New approaches in the Analysis of Dark Matter
direct detection data

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To compare direct detection results, Spin-independent or Spin-dependent scaling laws are usually assumed:

\[
v^2 \frac{d\sigma_{SI}}{dE_R} = \sigma_n \frac{\mu^2_{\chi,\text{nucleus}}}{\mu^2_{\chi,\text{nucleon}}} \frac{[Zf_p + (A - Z)f_n]^2}{f_p^2} |F(E_R)|^2
\]

\[
v^2 \frac{d\sigma_{SD}}{dE_R} = 32\mu^2G_F^2 \frac{J(J + 1)}{\pi} \left[ \frac{1}{J}(f_p\langle S_p \rangle + f_n\langle S_n \rangle) \right]^2
\]

and Maxwellian velocity distribution and elastic scattering.

Standard scaling laws (SI,SD) are motivated by High–energy physics. However, a model–independent bottom–up approach would be desirable. Similar strategy in the analysis of LHC data.
Experiments result status of the standard approach

Maxwellian distribution

Spin–independent

Spin–dependent

In the both cases, DAMA annual modulation result is excluded by other experiments.

E. Aprile et al. (The XENON100 Collaboration), Phys. Rev. Lett. 107, 131302

Parameter spaces searching

What is the most general parameter space for direct detection?

- Isovector coupling ($\text{WIMP–proton} \neq \text{WIMP–neutron}$)
- Inelastic scattering with mass splitting $\delta$.
- Ambiguity of WIMP velocity distribution which can have non-thermal components factorization of halo function.
- Bottom–up approach: Low energy effective theory WIMP–nucleus cross section from most general Galilean invariant Hamiltonian.
Inelastic scattering

An initial mass eigenstate $m_\chi$ scatters to final $m_{\chi'}$ with mass splitting:
Positive $\delta$: Kinetic energy needed to overcome step
Negative $\delta$: Metastable state.

Kinematical relation between energy and velocity is no more one-to-one mapping

$$v_{\text{min}} = \frac{1}{\sqrt{2M_NE_R}} \left| \frac{M_NE_R}{\mu^2} + \delta \right|.$$ 

Especially, $\delta > 0$ gives lower limit of $v_{\text{min}}$ in whole energy range. Indeed, self–consistency check of excess is possible using two different energy bins of the same experiment giving same $v_{\text{min}}$. 

![Graph showing scattering energy and velocity relationship]
Halo-independent approach

Dependence on WIMP velocity distribution is factorized in the halo function $\eta(v_{\text{min}})$:

$$\eta(v_{\text{min}}) = \int_{v_{\text{min}}} d^3 \vec{v} \frac{f(\vec{v})}{v}$$

Energy interval of one experiment can be mapped to another experiment using same $v_{\text{min}}$ interval. Estimations of quantity $\tilde{\eta} \equiv \rho_\chi \sigma_0 \eta(v_{\text{min}})/m_\chi$ in the same $v_{\text{min}}$ intervals from different experiments directly comparable.

($\rho_\chi$: local density, $\sigma_0$: point like cross section, $m_\chi$: WIMP mass)

P.J. Fox, J. Liu, N.Weiner, PRD83, 103514(2011)
By changing $\delta$ both estimations and bounds on the halo functions $\bar{\eta}$ shift (mainly) horizontally in a different way for different targets $\rightarrow$ allowed $m_\chi - \delta$ are only based on kinematics.

S.Scopel and K.H. Yoon, JCAP 1408, 060(2014)
Result

Standard Spin–independent interaction with inelastic scattering

Allowed regions in $m_\chi - \delta$ (only based on kinematics)

Low mass with Na in DAMA

High mass WIMP solution survives after CRESST 2014 data

S.Scopel and K.H. Yoon, JCAP 1408, 060(2014)
Result
Standard Spin–independent interaction with inelastic scattering

\[ m_\chi = 3 \text{ GeV}, \ \delta = -70 \text{ keV}, \ f_n/f_p = -0.79 \]

\[ m_\chi = 350 \text{ GeV}, \ \delta = 45 \text{ keV}, \ f_n/f_p = 1 \]

S.Scopel and K.H. Yoon, JCAP 1408, 060(2014)
General WIMP–nucleus cross section

Galilean invariant Hamiltonian

$$\mathcal{H} = \sum_i (c_i^0 + c_i^1 \tau_3) O_i,$$

$\tau_3$ is nuclear isospin operator and relation to the proton and neutron is $(c_i^0 \pm c_i^1)$, respectively.

Operators $O_i$ are Hermitian, if they are built out of the following four 3-vectors

$$i \frac{\hat{q}}{m_N}, \, \vec{v} \perp, \, \vec{S}_\chi, \, \vec{S}_N.$$

The WIMP-nucleus scattering amplitude can be written with WIMP response function $R$ and nuclear response function $W$

$$\frac{1}{2j_\chi + 1} \frac{1}{2j_T + 1} |\mathcal{M}| = \frac{4\pi}{2j_T + 1} \sum_{\tau, \tau' = 0, 1} \sum_k R^\tau_{T\tau'} \left[ c_k^\tau, (\vec{v}_T \perp)^2, \frac{q^2}{m_N^2} \right] W^\tau_{T\tau'} (q^2),$$

$$k = M, \Phi'', \Phi''', \tilde{\Phi}', \Sigma'', \Sigma', \Delta, \Delta\Sigma'.$$


General WIMP–nucleus cross section

The WIMP response function can be decomposed to

\[ R^\tau\tau'_{k} = R^\tau\tau'_{0k} + R^\tau\tau'_{1k} \left( v_T^2 - v_{\text{min}}^2 \right). \]

Halo functions of \( R_{0k} \) and \( R_{1k} \) are

\[ \tilde{\eta}(v_{\text{min}}) \sim \int_{v_{\text{min}}} d^3\vec{v}_T \frac{f(\vec{v}_T)}{v_T} \left( v_T \right) \left( v_{\perp}^2 \right) \] ("Usual" one)

\[ \tilde{\xi}(v_{\text{min}}) \sim \int_{v_{\text{min}}} d^3\vec{v}_T \frac{f(\vec{v}_T)}{v_T} \left( v_{\perp}^2 \right)^2 . \]

Generalized Spin–dependent scenario

Usual spin–dependent interaction

\[ S(q^2) = (c_4^0)^2 S_{00}(q^2) + c_4^0 c_4^1 S_{01}(q^2) + (c_4^1)^2 S_{11}(q^2) \]

can be written in terms of nuclear response function \( W_k^{\tau \tau'} \) as

\[
S_{00}(q^2) = W_{\Sigma''}^{00}(q^2) + W_{\Sigma'}^{00}(q^2) \\
S_{11}(q^2) = W_{\Sigma''}^{11}(q^2) + W_{\Sigma'}^{11}(q^2) \\
S_{01}(q^2) = 2[W_{\Sigma''}^{01}(q^2) + W_{\Sigma'}^{01}(q^2)]
\]

\( \Sigma'' \rightarrow \) component of the nucleon spin along the direction of the transferred momentum

\( \Sigma' \rightarrow \) component of the nucleon spin along perpendicular to the direction of the transferred momentum

\( (W_{\Sigma''}(q^2 \rightarrow 0) \simeq 2W_{\Sigma'}(q^2 \rightarrow 0)) \)

Generalized Spin–dependent scenario

: response functions suppressed for even-numbered nucleus Include non–standard dependences on momentum transfer $q$ and incoming WIMP velocity $v_{min}$

<table>
<thead>
<tr>
<th>coupling</th>
<th>$R_{0k}^{\tau\tau'}$</th>
<th>$R_{1k}^{\tau\tau'}$</th>
<th>coupling</th>
<th>$R_{0k}^{\tau\tau'}$</th>
<th>$R_{1k}^{\tau\tau'}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$M(q^0)$</td>
<td>-</td>
<td>3</td>
<td>$\Phi''(q^4)$</td>
<td>$\Sigma'(q^2)$</td>
</tr>
<tr>
<td>4</td>
<td>$\Sigma''(q^0),\Sigma'(q^0)$</td>
<td>-</td>
<td>5</td>
<td>$\Delta(q^4)$</td>
<td>$M(q^2)$</td>
</tr>
<tr>
<td>6</td>
<td>$\Sigma''(q^4)$</td>
<td>-</td>
<td>7</td>
<td>$\Sigma'(q^2)$</td>
<td>-</td>
</tr>
<tr>
<td>8</td>
<td>$\Delta(q^2)$</td>
<td>$M(q^0)$</td>
<td>9</td>
<td>$\Sigma'(q^2)$</td>
<td>-</td>
</tr>
<tr>
<td>10</td>
<td>$\Sigma''(q^2)$</td>
<td>-</td>
<td>11</td>
<td>$M(q^2)$</td>
<td>-</td>
</tr>
<tr>
<td>12</td>
<td>$\Phi''(q^2),\tilde{\Phi}'(q^2)$</td>
<td>$\Sigma''(q^0),\Sigma'(q^0)$</td>
<td>13</td>
<td>$\tilde{\Phi}'(q^4)$</td>
<td>$\Sigma''(q^2)$</td>
</tr>
<tr>
<td>14</td>
<td>-</td>
<td>$\Sigma'(q^2)$</td>
<td>15</td>
<td>$\Phi''(q^6)$</td>
<td>$\Sigma'(q^4)$</td>
</tr>
</tbody>
</table>

Couplings
4 ($q^0$), 6 ($q^4$), 9 ($q^2$) and 10 ($q^2$) are usual distribution function. 7 ($q^0$) and 14 ($q^2$) are velocity dependent distribution function. ⇒ 4–7 and 9–14 have same compatibility factors (Halo–independent approach).

### Generalized Spin–dependent scenario

#### Relativistic EFT Lagrangian of Spin–dependent interaction

<table>
<thead>
<tr>
<th></th>
<th>Relativistic EFT</th>
<th>Nonrelativistic limit</th>
<th>$\sum_i O_i$</th>
<th>cross section scaling</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\bar{\chi} \gamma^\mu \gamma^5 \chi \gamma_\mu \gamma^5 N$</td>
<td>$-4 \vec{S}_\chi \cdot \vec{S}_N$</td>
<td>$-4 O_4$</td>
<td>$W_{\Sigma \tau \tau'}(q^2) + W_{\Sigma}^2 \bar{W}_{\Sigma}(q^2)$</td>
</tr>
<tr>
<td>2</td>
<td>$2 \bar{\chi} \gamma^\mu \gamma_\mu \gamma^{5 \nu} \gamma^5 N + \bar{\chi} \gamma^\mu \gamma_\mu \gamma^{5 \nu} \gamma^5 N$</td>
<td>$-4 \vec{S}_N \cdot \vec{v}_T$</td>
<td>$-4 O_7$</td>
<td>$(v_T^\perp)^2 W_{\Sigma}^2 \bar{W}_{\Sigma}(q^2)$</td>
</tr>
<tr>
<td>3</td>
<td>$2 \bar{\chi} \gamma^\mu \gamma_\mu \gamma^{5 \nu} \gamma^5 N - \bar{\chi} \sigma_\mu \sigma_\nu \gamma^{5 \nu} \gamma^5 N$</td>
<td>$-4 \vec{S}_N \cdot \vec{v}_T$</td>
<td>$-4 O_7$</td>
<td>$(v_T^\perp)^2 W_{\Sigma}^2 \bar{W}_{\Sigma}(q^2)$</td>
</tr>
<tr>
<td>4</td>
<td>$\bar{\chi} \gamma^\mu \chi \tilde{N} \gamma_\mu \gamma^5 N$</td>
<td>$-2 \vec{S}<em>N \cdot \vec{v}<em>T + \frac{2}{m</em>{WIMP}} i \vec{S}</em>\chi \cdot (\vec{S}_N \times \vec{q})$</td>
<td>$-2 O_7 + \frac{2 m_N}{m_{WIMP}} O_{10}$</td>
<td>$\simeq q^2 W_{\Sigma}^2 \bar{W}_{\Sigma}(q^2)$</td>
</tr>
<tr>
<td>5</td>
<td>$\bar{\chi} \sigma_\mu \sigma_\nu \gamma^{5 \nu} \gamma_\mu \gamma^5 N$</td>
<td>$4 i(\frac{q}{m_N} \times \vec{q}) \cdot \vec{S}_N$</td>
<td>$4 m_N O_{10}$</td>
<td>$q^2 W_{\Sigma}^2 \bar{W}_{\Sigma}(q^2)$</td>
</tr>
<tr>
<td>6</td>
<td>$\bar{\chi} \gamma^\mu \gamma_\mu \gamma^{5 \nu} \gamma^5 N$</td>
<td>$4 i \vec{S}_\chi \cdot (\frac{q}{m_N} \times \vec{S}_N)$</td>
<td>$-4 m_N O_{10}$</td>
<td>$q^2 W_{\Sigma}^2 \bar{W}_{\Sigma}(q^2)$</td>
</tr>
<tr>
<td>7</td>
<td>$i \bar{\chi} \gamma_\mu \gamma^5 N$</td>
<td>$i \frac{q}{m_N} \cdot \vec{S}_N$</td>
<td>$O_{14}$</td>
<td>$q^4 W_{\Sigma}^2 \bar{W}_{\Sigma}(q^2)$</td>
</tr>
<tr>
<td>8</td>
<td>$i \bar{\chi} \sigma_\mu \sigma_\nu \gamma^{5 \nu} \gamma_\mu \gamma^5 N$</td>
<td>$-4 i \left(\frac{q}{m_N} \cdot \vec{S}_\chi\right)(\vec{v}_T^\perp \cdot \vec{S}_N)$</td>
<td>$-4 m_N O_{14}$</td>
<td>$(v_T^\perp)^2 q^2 W_{\Sigma}^2 \bar{W}_{\Sigma}(q^2)$</td>
</tr>
<tr>
<td>9</td>
<td>$\bar{\chi} \gamma_\mu \gamma_\mu \gamma^{5 \nu} \gamma^5 N$</td>
<td>$-\frac{q}{m_{WIMP}} \cdot \vec{S}_N$</td>
<td>$-\frac{m_N}{m_{WIMP}} O_{14}$</td>
<td>$q^4 W_{\Sigma}^2 \bar{W}_{\Sigma}(q^2)$</td>
</tr>
<tr>
<td>10</td>
<td>$\bar{\chi} \sigma_\mu \sigma_\nu \gamma^{5 \nu} \gamma_\mu \gamma^5 N$</td>
<td>$4 \frac{q}{m_N} \cdot \vec{S}_\chi \frac{q}{m_N} \cdot \vec{S}_N$</td>
<td>$4 \frac{m_N}{m_{WIMP}} O_{14}$</td>
<td>$q^4 W_{\Sigma}^2 \bar{W}_{\Sigma}(q^2)$</td>
</tr>
<tr>
<td>11</td>
<td>$\bar{\chi} \sigma_\mu \sigma_\nu \gamma^{5 \nu} \gamma_\mu \gamma^5 N$</td>
<td>$4 \left(\frac{q}{m_N} \times \vec{S}_\chi\right) \cdot \left(\frac{q}{m_N} \times \vec{S}_N\right)$</td>
<td>$4 \left(\frac{q^2}{m_N} O_4 - m_N^2 O_6\right)$</td>
<td>$q^4 W_{\Sigma}^2 \bar{W}_{\Sigma}(q^2)$</td>
</tr>
</tbody>
</table>

With the exception of the last column each scenario corresponds to one $O_i$, linear combination of $O_4$–$O_6$ in the last column.

Resulting spin–dependent scenarios :Since them the 4, 6, 9, 10 and 46. $(7 \text{ equivalent to } 4, 14 \text{ equivalent to } 9)$

Results

Generalized Spin–dependent interaction with elastic scattering

\[ D = 1 \]

\[ D = 1.7 \]

\[ \mathcal{O}_6, \mathcal{O}_{46} (q^4 \text{ momentum dependence}) \text{ and } \mathcal{O}_9, \mathcal{O}_{10} (q^2 \text{ momentum dependence}) \] are compatible, \( \mathcal{O}_4 \) (no momentum dependence) is not compatible.

\[ \mathcal{D} \equiv \text{compatibility factor} \ [1]\]

\[ \text{If } D < 1 \text{ all constraints are verified.} \]

\[ \text{Diagram (a) and (b) illustrate the impact of momentum dependence on } \mathcal{D}. \]


Results

Generalized Spin–dependent interaction with elastic scattering

$m_\chi = 10\text{GeV}$

$m_\chi = 30\text{GeV}$

\[ \frac{c_n}{c_p} = 0 \]

same compatibility factor as $\mathcal{O}_7$

**Results**

Generalized Spin–dependent interaction with elastic scattering

\[ m_\chi = 10 \text{GeV} \]

\[ m_\chi = 33 \text{GeV} \]

\[ \frac{c_n}{c_p} = 0 \]

same compatibility factor as \( \mathcal{O}_{10} \) and \( \mathcal{O}_{14} \)

Results

Generalized Spin–dependent interaction with elastic scattering

$m_\chi = 10\text{GeV}$

$m_\chi = 30\text{GeV}$

\[
\frac{c_n}{c_p} = 0
\]

same compatibility factor as $O_{46}$

Conclusions

The DAMA excess result is incompatible with other constraints if direct-detection data are analyzed with following assumptions:

1. Spin-dependent or isoscalar spin-independent cross section
2. Maxwellian velocity distribution in our Galaxy
3. WIMP elastic scattering

However, DAMA excess can compatible with constraints with:

1. Using non-relativistic EFT which introduces new response functions with explicit dependence on the transferred momentum and the WIMP incoming velocity
2. Factorizing the halo-function dependence
3. Allowing for inelastic scattering
4. Allowing for isovector couplings

In this way much wider parameter space opens up.
We are just starting to scratch below the surface of the most general WIMP parameter space.