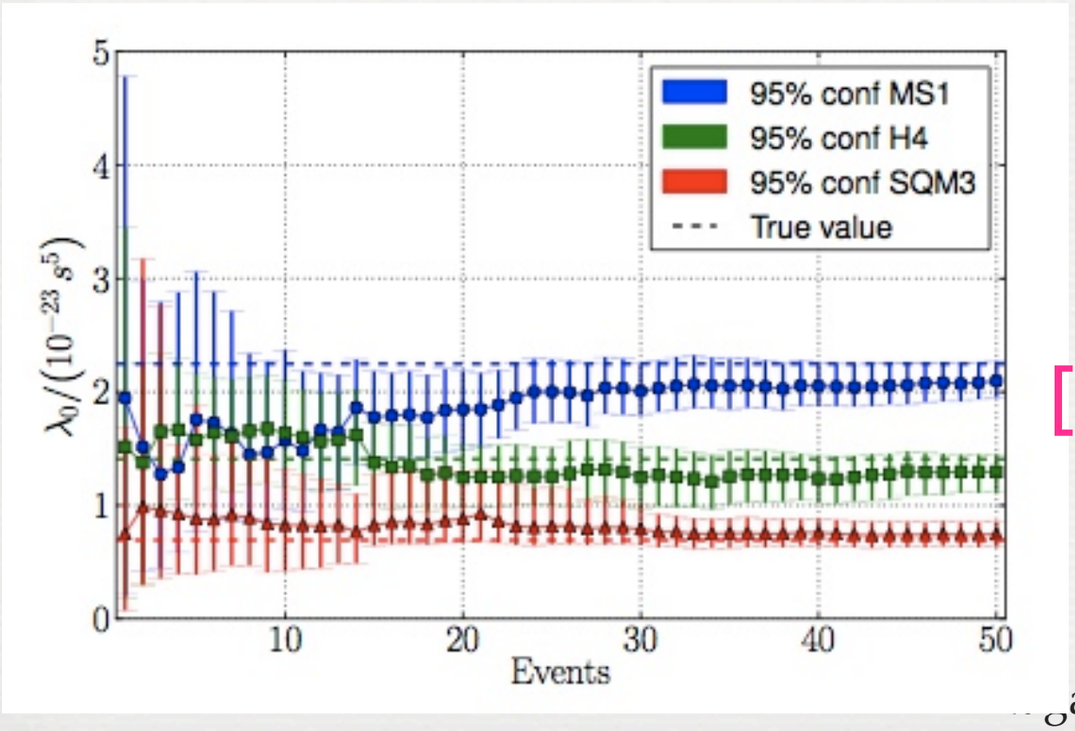
CRUCIAL RESULT

(Damour ,Nagar, Villain 2012)

Tidal polarizability parameters *can actually be measured* by adv LIGO with a reasonable SNR=16 (5 events per year)



Use EOB controlled, accurate, description of the phasing up to BNS merger!



[Del Pozzo et al. 2013]

THREE BNS RESULTS

1. Numerical-relativity matches effective-one-body (EOB) analytical-relativity waveforms and dynamics essentially up to merger. Method to compute GW templates for LIGO/Virgo to measure EOS out of tidal effects

S. Bernuzzi, A. Nagar, T. Dietrich & T. Damour, PRL 114 (2015), 161103

"Modeling the Dynamics of Tidally Interacting Binary Neutron Stars up to Merger"

[checked by Hotokezaka et al., PRD 91 (2015) 6, 064060, notably with reduced eccentricity]

2. Quasi-universality in BNS merger (binding energy, angular momentum, GW frequency vs tidal coupling constant): explained using EOB theory

S. Bernuzzi, A. Nagar, S. Balmelli, T. Dietrich & M. Ujevic, PRL 112 (2014), 201101

"Quasiuniversal properties of neutron star mergers"

3. Quasi-universality of post-merger $M f_2$ frequency vs tidal coupling constant

S. Bernuzzi, T. Dietrich & A. Nagar, PRL 115 (2015), 9, 091101

"Modeling the complete gravitational wave spectrum of neutron star mergers"

Unifying description of inspiral, merger and post-merger phases

BNS:ANALYTICAL NEEDS

- Study the response of each neutron star to the tidal field of the companion [theory of relativistic Love numbers (i.e. tidal polarizability coefficients) + tidal corrections to dynamics (beyond Newtonian accuracy)]
- Incorporate the corresponding tidal effects within a theoretical framework able to describe the gravitational wave signal emitted by inspiralling compact binaries (possibly up to merger): PN-expanded description vs EOB-resummed description.
- Compare analytical models against NR simulations, possibly calibrating high-order tidal corrections if needed
- Assess the measurability of tidal effects within the signal seen by interferometric detectors

TIDAL EFFECTS IN EOB FORMALISM

Tidal extension of EOB formalism: nonminimal worldline couplings

$$\Delta S_{\rm nonminimal} = \sum_A \frac{1}{4}\mu_2^A \int ds_A \left(u^\mu u^\nu R_{\mu\alpha\nu\beta}\right)^2 + \ldots$$

Damour&Esposito-Farèse96, Goldberger&Rothstein06, TD&AN09

Relativistic Love number

Modifications of the EOB effective metric...

$$\begin{aligned} A(r) &= A_r^0 + A^{\rm tidal}(r) \\ A^{\rm tidal}(r) &= -\kappa_2^T u^6 \left(1 + \bar{\alpha}_1 u + \bar{\alpha}_2 u^2 + \ldots\right) + \ldots \end{aligned}$$

And tidal modifications of GW waveform & radiation reaction

- Need analytical theory for computing $\mu_2,\ \kappa_2^T,\ \bar{\alpha}_1 \ldots$
- (?)Need accurate NR simulations to "calibrate" the higher-order PN tidal contributions, that may be quite important during the late inspiral

TIDAL INTERACTION POTENTIAL

Central tidal "coupling constant":

$$\kappa_\ell^T \equiv 2\left[\frac{1}{q}\left(\frac{X_A}{C_A}\right)^{2\ell+1} k_\ell^A + q\left(\frac{X_B}{C_B}\right)^{2\ell+1} k_\ell^B\right]$$

$X_{A,B} \equiv M_{A,B}/M \qquad C_{A,B} \equiv M_{A,B}/R_{A,B}$

$M_A = M_B$
$\ell = 2$
$k_2^A = k_2^B$

$$\kappa_2^T = \frac{1}{8}\frac{k_2}{\mathcal{C}^5}$$

Function of: mass ratio, compactnesses and relativistic Love numbers

In the dynamics:

$$\kappa_2^T \sim 100$$

$$A(u) = A^0(u) + A^{\rm tidal}$$

$$A^{\rm tidal} = \sum_{\ell\geq 2} -\kappa_\ell^T u^{2\ell+2}\hat{A}_\ell^{\rm tidal}(u)$$

"Newtonian" (LO) part
+ PN corrections (NLO, NNLO, ...)

NLO & NNLO tidal PN corrections known analytically

[Bini, Damour& Faye 2011]

$$\hat{A}_2^{\rm tidal} = 1 + \frac{5}{4}u + \frac{85}{14}u^2$$

TIDAL COUPLING CONSTANT

$$\kappa_\ell^T \equiv 2\left[\frac{1}{q}\left(\frac{X_A}{C_A}\right)^{2\ell+1} k_\ell^A + q\left(\frac{X_B}{C_B}\right)^{2\ell+1} k_\ell^B\right]$$

$$q = M_A/M_B \leq 1$$

$$X_A \equiv M_A/M = q/(1+q)$$

$$X_B \equiv M_B/M = 1/(1+q)$$

A tidally-interacting binary is dynamically characterized by:

- mass ratio
- Love number(s)
- Compactness(es)

$$\kappa_\ell^T$$

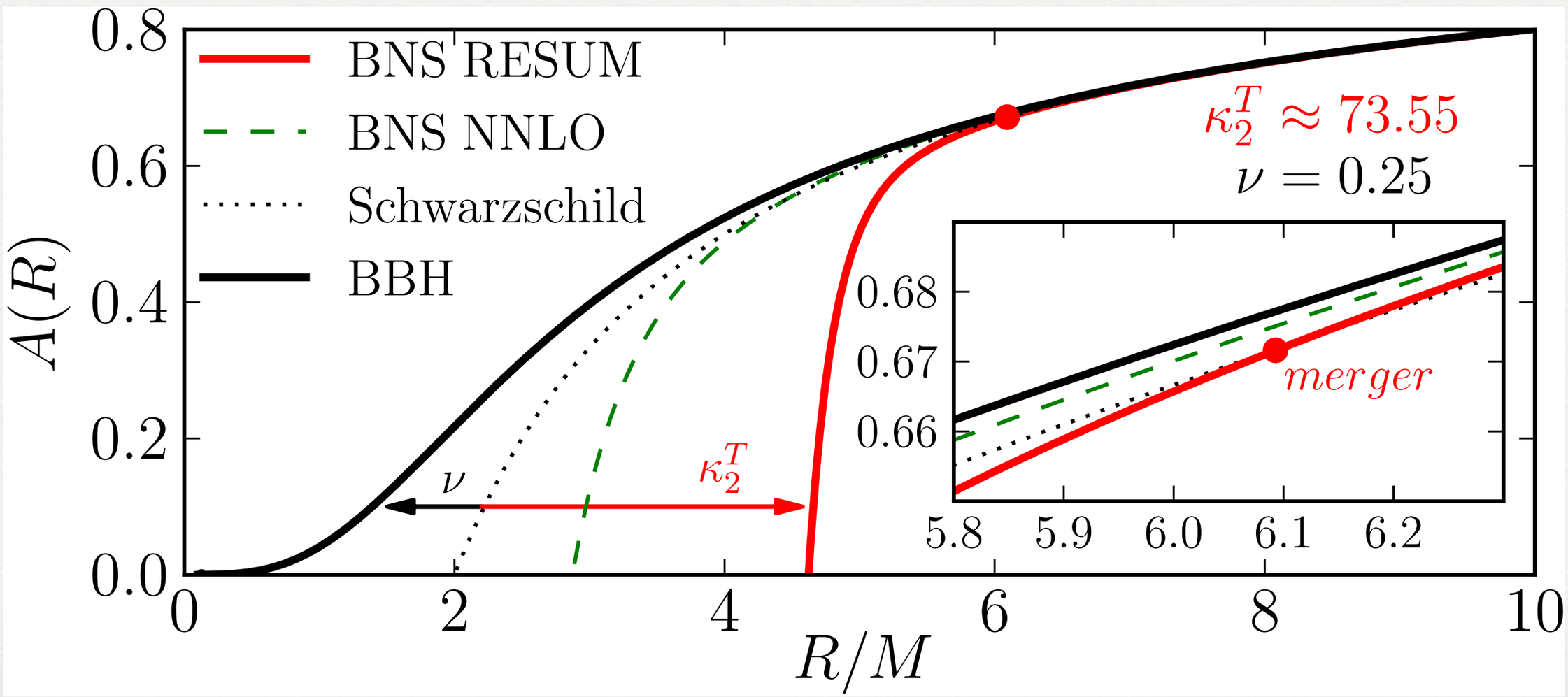
RESUMMED TIDAL INTERACTION

Bini&Damour (2015) resummed expression for $\hat{A}_\ell^{\rm tidal}$

Presence of a pole: potential strongly attractive @ mrg

$$A_T^{(+)}(u;\nu) \equiv -\sum_{\ell=2}^{4}\left[\kappa_A^{(\ell)} u^{2\ell+2}\hat{A}_A^{(\ell^+)} + (A \leftrightarrow B)\right]$$

$$\hat{A}_A^{(2^+)}(u) = 1 + \frac{3u^2}{1-r_{\rm LR}u} + \frac{X_A\tilde{A}_1^{(2^+)1{\rm SF}}}{(1-r_{\rm LR}u)^{7/2}} + \frac{X_A^2\tilde{A}_2^{(2^+)2{\rm SF}}}{(1-r_{\rm LR}u)^{p}}$$



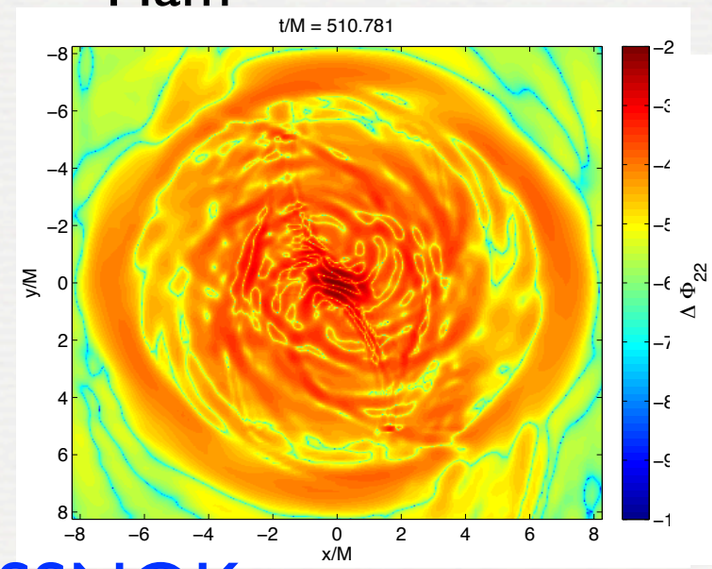
BNS SIMULATIONS: Z4C

Bernuzzi & Hilditch, PRD81 (2010) - Z4c - 1D (collapse and RNS)

Ruiz et al., PRD83 (2011) - 024025 - boundary conditions

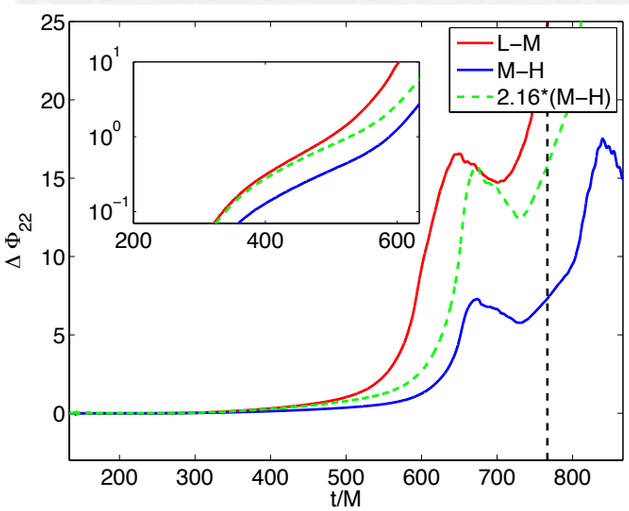
Hilditch, Bernuzzi et al., PRD88 (2013), 084057 - Z4c - 3D (BNS)

Ham

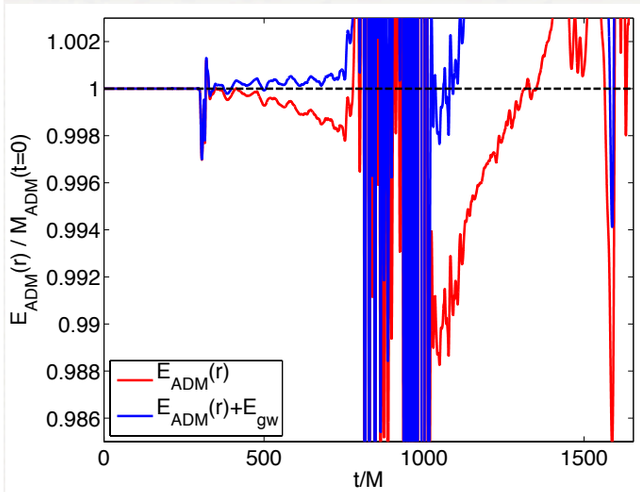


BSSNOK

GW phase conv.



ADM mass



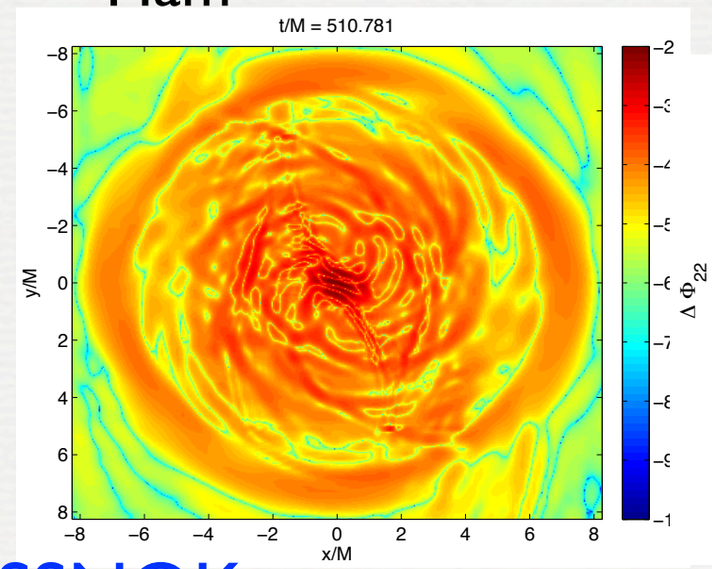
BNS SIMULATIONS: Z4C

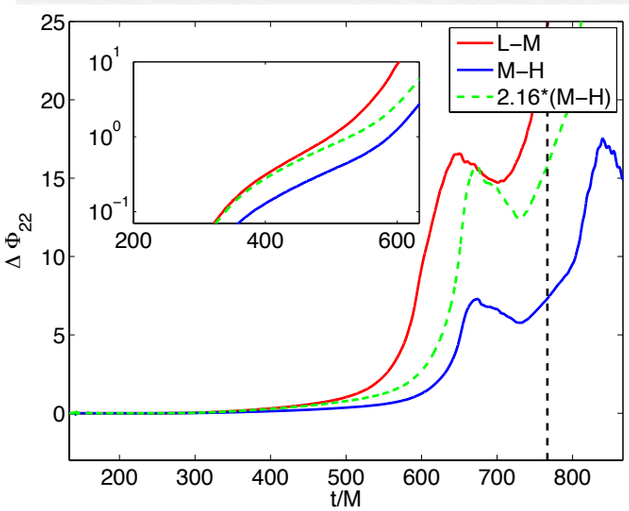
Bernuzzi & Hilditch, PRD81 (2010) - Z4c - 1D (collapse and RNS)

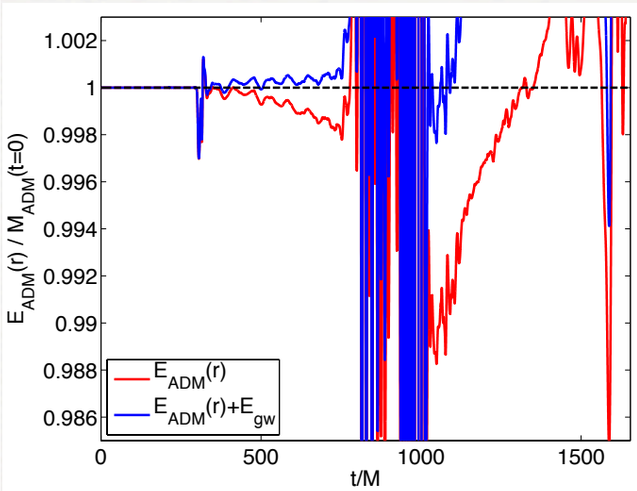
Ruiz et al., PRD83 (2011) - 024025 - boundary conditions

Hilditch, Bernuzzi et al., PRD88 (2013), 084057 - Z4c - 3D (BNS)

Ham | GW phase conv. | ADM mass







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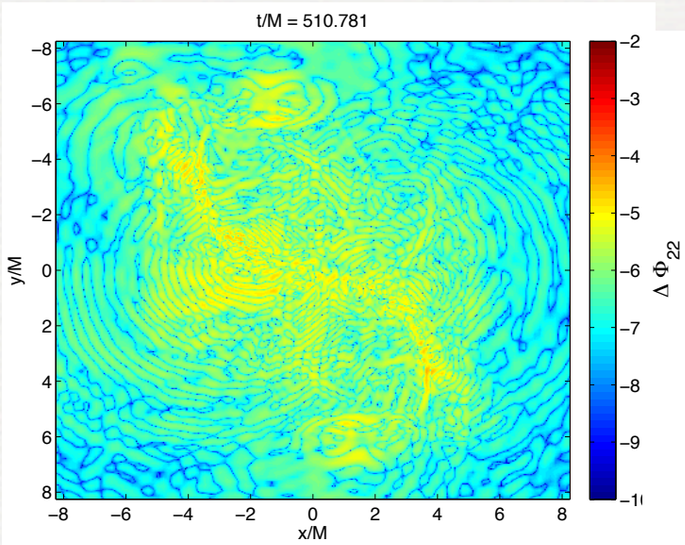
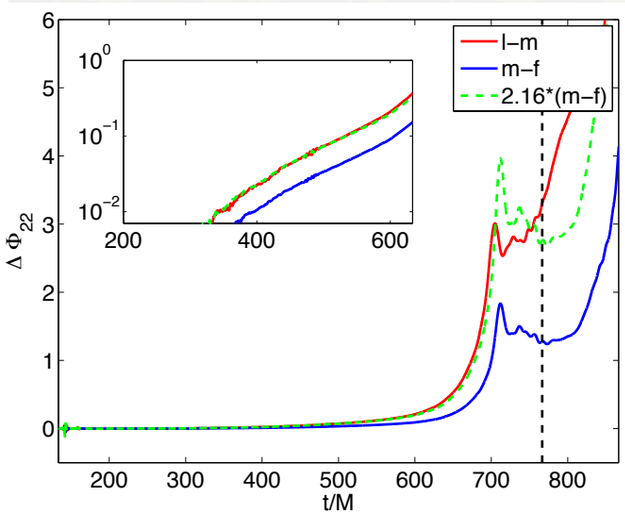
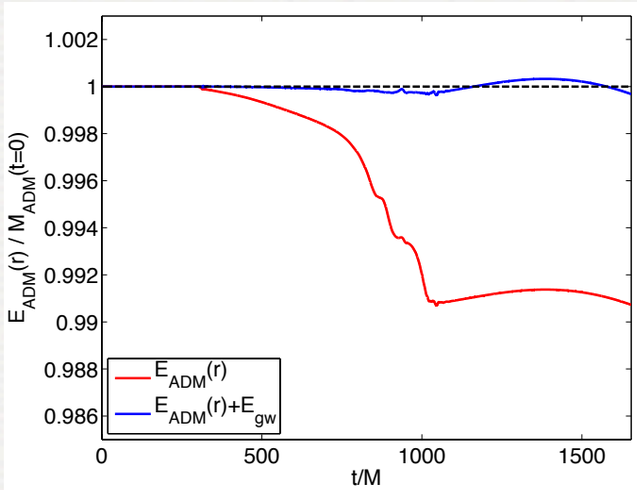


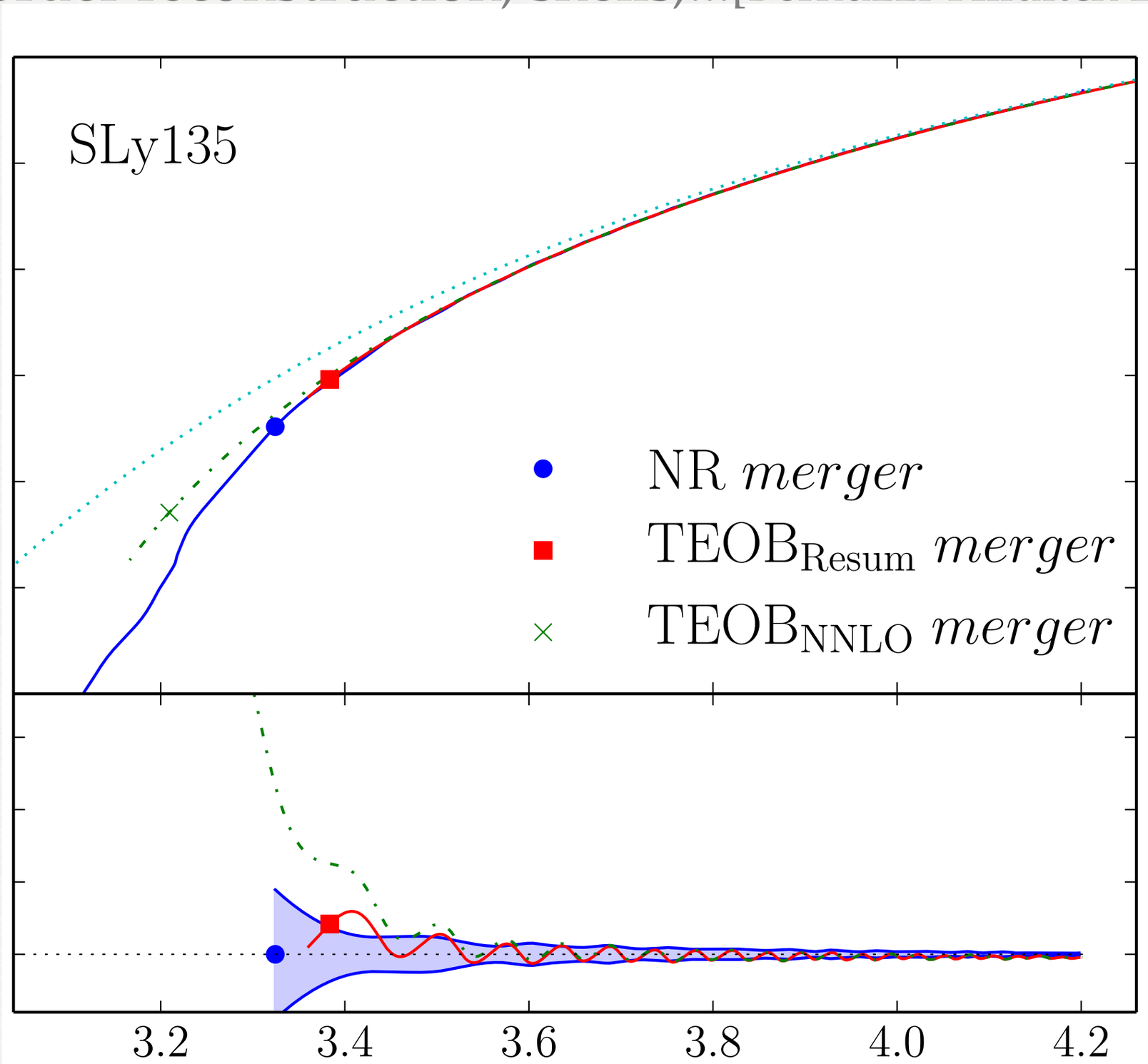




Z4c

NEW BNS SIMULATIONS

Z4c, high-order reconstruction, shells,...[Bernuzzi-Hilditch 2008 +]



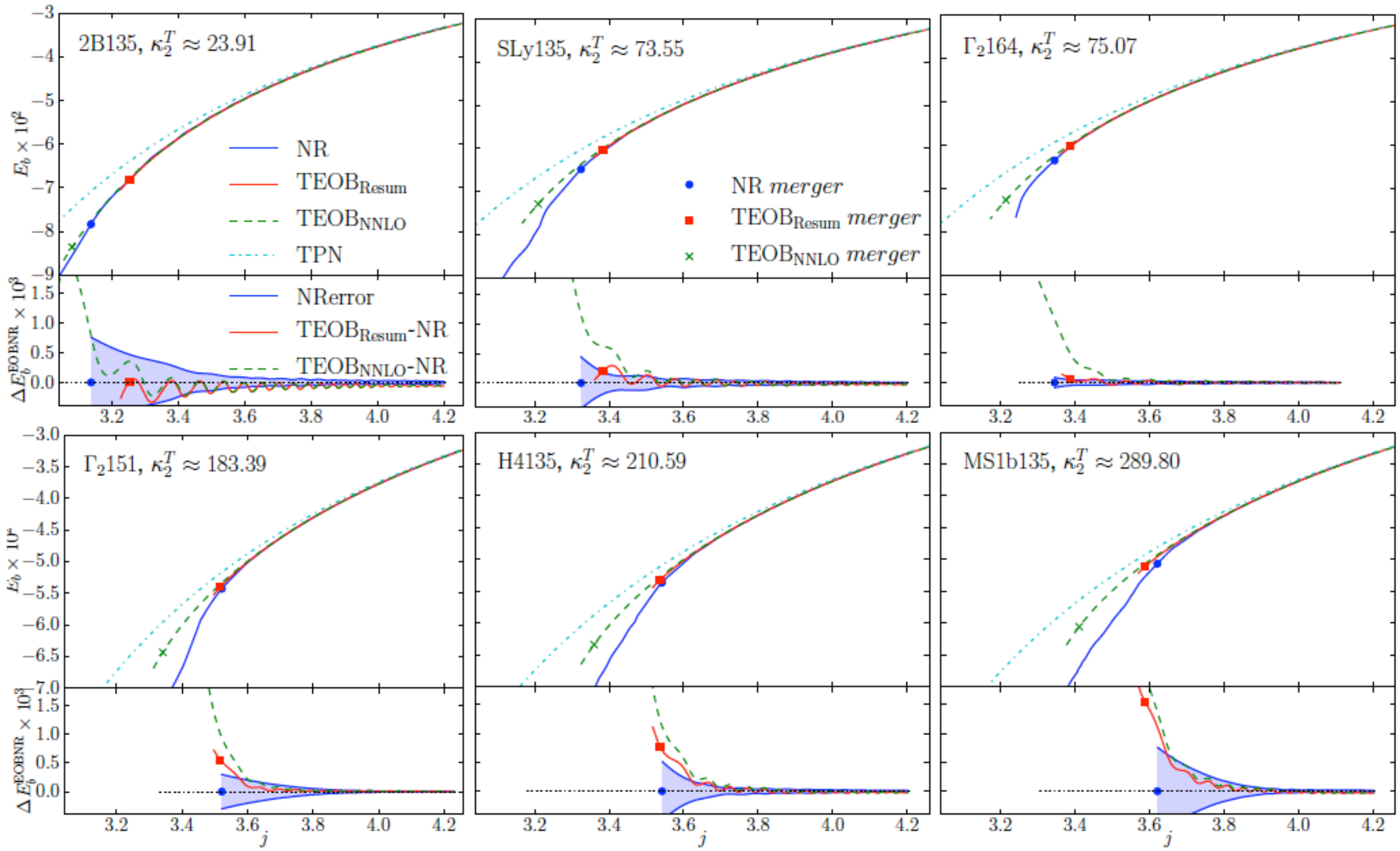


FIG. 2: Energetics: comparison between NR data, TEOB$_{\rm Resum}$, TEOB$_{\rm NNLO}$ and TPN. Each bottom panel shows the two EOB-NR differences. The filled circles locate the merger points (top) and the corresponding differences (bottom). The shaded area indicates the NR uncertainty. The TEOB$_{\rm Resum}$ model displays, globally, the smallest discrepancy with NR data (notably for merger quantities), supporting the theoretical, light-ring driven, amplification of the relativistic tidal factor.

S. Bernuzzi, A. Nagar, T. Dietrich & T. Damour, PRL 114 (2015), 161103

WAVEFORM

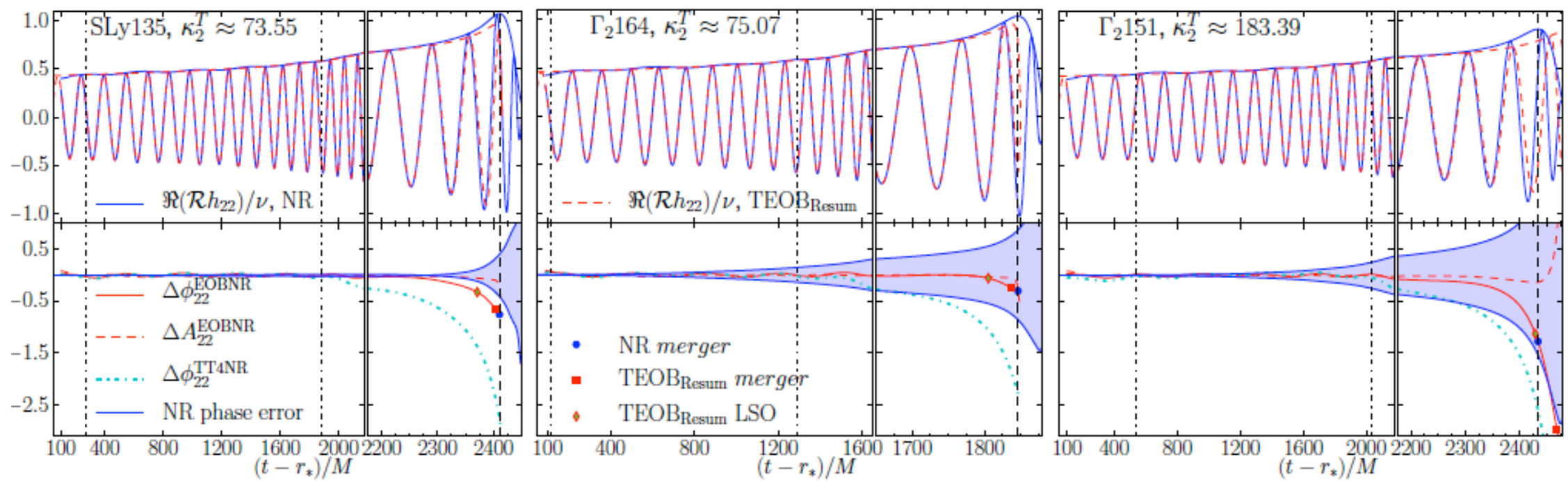


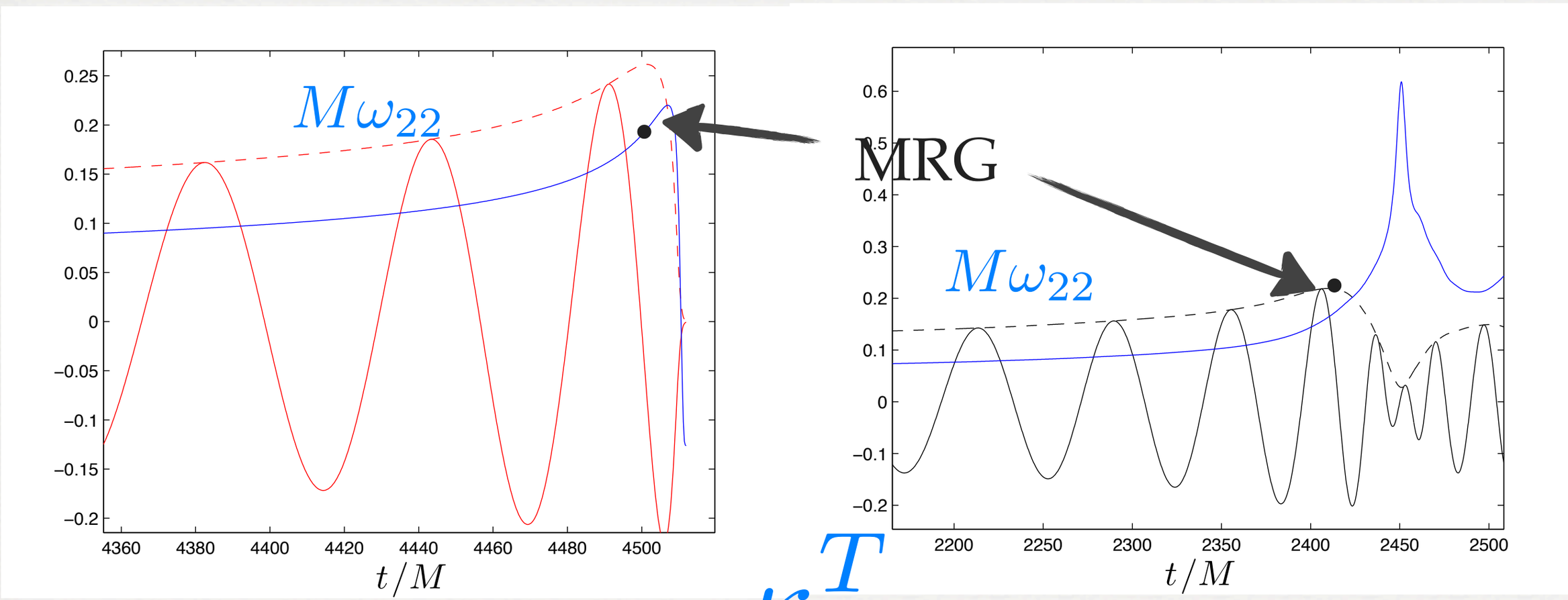
FIG. 3: Phasing and amplitude comparison (versus NR retarded time) between TEOB$_{\rm Resum}$, NR and the phasing of TT4 for three representative models. Waves are aligned on a time window (vertical dot-dashed lines) corresponding to $I_\omega \approx (0.04, 0.06)$. The markers in the bottom panels indicate: the crossing of the TEOB$_{\rm Resum}$ LSO radius; NR (also with a dashed vertical line) and EOB merger moments.

Name	EOS	κ_2^T	$r_{\rm LR}$	$\mathcal{C}_{A,B}$	$M_{A,B}[M_\odot]$	$M^0_{\rm ADM}[M_\odot]$	$\mathcal{J}^0_{\rm ADM}[M_\odot^2]$	$\Delta\phi^{\rm TT4}_{\rm NRmrg}$	$\Delta\phi^{\rm TEOB_{NNLO}}_{\rm NRmrg}$	$\Delta\phi^{\rm TEOB_{Resum}}_{\rm NRmrg}$	$\delta\phi^{\rm NR}_{\rm NRmrg}$
2B135	2B	23.9121	3.253	0.2049	1.34997	2.67762	7.66256	−1.25	−0.19	$+0.57^a$	±4.20
SLy135	SLy	73.5450	3.701	0.17381	1.35000	2.67760	7.65780	−2.75	−1.79	−0.75	±0.40
Γ_2164	$\Gamma = 2$	75.0671	3.728	0.15999	1.64388	3.25902	11.11313	−2.29	−1.36	−0.31	±0.90
Γ_2151	$\Gamma = 2$	183.3911	4.160	0.13999	1.51484	3.00497	9.71561	−2.60	−1.92	−1.27	±1.20
H4135	H4	210.5866	4.211	0.14710	1.35003	2.67768	7.66315	−3.02	−2.43	−1.88	±1.04
MS1b135	MS1b	289.8034	4.381	0.14218	1.35001	2.67769	7.66517	−3.25	−2.84	−2.45	±3.01

EOS quasi-UNIVERSALITY OF BNS MERGER

S. Bernuzzi, A. Nagar, S. Balmelli, T. Dietrich and M. Ujevic, PRL 112 (2014) 201101

- GW frequency
- binding energy
- angular momentum
- GW amplitude
- ....

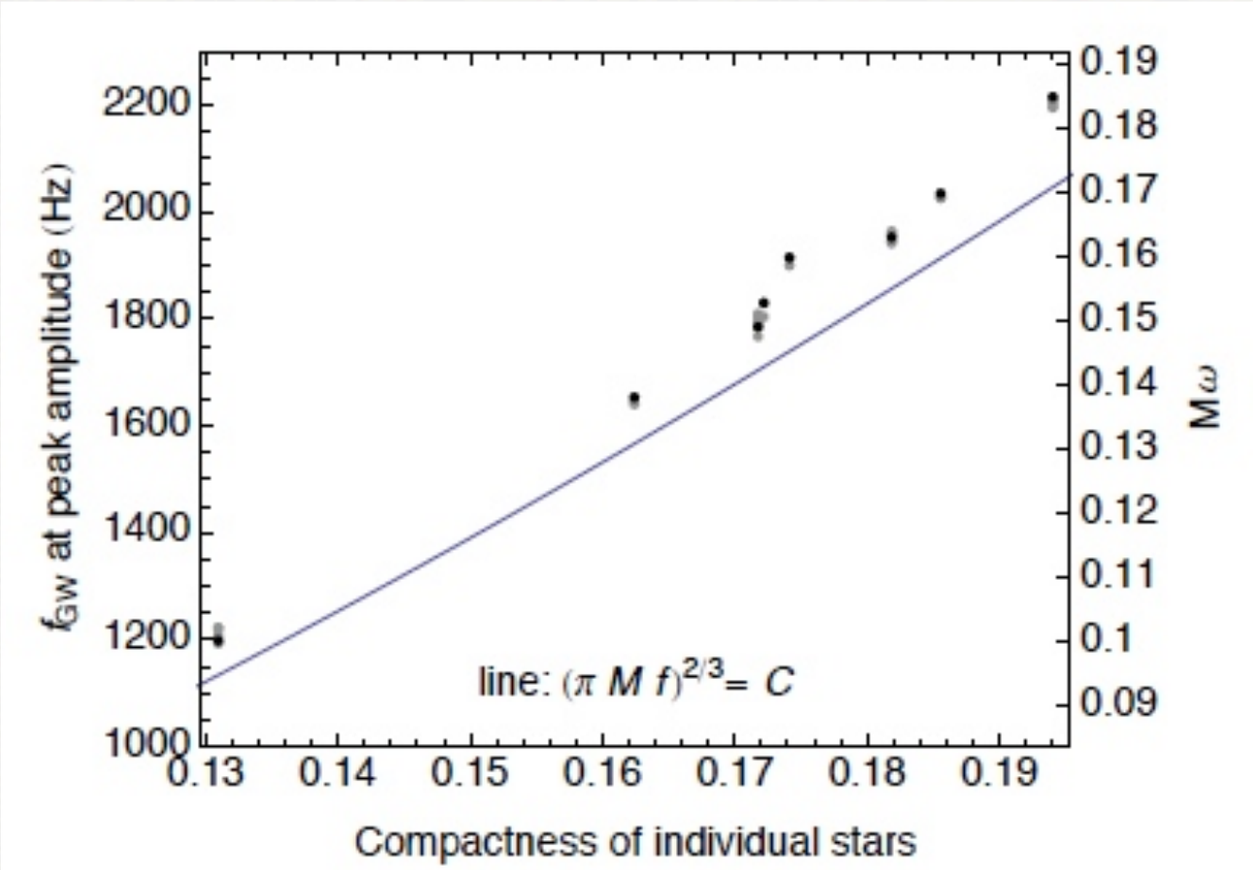


κ_2^T

EOS QUASI-UNIVERSALITY OF BNS MERGER

Matter effects on binary neutron star waveforms

Jocelyn S. Read,[1,2] Luca Baiotti,[3,4] Jolien D. E. Creighton,[5] John L. Friedman,[5] Bruno Giacomazzo,[6] Koutarou Kyutoku,[5] Charalampos Markakis,[7,9] Luciano Rezzolla,[8] Masaru Shibata,[4] and Keisuke Taniguchi[10]



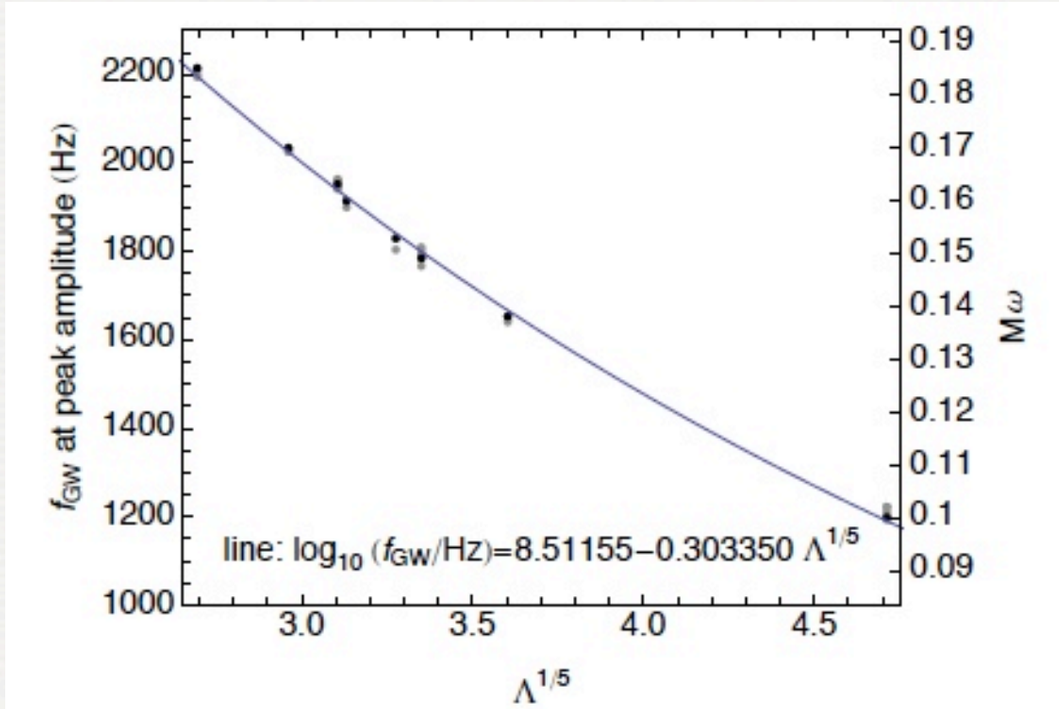


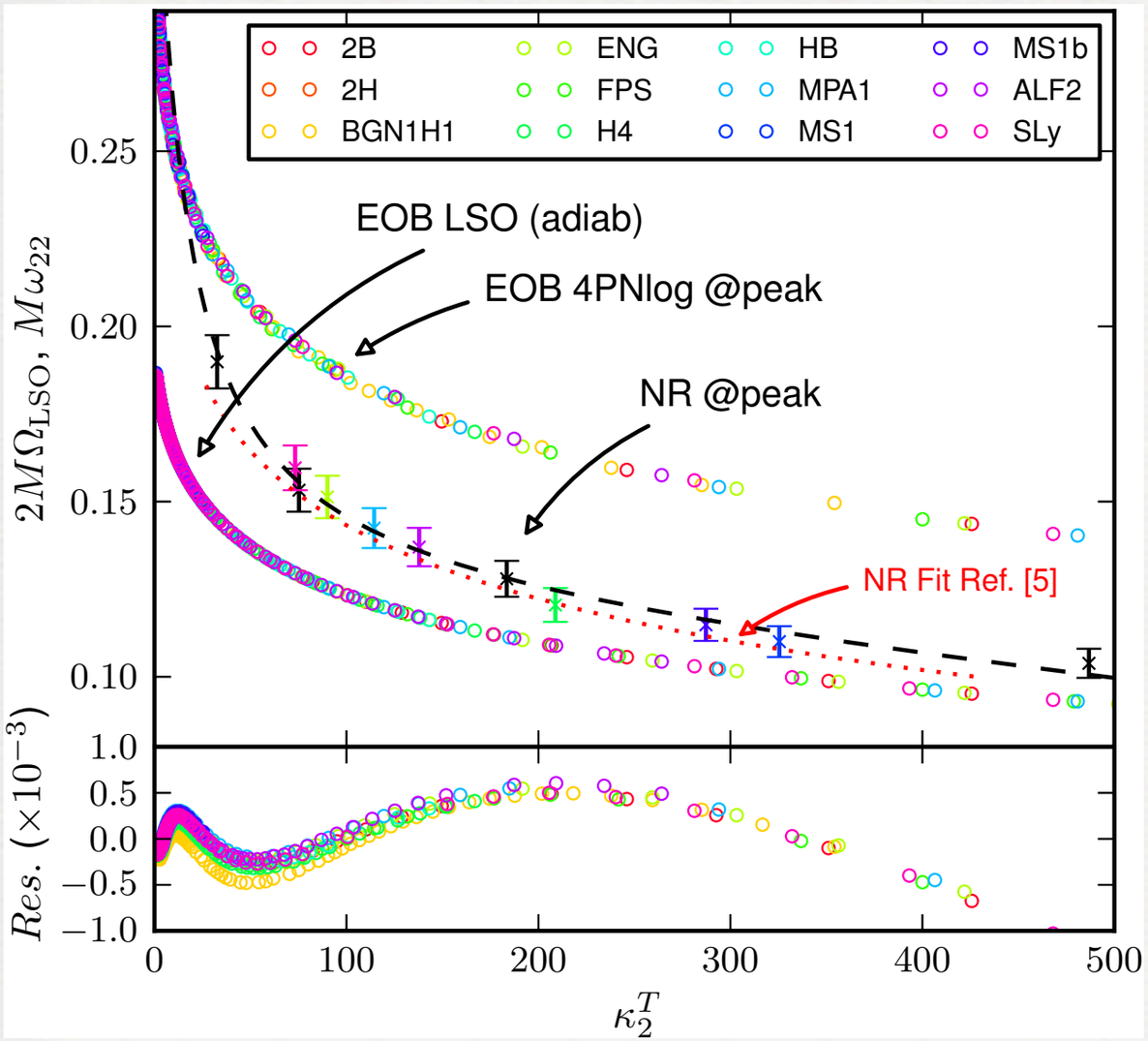
FIG. 4. Instantaneous gravitational-wave frequency at the point of peak amplitude, as a function of the tidal parameter $\Lambda^{1/5}$ (bottom panel) and as a function of individual star compactness C (top). For each model, the highest-resolution simulation for a given EOS is plotted in black, lower-resolution simulations in grey. The $x = (\pi M f)^{2/3} = C$ relation used in [15] to characterize merger frequency is shown in the compactness plot. An empirical fit using $\Lambda^{1/5}$ is shown in the bottom plot; the frequency of merger is more tightly correlated with Λ than with compactness/radius.

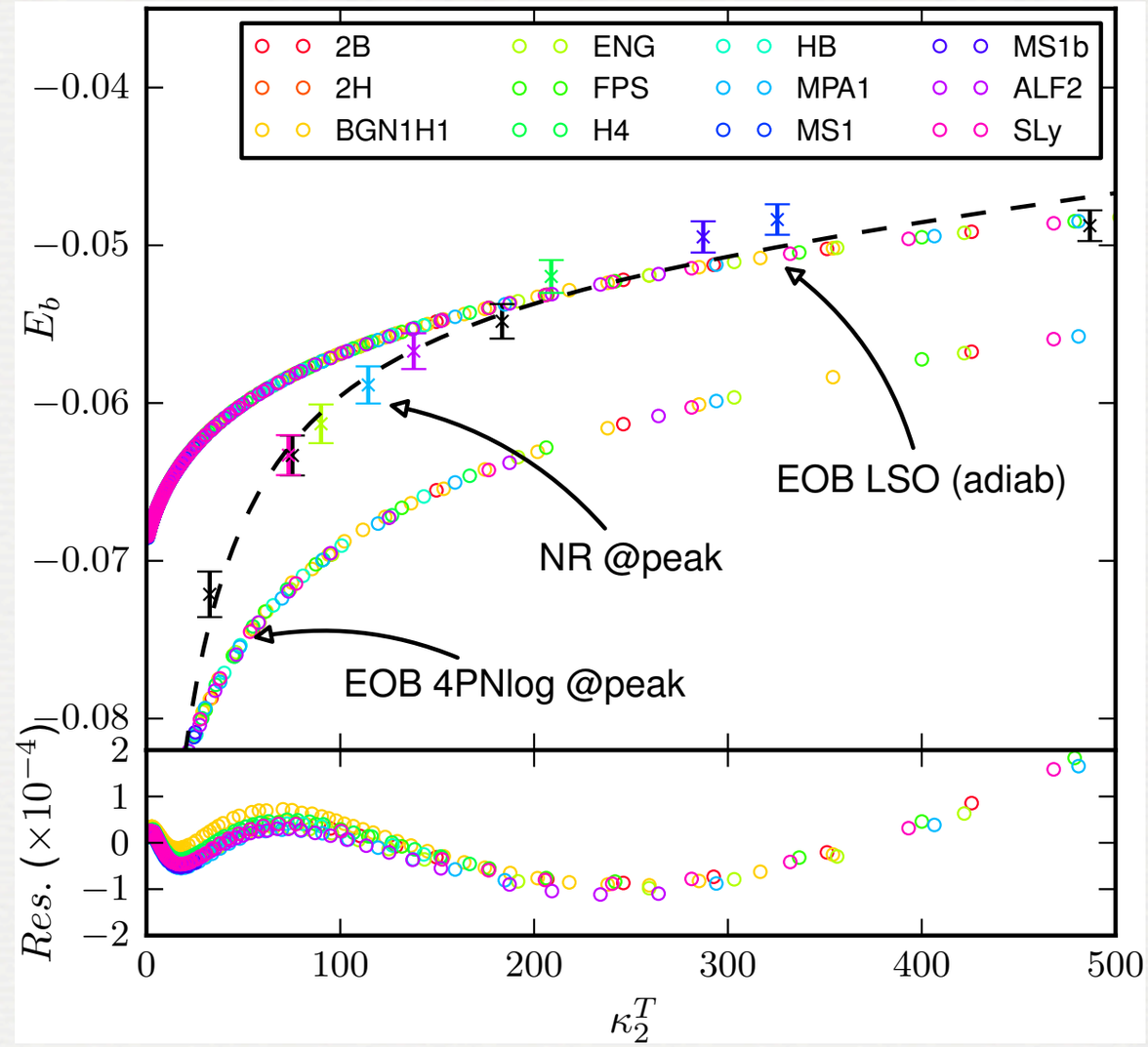
$$\Lambda = \frac{2}{3} k_2 \frac{1}{C^5}$$

empirical fit using $\Lambda^{1/5}$????

EOS QUASI-UNIVERSALITY OF BNS MERGER

Simple EOS-universal behavior: measure the frequency, constrain the EOS

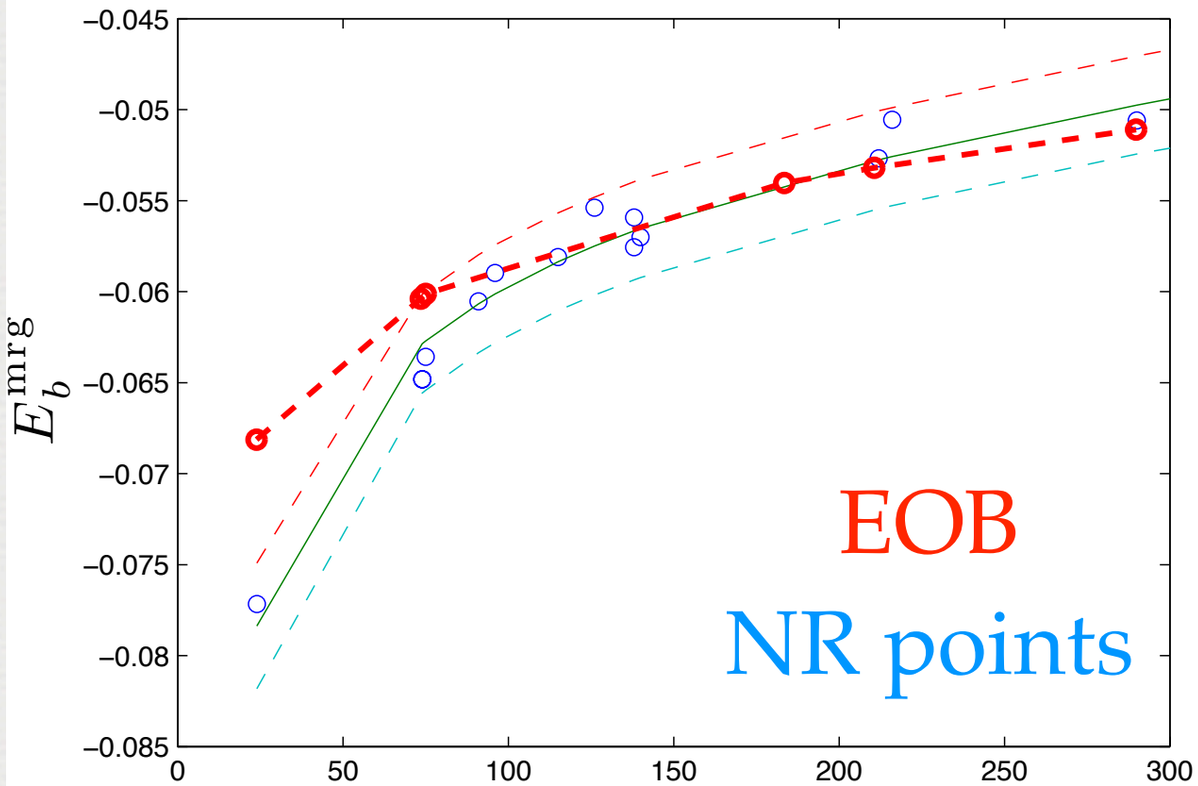


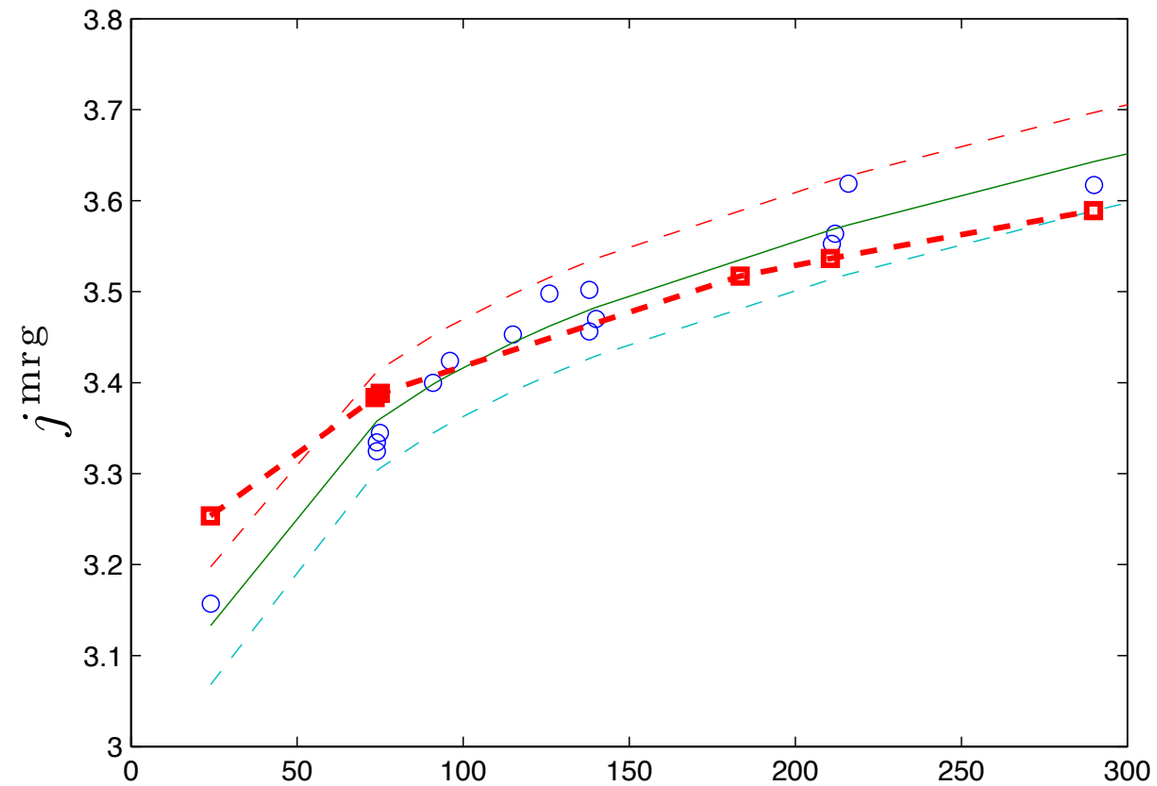


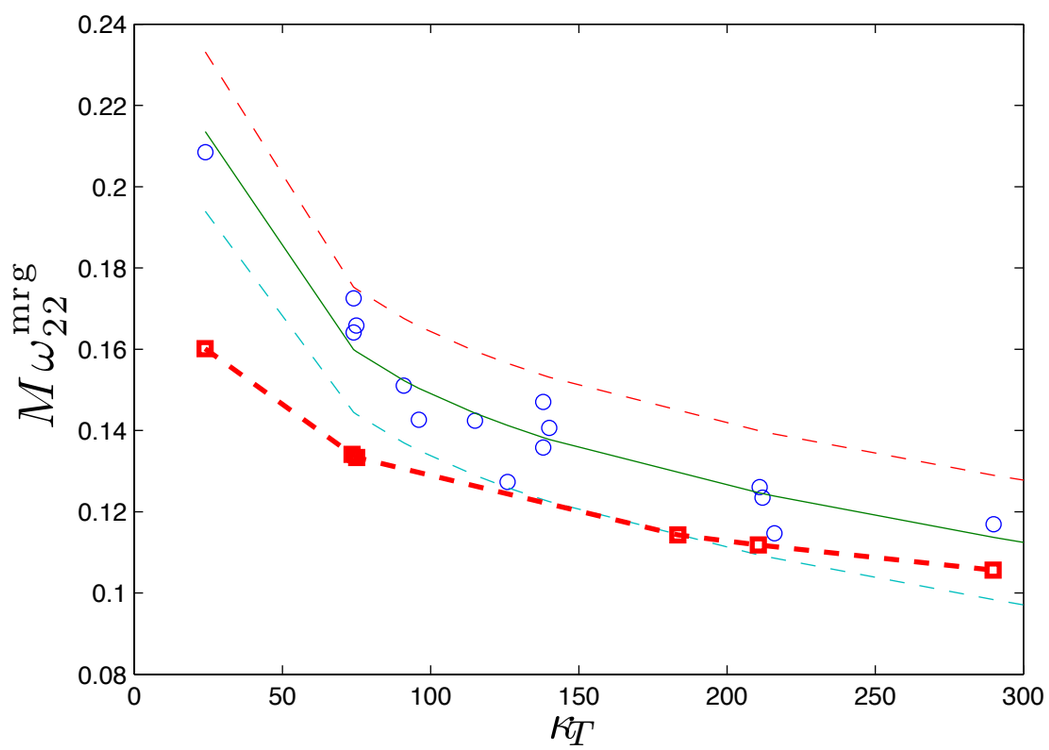
$$\kappa_\ell^T \equiv 2\left[\frac{1}{q}\left(\frac{X_A}{C_A}\right)^{2\ell+1} k_\ell^A + q\left(\frac{X_B}{C_B}\right)^{2\ell+1} k_\ell^B\right]$$

$$M\Omega = \dot{\Phi} = \partial_{p_\varphi} H_{\rm EOB}(p, q; \kappa_T)$$

NEW T-EOB MODEL





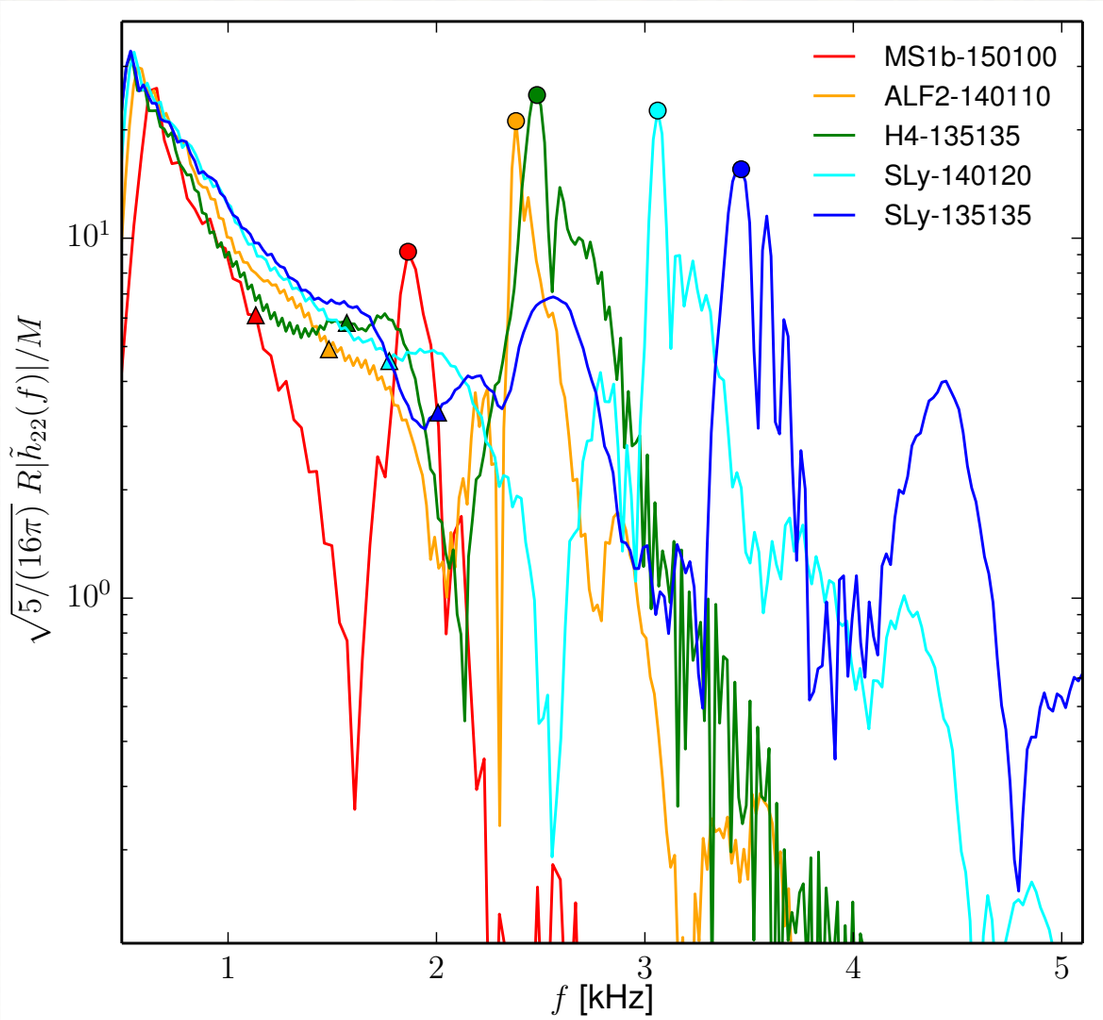


Good quantitative agreement

$$M\omega_{22}^{\rm mrg} \approx 2\Omega_{\rm mrg} = \frac{dE_b}{dj}\Big|_{j=j_{\rm mrg}}$$

POST MERGER SIGNAL

Bauswein-Janka, Phys. Rev. Lett. 108 (2012), 011101
f-mode frequency of HMNS



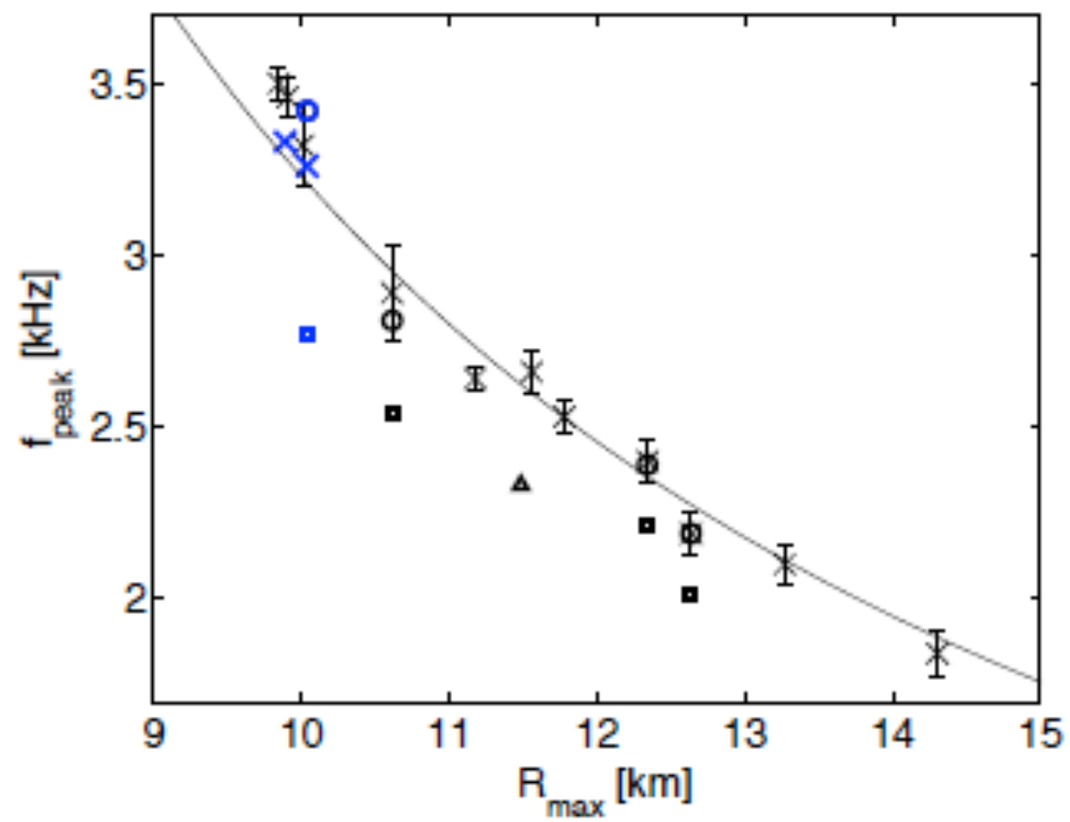


FIG. 3: Peak frequency of the postmerger GW emission vs. radius of the maximum-mass TOV solution. Blue cases are excluded by [3]. Error estimates are based on a Fisher matrix analysis for a source at 20 Mpc. The line is a least square fit $R_{\rm max} \propto f_{\rm peak}^{-2/3}$ for the accepted models. The triangle is an outlier (see text) and the squares correspond to models with lower $M_{\rm tot}$. See text for other symbols.

Empirical correlation with max radius of a TOV solution for given EOS

INSPIRAL-MERGER-POSTMERGER??

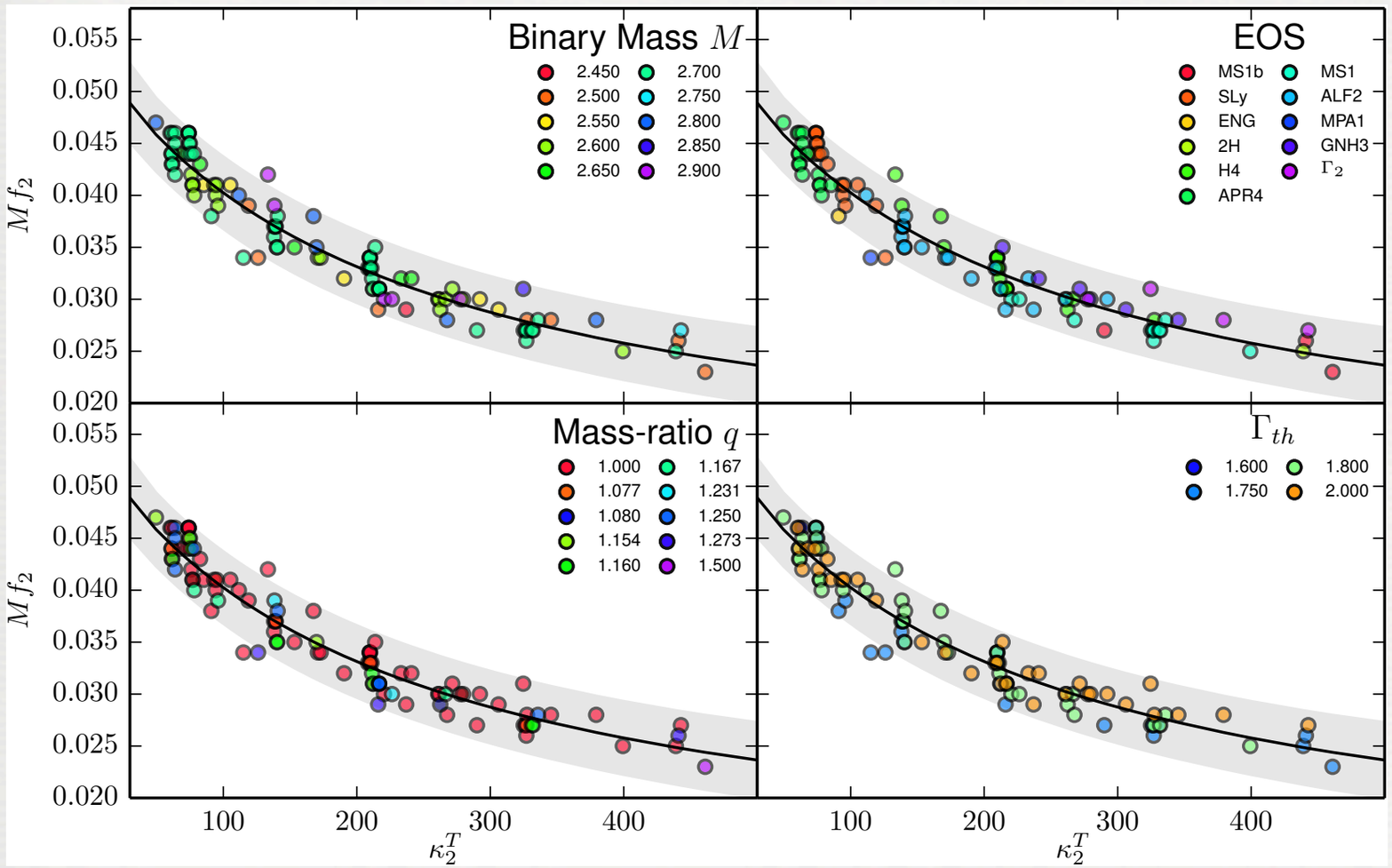


TABLE I: BNS configurations and data. Columns: Configuration name, equation of state, binary mass M, mass ratio q, f_2 frequency in kHz, tidal coupling constant κ_2^T.

Name	EOS	M $[M_\odot]$	q	f_2 [kHz]	κ_2^T
SLy-135135	SLy	2.70	1.00	3.48	74
SLy-145125	SLy	2.70	1.16	3.42	75
ENG-135135	ENG	2.70	1.00	2.86	91
SLy-140120	SLy	2.60	1.17	3.05	96
MPA1-135135	MPA1	2.70	1.00	2.57	115
SLy-140110	SLy	2.50	1.27	2.79	126
ALF2-135135	ALF2	2.70	1.00	2.73	138
ALF2-145125	ALF2	2.70	1.16	2.66	140
H4-135135	H4	2.70	1.00	2.50	211
H4-145125	H4	2.70	1.16	2.36	212
ALF2-140110	ALF2	2.50	1.27	2.38	216
MS1b-135135	MS1b	2.70	1.00	2.00	290
MS1-135135	MS1	2.70	1.00	1.95	327
MS1-145125	MS1	2.70	1.16	2.06	331
MS1b-140110	MS1b	2.50	1.27	2.08	441
2H-135135	2H	2.70	1.00	1.87	439
MS1b-150100	MS1b	2.50	1.50	1.87	461

S. Bernuzzi, T. Dietrich & A. Nagar, PRL 115 (2015), 9, 091101

CONCLUSIONS

1. Tidal effects are incorporated in EOB formalism.

2. NR/EOB waveforms and energetics essentially consistent up to merger.
We proved, out of question, the existence of strong amplification of tidal forces in the very-late inspiral phase, close to merger

3. The EOB theory easily allows to check the soundness of NR data. The simple theoretical understanding of BNS merger gives a paradigmatic example

4. Quasi-universal relations @BNS merger are found and physically understood thanks to EOB theory. This allows to understand better the phenomenology of BNS merger

3. Nowadays, the EOB model is the MORE ROBUST AND EFFICIENT METHOD to produce BNS templates waveforms up to MERGER with the hope of measuring the Love numbers, and thus the EOS, from the late-inspiral phase.

4. Further NR improvements (less eccentricity, more accuracy) are certainly useful but not essential (contrary to BBHs) for DA purposes [see Hotokezaka et al. paper]

5. EOB-based BNS templates should be implemented NOW in DA pipelines