Lattice computation of the nucleon sigma terms at the physical point

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TAUP 2015

Turin - September 8, 2015

Outline

1. Theoretical framework
2. Numerical setup and methodology
3. Data analysis - part I: fit strategy
4. Data analysis - part II: assessing statistical and systematic errors
5. Conclusions

The nucleon up-down and strange sigma terms $\sigma_{\pi N}$ and $\sigma_{\bar{s}sN}$, defined as

$$\sigma_{\pi N} \equiv m_{ud}\langle N|\bar{u}u + \bar{d}d|N\rangle \ , \qquad \sigma_{\bar{s}sN} \equiv 2m_s\langle N|\bar{s}s|N\rangle \ .$$

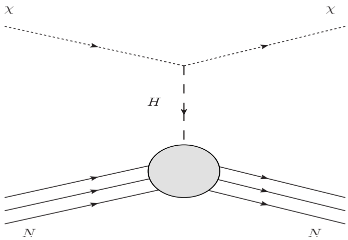
are observables of great interest given their relation to

- the quark-mass ratio m_{ud}/m_s;
- $\pi - N$ and $K - N$ scattering;
- counting rates in Higgs-Boson searches;
- direct detection of dark matter (DM).

Quantities related to the sigma terms are the so-called quark contents f_{ud}^N and f_s^N

$$f_{ud}^N = \frac{\sigma_{\pi N}}{M_N} \ , \qquad f_s^N = \frac{\sigma_{\bar{s}sN}}{2M_N} \ .$$

At a fundamental level, the WIMP-nucleon scattering ($\chi - N$) is a WIMP-quark interaction ($\chi - q$)



The nucleon quark contents are the major source of theoretical uncertainty together with the WIMP distribution of velocity.

Determinations from phenomenology have large uncertainties and are in conflict $\Rightarrow$ ***ab initio*** **computation of strong-interaction effects**.

As it is well known, in the Standard Model the fundamental theory of the strong force is quantum Chromodynamics (QCD).

Main features:

- fermionic d.o.f $\longrightarrow$ quarks divided into three families: (u,d), (c,s) and (t,b);
- intermediate vector bosons $\longrightarrow$ gluons carrying colour charge;
- $SU(3)$ symmetry group (non-Abelian);
- at low energy, perturbation theory cannot be applied.

Numerical simulations can be used to solve QCD in the non-perturbative regime $\longrightarrow$ **Lattice**.

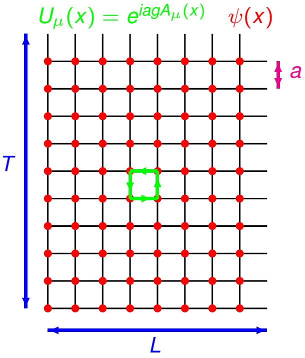
Lattice QCD (LQCD) in a nutshell:

Lattice gauge theory $\longrightarrow$ mathematically sound definition of NP QCD:

- UV (& IR) cutoff $\rightarrow$ well defined path integral in Euclidean spacetime:

$$\langle O \rangle = \int \mathcal{D}U\mathcal{D}\bar{\psi}\mathcal{D}\psi \, e^{-S_G - \int \bar{\psi} D[M] \psi} \, O[U, \psi, \bar{\psi}]$$
$$= \int \mathcal{D}U \, e^{-S_G} \, \det(D[M]) \, O[U]_{\text{Wick}}$$

- $\mathcal{D}U e^{-S_G} \det(D[M]) \geq 0$ & finite # of dofs
$\rightarrow$ evaluate numerically using stochastic methods



LQCD is QCD when $m_q \rightarrow m_q^{\text{phys}}$, $a \rightarrow 0$, $L \rightarrow \infty$ (and stats $\rightarrow \infty$)

HUGE conceptual and numerical ($\sim 10^9$ dofs) challenge

Some technical details:

- 3 fundamental parameters to be fixed: the coupling $\alpha_s(a)$ and the quark masses at the physical point $m_{ud}^{(\Phi)}$ and $m_s^{(\Phi)}$;
- $N_f = 2 + 1$;
- 47 ensembles corresponding to about 13000 overall configurations with
  - $0.054\ fm \lesssim a \lesssim 0.116\ fm$;
  - pion mass M_π down to $\lesssim 120$ MeV ($M_\pi^{(\Phi)} = 135$ MeV);
  - box size up to $\approx 6\ fm$ (protion radius $r_p \approx 0.8\ fm$);
- full non-perturbative renormalization of quark masses in the Renormalization Group Invariance (RGI) scheme (as in BMWc, JHEP 1108).

This setup allows for a **consistent control of systematic uncertainties**.

A possible strategy to compute them consists of relying on the Feynman-Hellman theorem, i.e.,

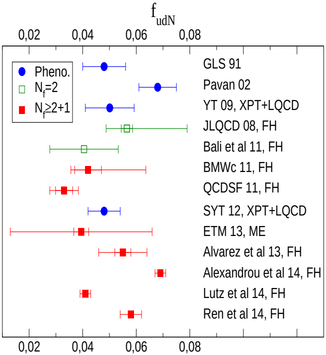
$$f_{ud}^N = \frac{m_{ud}}{M_N} \frac{\partial M_N}{\partial m_{ud}}\bigg|_\Phi \,, \qquad f_s^N = \frac{m_s}{M_N} \frac{\partial M_N}{\partial m_s}\bigg|_\Phi \,,$$

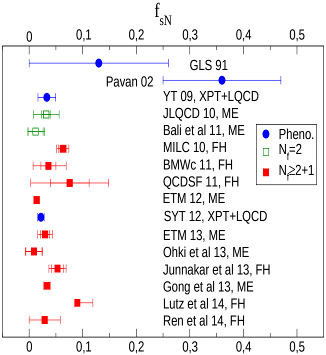
where derivatives have to be computed at the physical point (Φ).

Main advantages of this approach:

- need for 2-point functions only;
- no disconnected contributions.

Computations already exist (FH = Feynman-Hellmann, ME = Matrix Element):





However, most calculations employ model assumptions and/or have incomplete error analyses.

The mass of a particle p can be extracted from a time correlator $C_p(t, t_S)$

$$C_p(t, t_S) = a^3 \sum_{\vec{x}} \langle O_p(x) O_p^\dagger(x_S) \rangle ,$$

with a the lattice spacing, t the time component of point $x = (t, \vec{x})$ in the $4D$ discretized spacetime and where $O_p(x)$ is an interpolating operator capable of creating a hadron p out of the vacuum.

Asymptotically

$$\begin{aligned} C_p(t,0) &\propto cosh\left[\left(t - \frac{T}{2}\right) M_p\right] = cosh\left[\left(\tau - \frac{N_0}{2}\right) \widehat{M_p}\right] & (for\ mesons) , \\ C_p(t,0) &\propto e^{-M_p t} = e^{-\widehat{M_p}\tau} & (for\ baryons) , \end{aligned}$$

with $\tau = t/a$, $\widehat{M_p} = aM_p$, T the extent of the lattice in the time direction and $N_0 = T/a$.

In order to assess when the asymptotic behaviour sets in, it is useful to monitor the so-called effective mass $\widehat{E_p}(\tau)$ given by

$$\begin{aligned} \widehat{E_p}(\tau) &= arccosh\left[\frac{C_p(\tau-1,0) + C_p(\tau+1,0)}{2C_p(\tau)}\right] & (for\ mesons) , \\ \widehat{E_p}(\tau) &= log\left[\frac{C_p(\tau,0)}{C_p(\tau+1,0)}\right] & (for\ baryons) . \end{aligned}$$

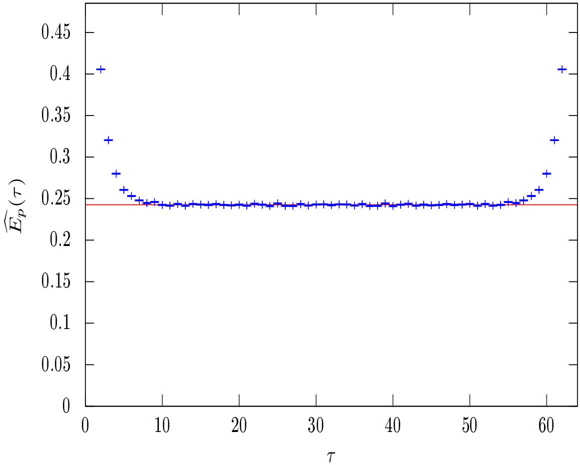


Figure 1: typical graph of $\widehat{E_p}(\tau)$ vs. τ for mesons: here the asymptotic behaviour sets in at $\tau \approx 10$. The red line corresponds to the fitted $\widehat{M_p}$.

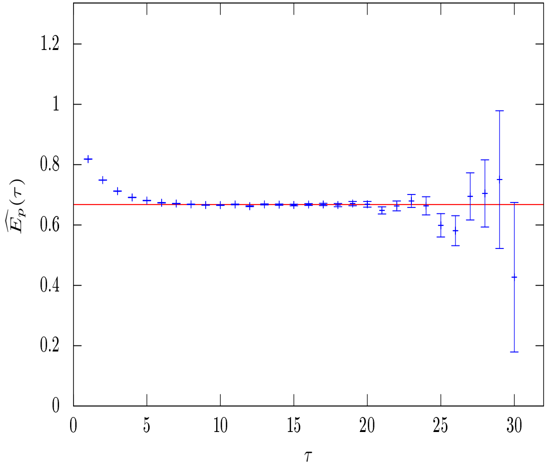


Figure 2: same as Fig. 2 but for baryons. Here the asymptotic behaviour sets in at $\tau \approx 10$. Points with percentage error larger than 0.25% are excluded from the fit.

An example of functional form — with **experimental input** to set $\alpha_s(a)$, $m_{ud}^{(\Phi)}$ and $m_s^{(\Phi)}$ and **fit parameters** — employed in the fit reads

$$aM_X = a\left\{ M_X^{(\Phi)} + \sum_i c_{X,ud,i} \left[\frac{am_{ud} Z_s^{-1}(\beta)}{a(1+d_{ud}a^2)} - m_{ud}^{(\Phi)} \right]^i + \right.$$
$$\left. + \sum_j c_{X,s,j} \left[\frac{am_s Z_s^{-1}(\beta)}{a(1+d_s a^2)} - m_s^{(\Phi)} \right]^j \right\} ,$$

with $X = \Omega$ (for scale setting), N, π and K^χ with $m_{K^\chi}^2 = m_K^2 - \frac{1}{2} m_\pi^2$. $M_N^{(\Phi)}$ was actually also considered as a fit parameter.

The masses of these four particles are fitted at the same time, i.e. the corresponding functionals share the same fit parameters - with the exception of the $c_{X,ud,i}$'s and $c_{X,s,i}$'s.

Quark masses in the functional above are obtained through the ratio-difference method (BMWc, JHEP 1108).

Fit parameters $c = \{a, m_{ud}^{(\phi)}, m_s^{(\phi)}, \ldots\}$ of functions $f^{(i)}(c, x)$ — with $i = 1, 2, 3, 4$ and $x = \{am_{ud}, am_s\}$ — are determined by minimizing a χ^2 function defined as

$$\chi^2 = V^T C^{-1} V \ ,$$

where C is the covariance matrix associated to the entries of the column vector V whose structure reads

$$V = (y_1^{(1)} - f^{(1)}(c, x_1) \ , \ \ldots \ , \ y_n^{(4)} - f^{(4)}(c, x_n) \ , \ x_1 - q_1 \ , \ x_2 - q_2 \ , \ \ldots \ , \ x_n - q_n) \ ,$$

where q_i is the value of variable x_i obtained in simulation i.

Entries of matrix C are obtained via a bootstrap procedure with $n_{boot} = 2000$.

All fits are correlated.

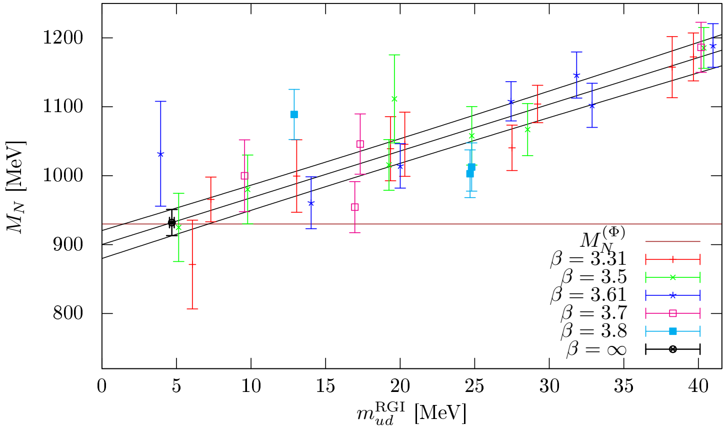


Figure 3: typical dependence of M_N vs. m_{ud}^{RGI}. The black point corresponds to the physical point while the horizontal line in brown to $M_N^{(\Phi)}$.

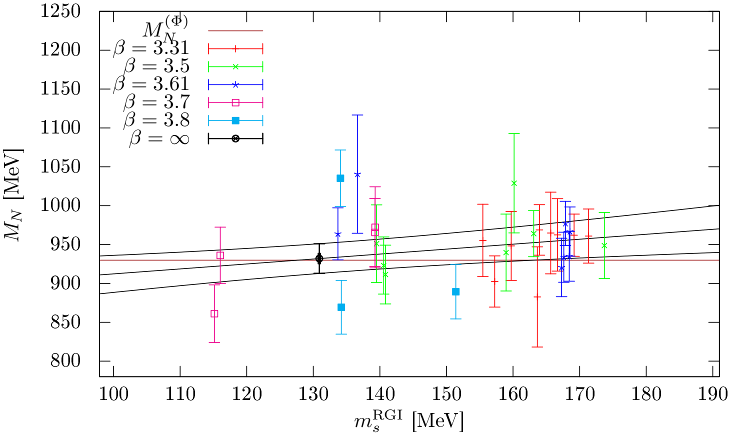


Figure 4: same as Fig. 3 but vs. m_s^{RGI}.

The sources of systematic uncertainty taken into account in this study are the following:

- contributions from excited states in the fit of $C_p(t, 0)$;
- truncation errors in the power series;
- different definitions of Z_S (as in BMWc, JHEP 1108);
- discretization artifacts that might go either as a^2 or as $\alpha_s a$.

Further sources are taken into account by considering different parametrizations for the m_{ud} and m_s dependence of baryons (polynomials vs. Padé).

This results in $2 \cdot 2 \cdot 6 \cdot 2 \cdot 2 =$ **96** fitting strategies altogether.

The systematics is subsequently evaluated by means of the **Akaike Information Criterion**, i.e. for the i^{th} fitting procedure the AIC value AIC_i

$$AIC_i = 2k_i - 2ln(L_i) = 2k_i + \chi^2 \ ,$$

is computed, being k_i the number of fit parameters and L_i the value of the likelihood function at its minimum.

The mean value and systematic error of a generic fit parameter c_i are obtained by computing, respectively, the weighted mean standard deviation of the values of c_i resulting from the different fitting procedures with the weight ω_i given by

$$\omega_i = exp[(AIC_{min} - AIC_i)/2] \ ,$$

where AIC_{min} is the lowest of the AIC_i's.

The bootstrap error on the mean provides the statistical error.

Preliminary results for $f_{udN} \equiv \sigma_{\pi N}/M_N$ and $f_{sN} \equiv \sigma_{\bar{s}sN}/(2M_N)$ read

$$f_{udN} = 0.0393(34)(19) \; , \qquad f_{sN} = 0.101(42)(2) \; ,$$

while the estimate obtained for M_N is given by

$$M_N = 939(13)(2) \, \mathrm{MeV} \; ,$$

in excellent agreement with the experimental value $M_N = 938.9 \, \mathrm{MeV}$.

The study of systematic errors is still in progress, though.

Conclusions

- f_{ud}^N and f_s^N are being computed directly in QCD without additional assumptions that are difficult to justify;
- a complete investigation of different sources of systematic uncertainties is underway;
- errorbars are still a bit large, at least on f_{sN}.

Outlook

- more lever arm on m_s and smaller statistical errors are needed to improve the precision on f_{sN};
- work on spin-dependent couplings has begun.