Boltzmann hierarchy for interacting neutrinos

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massless neutrinos ⚡ observation of neutrino oscillations

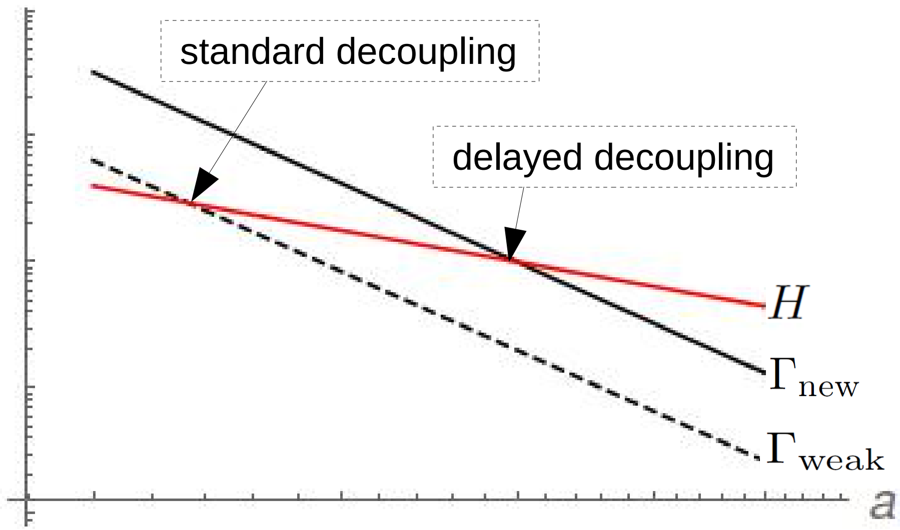
→ Models of neutrino mass generation, “Majoron models“

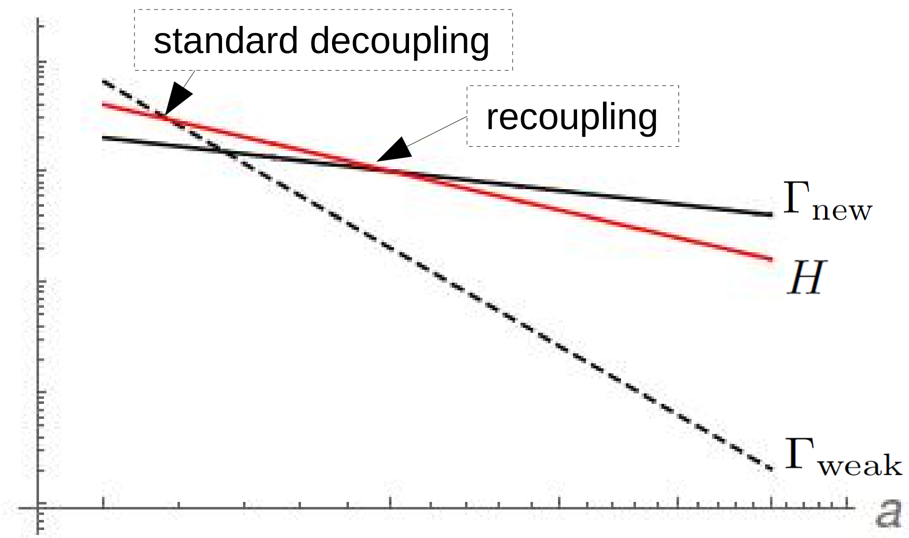
$$\mathcal{L}_{\rm int} = \mathfrak{g}_{ij}\bar{\nu}_i\nu_j\phi + \mathfrak{h}_{ij}\bar{\nu}_i\gamma_5\nu_j\phi$$

→ **non-standard neutrino interactions**

Massive scalar → $\Gamma_{\rm new} \sim G_{\rm eff}^2 T^5$

Massless scalar → $\Gamma_{\rm new} \sim \mathfrak{g}T$





→ **cosmological signature?**

Einstein equation: $\bar{G}_{\mu\nu} + \delta G_{\mu\nu} = 8\pi G(\bar{T}_{\mu\nu} + \delta T_{\mu\nu})$

$$ds^2 = a(\tau)^2 \left(-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j\right)$$

Boltzmann equation: $P^\alpha \frac{\partial f}{\partial x^\alpha} - \Gamma^\gamma_{\alpha\beta} P^\alpha P^\beta \frac{\partial f}{\partial P^\gamma} = \left(\frac{\partial f}{\partial \tau}\right)_{\mathrm{coll}}$

- Perturb $f_i(\boldsymbol{x}, \boldsymbol{P}, \eta) = \bar{f}_i(|\boldsymbol{q}|, \eta) + F_i(\boldsymbol{x}, \boldsymbol{q}, \eta)$
- Fourier space

$$\dot{F}(\boldsymbol{k}, \boldsymbol{q}, \eta) + \mathrm{i}\frac{|\boldsymbol{q}||\boldsymbol{k}|}{\epsilon}(\hat{k} \cdot \hat{q})\, F(\boldsymbol{k}, \boldsymbol{q}, \eta) + \frac{\partial \bar{f}(|\boldsymbol{q}|)}{\partial \ln |\boldsymbol{q}|}\left[\dot{\tilde{\eta}} - (\hat{k} \cdot \hat{q})^2 \frac{\dot{h} + 6\dot{\tilde{\eta}}}{2}\right] = \left(\frac{\partial f}{\partial \eta}\right)^{(1)}_{\mathrm{coll}}$$

Apply on all relevant particle species:

	interacting	non-interacting
relativistic	photons	neutrinos ???
non-relativistic	baryons	CDM

$$F(|\boldsymbol{k}|, |\boldsymbol{q}|, \hat{k} \cdot \hat{q}) = \sum_{\ell=0}^{\infty} (-\mathrm{i})^{\ell} (2\ell + 1)\, F_{\ell}(|\boldsymbol{k}|, |\boldsymbol{q}|)\, P_{\ell}(\hat{k} \cdot \hat{q})$$ → Taking moments

→ Boltzmann hierarchy (no interactions):

$$\dot{\delta}_\nu = -\frac{4}{3}\theta_\nu - \frac{2}{3}\dot{h},$$
$$\dot{\theta}_\nu = k^2\left(\frac{1}{4}\delta_\nu - \sigma_\nu\right),$$
$$\dot{F}_{\nu 2} = 2\dot{\sigma}_\nu = \frac{8}{15}\theta_\nu - \frac{3}{5}kF_{\nu 3} + \frac{4}{15}\dot{h} + \frac{8}{5}\dot{\eta},$$
$$\dot{F}_{\nu l} = \frac{k}{2l+1}\left[lF_{\nu(l-1)} - (l+1)F_{\nu(l+1)}\right],\; l \geq 3$$

Neutrino interactions???

1.) Tightly coupled limit: $F_{\nu l \geq 2} = 0$

$$\dot{\delta}_\nu = -\tfrac{4}{3}\theta_\nu - \tfrac{2}{3}\dot{h}$$
$$\dot{\theta}_\nu = \tfrac{1}{4}k^2\delta_\nu$$

→ only valid for sufficiently strong couplings

*Hannestad, astro-ph/0411475, ...

2.) Adding damping term:

$$\dot{\mathcal{F}}_{\nu 2} = \frac{8}{15}\theta_\nu - \frac{3}{5}k\mathcal{F}_{\nu 3} + \frac{4}{15}\dot{h} + \frac{8}{5}\dot{\tilde{\eta}} + \frac{9}{10}\alpha_2\dot{\tau}_\nu\mathcal{F}_{\nu 2} \ ,$$

$$\dot{\mathcal{F}}_{\nu\ell} = \frac{k}{2\ell+1}\left[\ell\mathcal{F}_{\nu(\ell-1)} - (\ell+1)\mathcal{F}_{\nu(\ell+1)}\right] + \alpha_\ell\dot{\tau}_\nu\mathcal{F}_{\nu\ell} \ , \quad \ell \geq 3$$

→ motivated from the photon hierachy

*Cyr-Racine, Sigurdson, astro-ph/1306.1536

3.) Parametrisation used to fit cosmologocal data:

$$\dot{\delta}_\nu = -\frac{4}{3}\theta_\nu - \frac{2}{3}\dot{h} + \frac{\dot{a}}{a}(1-3c_{\rm eff}^2)\left(\delta_\nu + 4\frac{\dot{a}}{a}\frac{\theta_\nu}{k^2}\right) \ ,$$

$$\dot{\theta}_\nu = k^2\left(\frac{1}{4}\delta_\nu - \sigma_\nu\right) - \frac{k^2}{4}(1-3c_{\rm eff}^2)\left(\delta_\nu + 4\frac{\dot{a}}{a}\frac{\theta_\nu}{k^2}\right) \ ,$$

$$\dot{\mathcal{F}}_{\nu 2} = 2\dot{\sigma}_\nu = \frac{8}{15}\theta_\nu - \frac{3}{5}k\mathcal{F}_{\nu 3} + \frac{4}{15}\dot{h} + \frac{8}{5}\dot{\tilde{\eta}} - (1-3c_{\rm vis}^2)\left(\frac{8}{15}\theta_\nu + \frac{4}{15}\dot{h} + \frac{8}{5}\dot{\tilde{\eta}}\right) \ ,$$

$$\dot{\mathcal{F}}_{\nu\ell} = \frac{k}{2\ell+1}\left[\ell\mathcal{F}_{\nu(\ell-1)} - (\ell+1)\mathcal{F}_{\nu(\ell+1)}\right] \ , \quad \ell \geq 3 \ ,$$

$$(c_{\rm eff}^2, c_{\rm vis}^2) = (\tfrac{1}{3}, \tfrac{1}{3})$$

→ standard case

$c_{\rm vis}^2 = 0 \qquad c_{\rm eff}^2 < \dfrac{1}{3}$ → tightly coupled limit...???

*e.g. Melchiorri, arxiv:1109.2767, ...

!

- No proof.
- Accurate treatment of interacting neutrinos includes calculation of the collision integral.

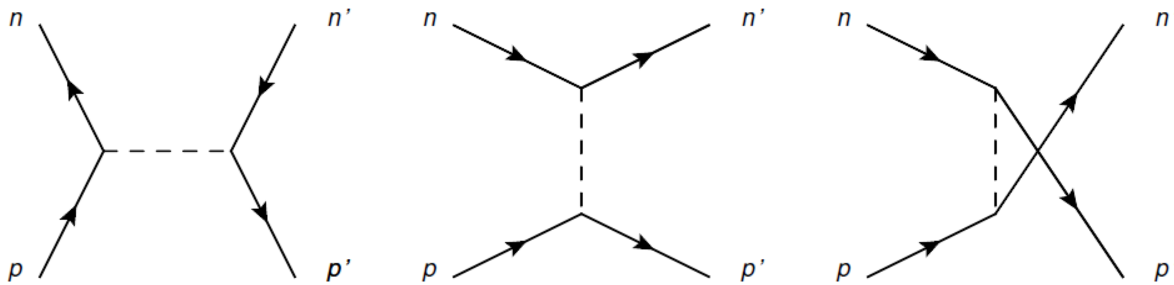
$$\rightarrow \dot{F}(\boldsymbol{k},\boldsymbol{q},\eta) + \mathrm{i}\frac{|\boldsymbol{q}||\boldsymbol{k}|}{\epsilon}(\hat{k}\cdot\hat{q})\,F(\boldsymbol{k},\boldsymbol{q},\eta) + \frac{\partial \bar{f}(|\boldsymbol{q}|)}{\partial \ln|\boldsymbol{q}|}\left[\dot{\tilde{\eta}} - (\hat{k}\cdot\hat{q})^2\,\frac{\dot{h}+6\dot{\tilde{\eta}}}{2}\right] = \left(\frac{\partial f}{\partial \eta}\right)^{(1)}_{\mathrm{coll}}$$

difference to photon case: Thomson scattering = low energy transfer

Approximations: Neglect of quantum statistical effects, zero neutrino masses, zero chemical potentials

$$\left(\frac{\partial f_i}{\partial \eta}\right)^{(1)}_{ij\leftrightarrow kl}(\boldsymbol{k},\boldsymbol{q},\eta) = \frac{g_j g_k g_l}{2|\boldsymbol{q}|(2\pi)^5}\int\frac{\mathrm{d}^3\boldsymbol{q}'}{2|\boldsymbol{q}'|}\int\frac{\mathrm{d}^3\boldsymbol{l}}{2|\boldsymbol{l}|}\int\frac{\mathrm{d}^3\boldsymbol{l}'}{2|\boldsymbol{l}'|}\,\delta^{(4)}_{\mathrm{D}}(q+l-q'-l')$$
$$\times\,|\mathcal{M}_{ij\leftrightarrow kl}|^2\Big(\bar{f}_k(|\boldsymbol{q}'|,\eta)\,F_l(\boldsymbol{k},\boldsymbol{l}',\eta) + \bar{f}_l(|\boldsymbol{l}'|,\eta)\,F_k(\boldsymbol{k},\boldsymbol{q}',\eta)$$
$$-\,\bar{f}_i(|\boldsymbol{q}|,\eta)\,F_j(\boldsymbol{k},\boldsymbol{l},\eta) + \bar{f}_j(|\boldsymbol{l}|,\eta)\,F_i(\boldsymbol{k},\boldsymbol{q},\eta)\Big)$$

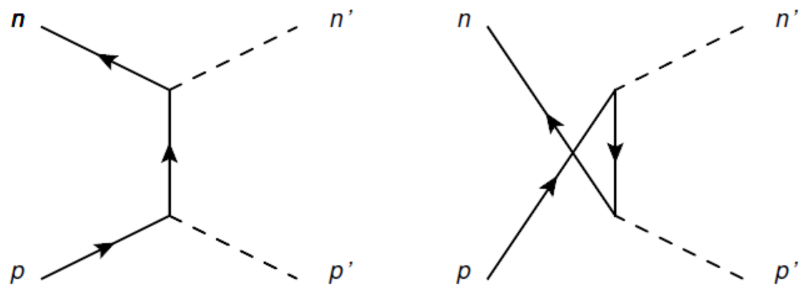
$\nu\nu \leftrightarrow \nu\nu$:



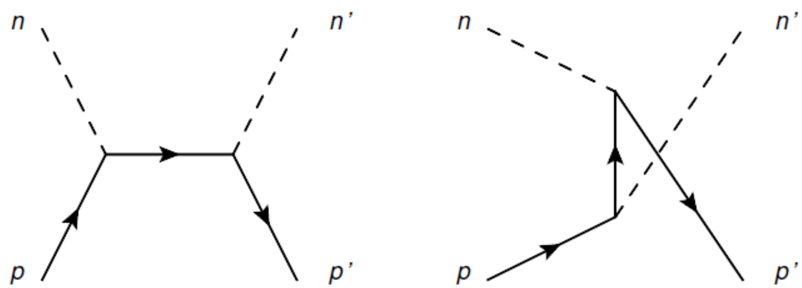
massive case:

only neutrino self-interactions

$\nu\nu \leftrightarrow \phi\phi$:



$\nu\phi \leftrightarrow \nu\phi$:



massless case:

need to include new hierarchy for scalar particle as well

$$
\begin{aligned}
\dot{F}_{\nu,0}(q) =& -kF_{\nu,1}(q) + \frac{1}{6}\frac{\partial \bar{f}_\nu}{\partial \ln q}\dot{h} - \frac{40}{3}G^{\rm m} q\, T_{\nu,0}^4\, F_{\nu,0}(q) \\
&+ G^{\rm m}\int \mathrm{d}q'\,\frac{q'}{q}\left[K_0^{\rm m}(q,q') - \frac{20}{9}q^2\, q'^2 e^{-q/T_{\nu,0}}\right] F_{\nu,0}(q')\,, \\
\dot{F}_{\nu,1}(q) =& -\frac{2}{3}kF_{\nu,2}(q) + \frac{1}{3}kF_{\nu,0}(q) - \frac{40}{3}G^{\rm m} q\, T_{\nu,0}^4\, F_{\nu,1}(q) \\
&+ G^{\rm m}\int \mathrm{d}q'\,\frac{q'}{q}\left[K_1^{\rm m}(q,q') + \frac{10}{9}q^2\, q'^2 e^{-q/T_{\nu,0}}\right] F_{\nu,1}(q')\,, \\
\dot{F}_{\nu,2}(q) =& -\frac{3}{5}kF_{\nu,3}(q) + \frac{2}{5}kF_{\nu,1}(q) - \frac{\partial \bar{f}_\nu}{\partial \ln q}\left(\frac{2}{5}\dot{\tilde{\eta}} + \frac{1}{15}\dot{h}\right) - \frac{40}{3}G^{\rm m} q\, T_{\nu,0}^4\, F_{\nu,2}(q) \\
&+ G^{\rm m}\int \mathrm{d}q'\,\frac{q'}{q}\left[K_2^{\rm m}(q,q') - \frac{2}{9}q^2\, q'^2 e^{-q/T_{\nu,0}}\right] F_{\nu,2}(q')\,, \\
\dot{F}_{\nu,\ell>2}(q) =& \frac{k}{2\ell+1}\left[\ell F_{\nu,\ell-1}(q) - (\ell+1)F_{\nu,\ell+1}(q)\right] - \frac{40}{3}G^{\rm m} q\, T_{\nu,0}^4\, F_{\nu,\ell}(q) \\
&+ G^{\rm m}\int \mathrm{d}q'\,\frac{q'}{q}K_\ell^{\rm m}(q,q')\, F_{\nu,\ell}(q')\,,
\end{aligned}
$$

- **momentum-dependence reflects non-negligible energy transfer**
- integration over momentum yields $c_{\rm eff}^2 = \frac{1}{3}$ and $c_{\rm vis}^2 = ???$

$$
\begin{aligned}
\dot{F}_{\nu,0}(q) =& -kF_{\nu,1}(q) + \frac{1}{6}\frac{\partial f_\nu}{\partial \ln q}\dot{h} - G^0\mathcal{X}^\nu(q)\,F_{\nu,0}(q) \\
&+ G^0\int \mathrm{d}q'\,\frac{q'}{q}\,\mathcal{K}_0^\nu(q,q')\,F_{\nu,0}(q') + G^0\int \mathrm{d}q'\,\frac{q'}{q}\,\mathfrak{K}_0^\nu(q,q')\,F_{\phi,0}(q')\,, \\
\dot{F}_{\nu,1}(q) =& -\frac{2}{3}kF_{\nu,2}(q) + \frac{1}{3}kF_{\nu,0}(q) - G^0\mathcal{X}^\nu(q)\,F_{\nu,1}(q) \\
&+ G^0\int \mathrm{d}q'\,\frac{q'}{q}\,\mathcal{K}_1^\nu(q,q')\,F_{\nu,1}(q') + G^0\int \mathrm{d}q'\,\frac{q'}{q}\,\mathfrak{K}_1^\nu(q,q')\,F_{\phi,1}(q')\,, \\
\dot{F}_{\nu,2}(q) =& -\frac{3}{5}kF_{\nu,3}(q) + \frac{2}{5}kF_{\nu,1}(q) - \frac{\partial \bar{f}_\nu}{\partial \ln q}\left(\frac{2}{5}\dot{\tilde{\eta}} + \frac{1}{15}\dot{h}\right) - G^0\mathcal{X}^\nu(q)\,F_{\nu,2}(q) \\
&+ G^0\int \mathrm{d}q'\,\frac{q'}{q}\,\mathcal{K}_2^\nu(q,q')\,F_{\nu,2}(q') + G^0\int \mathrm{d}q'\,\frac{q'}{q}\,\mathfrak{K}_2^\nu(q,q')\,F_{\phi,2}(q')\,, \\
\dot{F}_{\nu,\ell>2}(q) =& \frac{k}{2\ell+1}\left[\ell F_{\nu,\ell-1}(q) - (\ell+1)F_{\nu,\ell+1}(q)\right] - G^0\mathcal{X}^\nu(q)\,F_{\nu,\ell}(q) \\
&+ G^0\int \mathrm{d}q'\,\frac{q'}{q}\,\mathcal{K}_\ell^\nu(q,q')\,F_{\nu,\ell}(q') + G^0\int \mathrm{d}q'\,\frac{q'}{q}\,\mathfrak{K}_\ell^\nu(q,q')\,F_{\phi,\ell}(q')\,,
\end{aligned}
$$

+ similar hierarchy for massless scalar

- time-dependent integral kernels
- coupling to scalar hierarchy at each multipole order!

<u>Conclusion:</u> We have for the first time calculated the Boltzmann hierarchy for interacting neutrinos from first principles. Compared to previous approaches in the literature, our results reveal a much richer structure for the collsion terms.

<u>Outlook:</u> However, to study the precise phenomenological impact of our findings, we have to implement our Boltzmann hierarchy into Boltzmann solver.

Conclusion: We have for the first time calculated the Boltzmann hierarchy for interacting neutrinos from first principles. Compared to previous approaches in the literature, our results reveal a much richer structure for the collsion terms.

Outlook: However, to study the precise phenomenological impact of our findings, we have to implement our Boltzmann hierarchy into Boltzmann solver.

Thank you for your attention!