GRAVITATIONAL WAVES FROM COALESCING COMPACT BINARIES

INTERFACING NUMERICAL AND ANALYTICAL RESULTS

Alessandro Nagar
Institut des Hautes Etudes Scientifiques (IHES)


A. Nagar, TAUP - Torino 2015
Coalescing (stellar-mass) relativistic binaries: most promising sources for ground-based gravitational wave (GW) detectors: strong fields, high velocities

Need waveform templates for detection

Numerical modelizations (BH-BH, BH-NS, NS-NS)

Analytical modelizations: predictions, interpretation of numerical results & construction of templates for GW detection
MATCHED FILTERING TECHNIQUE

To extract GW signal from detector’s output (lost in broadband noise $S_n(f)$)

$$\langle output|h_{\text{template}}\rangle = \int \frac{df}{S_n(f)} o(f) h^*_{\text{template}}(f)$$

Detector’s output

Template of expected GW signal
The importance of an analytical formalism

- **Theoretical**: physical understanding of the coalescence process, especially in complicated situations (e.g., precessing spins).

- **Practical**: need many thousands of accurate GWs templates for detection and data analysis. Need analytical templates: \( h \left( m_1, m_2, \vec{S}_1, \vec{S}_2 \right) \)

- **Solution**: synergy between analytical & numerical relativity

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Perturbation Theory

PN

Resummed perturbation theory:

EOB

Numerical Relativity

Non-perturbative information
TEMPLATES FOR GWS FROM BBH COALESCENCE

Brady, Craighton & Thorne, 1998

Numerical Relativity: \( \geq 2005 \) (Pretorius, Campanelli et al., Baker et al.)

Most accurate data: Caltech-Cornell spectral code (with some caveats): M. Scheel et al., 2008

Spectral code

Extrapolation (radius & resolution)

Phase error:

\(< 0.02 \) rad (inspiral)

\(< 0.1 \) rad (ringdown)
A catalog of 171 high-quality binary black-hole simulations for gravitational-wave astronomy  [PRL 111 (2013) 241104]

Abdul H. Mroué,1 Mark A. Scheel,2 Béla Szilágyi,2 Harald P. Pfeiffer,1 Michael Boyle,3 Daniel A. Hemberger,3 Lawrence E. Kidder,3 Geoffrey Lovelace,4,2 Sergei Ossokine,1,5 Nicholas W. Taylor,2 Anil Zenginoğlu,2 Luisa T. Buchman,2 Tony Chu,1 Evan Foley,1 Matthew Giesler,4 Robert Owen,6 and Saul A. Teukolsky3

FIG. 3: Waveforms from all simulations in the catalog. Shown here are $h_+$ (blue) and $h_\times$ (red) in a sky direction parallel to the initial orbital plane of each simulation. All plots have the same horizontal scale, with each tick representing a time interval of 2000$M$, where $M$ is the total mass.

www.black-holes.org

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SUMMARY OF BBH/BNS NR/AR RECENT RESULTS

1. **NEW EOB model** for coalescing [NONPRECESSING] spinning BBHs
   “Energetic and phasing of nonprecessing spinning black hole binaries”

2. **BNS: Numerical-relativity** matches **EOB analytical-relativity** waveforms and dynamics essentially up to merger. GW templates for LIGO/Virgo to measure EOS out of tidal effects
   S. Bernuzzi, A. Nagar, T. Dietrich & T. Damour, PRL 114 (2015), 161103
   “Modeling the Dynamics of Tidally Interacting Binary Neutron Stars up to Merger”
   [verified/improved by Hotokezaka et al., PRD 91 (2015) 6, 064060, notably with reduced eccentricity]

3. **Quasi-universality** in BNS merger (binding energy, angular momentum, GW frequency vs tidal coupling constant): explained using EOB theory
   S. Bernuzzi, A. Nagar, S. Balmelli, T. Dietrich & M. Ujevic, PRL 112 (2014), 201101
   “Quasiuniversal properties of neutron star mergers”

4. **Quasi-universality** of post-merger $M f_2$ frequency vs tidal coupling constant
   S. Bernuzzi, A. Nagar & T. Dietrich, arXiv:1504.01764
   “Modeling the complete gravitational wave spectrum of neutron star mergers”
   Unifying description of inspiral, merger and post-merger phases

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EOB APPROACH IN A NUTSHELL

(Buonanno-Damour 99, 00, Damour-Jaranowski-Schäfer 00, Damour 01, Damour-Nagar 07, Damour-Iyer-Nagar 08)

key ideas:

1. Replace two-body dynamics \((m_1, m_2)\) by dynamics of a particle \((\mu \equiv m_1 m_2/(m_1 + m_2))\) in an effective metric \(g_{\mu\nu}^{\text{eff}}(u)\), with

\[
u \equiv \mu/M \equiv m_1 m_2/(m_1 + m_2)^2 \text{ in the interval } 0 \leq \nu \leq \frac{1}{4}\]
STRUCTURE OF THE EOB FORMALISM

EOB Hamiltonian
\[ H_{\text{EOB}} \]

EOB Rad. Reac. force
\[ \hat{F}_\varphi \]

Factorized waveform
\[ h_{\ell m} = h^{(N,\epsilon)}_{\ell m} \hat{h}^{(\epsilon)}_{\ell m} \]
\[ \hat{h}^{(\epsilon)}_{\ell m} = \hat{S}^{(\epsilon)} T_{\ell m} e^{i \delta_{\ell m}} \rho_{\ell m} \]

Matching at merger time
\[ h_{\text{ringdown}}(t) = \sum N C_N^+ e^{-\sigma_N^+ (t-t_m)} \]

PN dynamics
\[ \left( \frac{d}{dt} \right)^{1/2} \frac{\partial \hat{H}_{\text{EOB}}}{\partial p_{r^*}} , \]
\[ \left( \frac{d}{dt} \right)^{1/2} \frac{\partial \hat{H}_{\text{EOB}}}{\partial r} , \]
\[ \Omega = \frac{d \varphi}{dt} = \frac{\partial \hat{H}_{\text{EOB}}}{\partial p_\varphi} , \]
\[ \frac{d}{dt} \frac{\partial \hat{F}_\varphi}{\partial p_\varphi} = \hat{F}_\varphi . \]

EOB waveform
\[ h_{\text{EOB}}^{\text{insplunge}}(t) = \theta(t_m - t) h_{\ell m}^{\text{insplunge}}(t) + \theta(t - t_m) h_{\ell m}^{\text{ringdown}}(t) \]

PN rad losses
Resummed (BD99)
Resummed (DIS98)
Resummed (DN07,DIN08)

PN waveforms
BD89, B95\&05, ABIQ04,

BDIF04

Resummed (DN07, DIN08)

BH perturbations
Resummed (BD99)
Resummed (DIS98)
Resummed (DN07, DIN08)

QNMs spectrum
\[ \sigma_N = \alpha_N + i \omega_N \]

Matching at merger time

BNS: tides
(Love numbers)

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EXPLICIT FORM OF THE EOB HAMILTONIAN

EOB Hamiltonian

\[ H_{\text{EOB}} = M \sqrt{1 + 2\nu \left( \hat{H}_{\text{eff}} - 1 \right)} \]

All Functions are a \( \nu \)-dependent deformation of the Schwarzschild ones

\[ A(r) = 1 - 2u + 2\nu u^3 + a_4 \nu u^4 \]

\[ A(r)B(r) = 1 - 6\nu u^2 + 2(3\nu - 26)\nu u^3 \]

Simple effective Hamiltonian:

\[ \hat{H}_{\text{eff}} \equiv \sqrt{p_{r*}^2 + A(r) \left( 1 + \frac{p_\varphi^2}{r^2} + z_3 \frac{p_{r*}^4}{r^2} \right)} \]

\[ p_{r*} = \left( \frac{A}{B} \right)^{1/2} p_r \]

\[ u = \frac{GM}{c^2 R} \]

\[ a_4 = \frac{94}{3} - \frac{41}{32} \pi^2 \approx 18.6879027 \]

Crucial EOB radial potential

Contribution at 3PN
HAMILTON’S EQUATIONS & RADIATION REACTION

\[ \dot{r} = \left( \frac{A}{B} \right)^{1/2} \frac{\partial \hat{H}_{\text{EOB}}}{\partial p_{r^*}} \]

\[ \dot{\phi} = \frac{\partial \hat{H}_{\text{EOB}}}{\partial p_\phi} \equiv \Omega \]

\[ \dot{p}_{r^*} = - \left( \frac{A}{B} \right)^{1/2} \frac{\partial \hat{H}_{\text{EOB}}}{\partial r} + \hat{F}_{r^*} \]

\[ \dot{p}_\phi = \hat{F}_\phi \]

- The system must radiate angular momentum
- How? Use PN-based (Taylor-expanded) radiation reaction force (ang-mom flux)
- Need flux resummation

Circular orbit

Last-Stable-Orbit (LSO): \( r < 6M \)

Plunge

\[ \hat{H}_{\text{eff}}(r, p_\phi; \nu) = A(r; \nu) \left( 1 + \frac{p_\phi^2}{r^2} \right) \]

\[ \hat{F}_{\phi}^{\text{Taylor}} = - \frac{32}{5} \nu \Omega^5 r_\Omega^4 \hat{F}_{\phi}^{\text{Taylor}}(v_\phi) \]

Plus horizon contribution [AN&Akcay2012]

Resummation multipole by multipole (Damour, Iyer & Nagar 2008, Damour & Nagar, 2009)
4PN analytically complete + 5PN logarithmic term in the $A(u)$ function:

$A_{5PN}^{\text{Taylor}} = 1 - 2u + 2v u^3 + \left(\frac{94}{3} - \frac{41}{32} \pi^2\right) v u^4 + v [a_5^c(v) + a_5^\ln \ln u] u^5 + v [a_6^c(v) + a_6^\ln \ln u] u^6$

$A(u; \nu, a_6^c) = P^1_5 [A_{5PN}^{\text{Taylor}} (u; \nu, a_6^c)]$

NEED ONE "effective" 5PN parameter from NR waveform data: $a_6^c(v)$

The knowledge of the central A potential today

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THE EOB[NR] POTENTIAL

Effect of finite-mass corrections: system is more bound!

From (6 nonspinning) NR data sets one gets:

\[ a_6^c(\nu) = 3097.3\nu^2 - 1330.6\nu + 81.3804 \]

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NDRP, arXiv:1506.08457
RESULTS: WAVEFORMS (NO SPIN)

\[ \chi = 0 \]

equal-mass case

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NDRP, arXiv:1506.08457
ENERGETICS - NONSPINNING

Binding energy vs angular momentum (Llama NR data)

\[ E_b = \frac{E - M c^2}{\mu} \]

NDRP, arXiv:1506.08457
EOB APPROACH TO THE DYNAMICS OF TWO SPINNING BLACK HOLES

Damour01, Buonanno-Chen-Damour06, Damour-Jaranowski-Schafer08, Barausse&Buonanno10, Nagar11, Barausse&Buonanno2011, Taracchini et al. 12, Balmelli&Jetzer2013, Pan et al. 2013

Nonspinning case: EOB description = deformation of test-particle dynamics in a Schwarzschild background

Spinning case: EOB description = deformation of (spinning) test-particle dynamics in a Kerr background

Deformation parameters:

\[ \nu = \frac{\mu}{M} \]
and “effective test spin” \[ S^* \]

Based on Hamiltonian formulation in the center of mass frame
UNFAITHFULNESS

38 spin-aligned SXS
NR waveforms

\[ \bar{F} \equiv 1 - \max_{t_0, \phi_0} \frac{\langle h_{22}^{\text{EOB}}, h_{22}^{\text{NR}} \rangle}{||h_{22}^{\text{EOB}}|| \cdot ||h_{22}^{\text{NR}}||} \]

\[ \langle h_1, h_2 \rangle \equiv 4\Re \int_{f_{\text{min}}}^{\infty} \bar{h}_1(f)\bar{h}_2^*(f)/S_n(f) \, df \]

\( \bar{F} \) as a function of \( M/M_\odot \) for different configurations. The diagram shows the unfaithfulness for several configurations, with the configuration \((8, +0.50, 0)\) highlighted, which has a lower unfaithfulness compared to others.

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NDRP, arXiv:1506.08457
COMPARISON WITH SEOBNRv2

Different EOB Hamiltonian [Barausse & Buonanno11, Taracchini et al.12]
Different choices for the analytic freedom (as well as spin gauge)

Taracchini, Buonanno et al., PRD 89, 061502 (R), 2014
BBH TAKE-AWAY

1. New way of blending finite-mass and spin effect in an EOB Hamiltonian based on the structure of the Hamiltonian of a (spinning) particle on a Kerr background

2. Analytical freedom: only two flexibility parameters that are extracted from NR data as simple (separate) functions of symmetric mass ratio and spin magnitude

3. Compatibility (within NR errors) between such EOBNR model and state-of-the art NR data over mass ratio and spin (no precession for the moment)
BINARY NEUTRON STARS
**BNS: EOS detection**

Induced polarizability coefficient: the Love number

\[ M_{ij} = \mu_2 G_{ij} \]

**EOS detection by measuring the tidal polarizability coefficient**
CRUCIAL RESULT
(Damour, Nagar, Villain 2012)

Tidal polarizability parameters can actually be measured by adv LIGO with a reasonable SNR=16 (5 events per year)

Use EOB controlled, accurate, description of the phasing up to BNS merger!

[Del Pozzo et al. 2013]
THREE BNS RESULTS

1. **Numerical-relativity** matches effective-one-body (EOB) analytical-relativity waveforms and dynamics essentially up to merger. Method to compute GW templates for LIGO/Virgo to measure EOS out of tidal effects

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   Unifying description of inspiral, merger and post-merger phases
BNS: ANALYTICAL NEEDS

- Study the response of each neutron star to the tidal field of the companion [theory of relativistic Love numbers (i.e. tidal polarizability coefficients) + tidal corrections to dynamics (beyond Newtonian accuracy)]

- Incorporate the corresponding tidal effects within a theoretical framework able to describe the gravitational wave signal emitted by inspiralling compact binaries (possibly up to merger): PN-expanded description vs EOB-resummed description.

- Compare analytical models against NR simulations, possibly calibrating high-order tidal corrections if needed

- Assess the measurability of tidal effects within the signal seen by interferometric detectors
TIDAL EFFECTS IN EOB FORMALISM

Tidal extension of EOB formalism: nonminimal worldline couplings

\[ \Delta S_{\text{nonminimal}} = \sum_A \frac{1}{4} \mu_2^A \int ds_A \left( u^\mu u^\nu R_{\mu\alpha\nu\beta} \right)^2 + \ldots \]

Damour\&Esposito-Farèse\text{96}, Goldberger\&Rothstein\text{06}, TD\&AN\text{09}

Modifications of the EOB effective metric...

\[ A(r) = A^0_r + A^{\text{tidal}}(r) \]

\[ A^{\text{tidal}}(r) = -\kappa_2^T u^6 \left( 1 + \bar{\alpha}_1 u + \bar{\alpha}_2 u^2 + \ldots \right) + \ldots \]

And tidal modifications of GW waveform \& radiation reaction

- Need analytical theory for computing \( \mu_2, \kappa_2^T, \bar{\alpha}_1 \ldots \)

- (?) Need accurate NR simulations to "calibrate" the higher-order PN tidal contributions, that may be quite important during the late inspiral
TIDAL INTERACTION POTENTIAL

Central tidal “coupling constant”:

\[ \kappa_T^T \equiv 2 \left[ \frac{1}{q} \left( \frac{X_A}{C_A} \right)^{2\ell+1} k^A_{\ell} + q \left( \frac{X_B}{C_B} \right)^{2\ell+1} k^B_{\ell} \right] \]

\[ X_{A,B} \equiv M_{A,B}/M \quad C_{A,B} \equiv M_{A,B}/R_{A,B} \]

Function of: mass ratio, compactnesses and relativistic Love numbers

In the dynamics:

\[ A(u) = A^0(u) + A^{\text{tidal}} \]

\[ A^{\text{tidal}} = \sum_{\ell \geq 2} -\kappa_T^T u^{2\ell+2} A^{\text{tidal}}_\ell(u) \]

NLO & NNLO tidal PN corrections known analytically [Bini, Damour & Faye 2011]

\[ A^{\text{tidal}}_2 = 1 + \frac{5}{4} u + \frac{85}{14} u^2 \]

\[ \kappa_T^T \sim 100 \]

“Newtonian” (LO) part + PN corrections (NLO, NNLO, ...)

\[ \kappa_2^T = \frac{1}{8} \frac{k_2}{C^5} \]
TIDAL COUPLING CONSTANT

$$\kappa_\ell^T = 2 \left[ \frac{1}{q} \left( \frac{X_A}{C_A} \right)^{2\ell+1} \kappa_\ell^A + q \left( \frac{X_B}{C_B} \right)^{2\ell+1} \kappa_\ell^B \right]$$

$$q = \frac{M_A}{M_B} \leq 1$$

$$X_A \equiv \frac{M_A}{M} = q/(1+q)$$

$$X_B \equiv \frac{M_B}{M} = 1/(1+q)$$

A tidally-interacting binary is dynamically characterized by:
- mass ratio
- Love number(s)
- Compactness(es)
Bini&Damour (2015) resummed expression for $\hat{A}^{\text{tidal}}_\ell$

Presence of a pole: potential strongly attractive @ mrg

\[
A_T^{(+)}(u; \nu) \equiv - \sum_{\ell=2}^{4} \left[ \kappa^{(\ell)}_A u^{2\ell+2} \hat{A}^{(\ell^+)}_A + (A \leftrightarrow B) \right].
\]

\[
\hat{A}^{(2^+)}_A(u) = 1 + \frac{3u^2}{1 - r_{LR}u} + \frac{X_A\tilde{A}_1^{(2^+)}1SF}{(1 - r_{LR}u)^{7/2}} + \frac{X_A^2\tilde{A}_2^{(2^+)}2SF}{(1 - r_{LR}u)^p}
\]

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**Graph**

- **BNS RESUM**
- **BNS NNLO**
- **Schwarzschild**
- **BBH**

\[ \nu = 0.25 \]

\[ \kappa_T^2 \approx 73.55 \]

**merger**
BNS SIMULATIONS: Z4C

Bernuzzi & Hilditch, PRD81 (2010) - Z4c - 1D (collapse and RNS)
Ruiz et al., PRD83 (2011) - 024025 - boundary conditions
Hilditch, Bernuzzi et al., PRD88 (2013), 084057 - Z4c - 3D (BNS)
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Hilditch, Bernuzzi et al., PRD88 (2013), 084057 - Z4c - 3D (BNS)
NEW BNS SIMULATIONS

Z4c, high-order reconstruction, shells,...[Bernuzzi-Hilditch 2008 + ]
FIG. 2: Energetics: comparison between NR data, TEOB_{Resum}, TEOB_{NNLO} and TPN. Each bottom panel shows the two EOB-NR differences. The filled circles locate the merger points (top) and the corresponding differences (bottom). The shaded area indicates the NR uncertainty. The TEOB_{Resum} model displays, globally, the smallest discrepancy with NR data (notably for merger quantities), supporting the theoretical, light-ring driven, amplification of the relativistic tidal factor.

S. Bernuzzi, A. Nagar, T. Dietrich & T. Damour, PRL 114 (2015), 161103
A. Nagar - TAUP 2015 - Torino

mercoledì 9 settembre 15
FIG. 3: Phasing and amplitude comparison (versus NR retarded time) between TEOBResum, NR and the phasing of TT4 for three representative models. Waves are aligned on a time window (vertical dot-dashed lines) corresponding to $I_{\psi} \approx (0.04, 0.06)$. The markers in the bottom panels indicate: the crossing of the TEOBResum LSO radius; NR (also with a dashed vertical line) and EOB merger moments.

<table>
<thead>
<tr>
<th>Name</th>
<th>EOS</th>
<th>$\kappa_1^T$</th>
<th>$r_{LR}$</th>
<th>$C_{A,B}$</th>
<th>$M_{A,B}[M_\odot]$</th>
<th>$M_{ADM}^0[M_\odot]$</th>
<th>$J_{ADM}^0[M_\odot]^2$</th>
<th>$\Delta\phi_{TT4 NRmrg}$</th>
<th>$\Delta\phi_{TEOBNNLO NRmrg}$</th>
<th>$\Delta\phi_{NRmrg}$</th>
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</table>
EOS QUASI-UNIVERSALITY OF BNS MERGER

S. Bernuzzi, A. Nagar, S. Balmelli, T. Dietrich and M. Ujevic, PRL 112 (2014) 201101

- GW frequency
- binding energy
- angular momentum
- GW amplitude
- ....

\[ M \omega_{22} \]

\[ MRG \]

\[ \kappa_2 T \]
Matter effects on binary neutron star waveforms

Jocelyn S. Read,1,2 Luca Baiotti,3,4 Jolien D. E. Creighton,5 John L. Friedman,5 Bruno Giacomazzo,6 Koutarou Kyutoku,5 Charalampos Markakis,7,9 Luciano Rezzolla,8 Masaru Shibata,4 and Keisuke Taniguchi10

\[ \Lambda = \frac{2}{3} k^2 \frac{1}{C^5} \]

empirical fit using \( \Lambda^{1/5} \) ???

FIG. 4. Instantaneous gravitational-wave frequency at the point of peak amplitude, as a function of the tidal parameter \( \Lambda^{1/5} \) (bottom panel) and as a function of individual star compactness \( C \) (top). For each model, the highest-resolution simulation for a given EOS is plotted in black, lower-resolution simulations in grey. The \( x = (\pi M f)^{2/3} = C \) relation used in [15] to characterize merger frequency is shown in the compactness plot. An empirical fit using \( \Lambda^{1/5} \) is shown in the bottom plot; the frequency of merger is more tightly correlated with \( \Lambda \) than with compactness/radius.
Simple EOS-universal behavior:
measure the frequency, constrain the EOS

$$\kappa_l^T = 2 \left[ \frac{1}{q} \left( \frac{X_A}{C_A} \right)^{2\ell+1} k_l^A + q \left( \frac{X_B}{C_B} \right)^{2\ell+1} k_l^B \right]$$

$$M \Omega = \dot{\Phi} = \partial_{p \varphi} H_{\text{EOB}}(p, q; \kappa_T)$$
NEW T-EOB MODEL

EOB
NR points

Good quantitative agreement

\[ M \omega_{22}^{\text{mrg}} \approx 2 \Omega_{\text{mrg}} = \frac{dE_b}{dj} \bigg|_{j=j_{\text{mrg}}} \]
**POST MERGER SIGNAL**


f-mode frequency of HMNS

Empirical correlation with max radius of a TOV solution for given EOS

A. Nagar - TAUP 2015 - Torino
S. Bernuzzi, T. Dietrich & A. Nagar, PRL 115 (2015), 9, 091101
CONCLUSIONS

1. Tidal effects are incorporated in EOB formalism.

2. NR/EOB waveforms and energetics essentially consistent up to merger.
   We proved, out of question, the existence of strong amplification of tidal forces in the very-late inspiral phase, close to merger.

3. The EOB theory easily allows to check the soundness of NR data. The simple theoretical understanding of BNS merger gives a paradigmatic example.

4. Quasi-universal relations @BNS merger are found and physically understood thanks to EOB theory. This allows to understand better the phenomenology of BNS merger.

5. Nowadays, the EOB model is the MORE ROBUST AND EFFICIENT METHOD to produce BNS templates waveforms up to MERGER with the hope of measuring the Love numbers, and thus the EOS, from the late-inspiral phase.

4. Further NR improvements (less eccentricity, more accuracy) are certainly useful but not essential (contrary to BBHs) for DA purposes [see Hotokezaka et al. paper].

5. EOB-based BNS templates should be implemented NOW in DA pipelines.