Theoretical challenges in Double Beta Decay

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Introduction

Double beta decay is the nuclear process with the longest lifetime measured so far, which presents a great interest especially for testing LNV and understanding the neutrino properties. Moreover it has a broader potential to provide info on BSM processes.

According to the number and type of the released leptons we may have the following DBD modes: ***i) 2νβ⁻β⁻; ii) 0νβ⁻β⁻; iii) 2νβ⁺β⁺; iv) 0νβ⁺β⁺; v) 2νECβ⁺; vi) 0νECβ⁺; vii) 2νECEC; viii) 0νECEC ; ix) 0νβ⁻β⁻ M***

$$2\nu\beta\beta \;:\; (\mathbf{A},\mathbf{Z}) \rightarrow (\mathbf{A},\mathbf{Z}+\mathbf{2}) + \mathbf{2e}^- + \mathbf{2}\bar{\nu} \qquad \left(T^{2\nu}_{1/2}\right)^{-1} = G^{m}_{2\nu}(E_0,Z)\, g_A^4 \mid m_e c^2 M_{2\nu} \mid^2$$

$$0\nu\beta\beta \;:\; (\mathbf{A},\mathbf{Z}) \rightarrow (\mathbf{A},\mathbf{Z}+\mathbf{2}) + \mathbf{2e}^- \qquad \left(T^{0\nu}_{1/2}\right)^{-1} = G^{m}_{0\nu}(E_0,Z)\, g_A^4 \mid M_{0\nu} \mid^2 (< m_\nu > /m_e)^2$$

Precise calculations of both ***PSF*** and ***NME*** are required to predict lifetimes, derive nuclear input parameters and neutrino parameters, and other BSM info.

NME: many works devoted to their calculations: different methods, different groups

PSF: less attention (considered to be calculated with enough accuracy). Recently, they were re-computed more accurately and one found relevant differences as compared with previous values in several cases.

Nuclear Matrix Elements (NME)

Are currently computed with several methods, each with its advantages and disadvantages

1) pnQRPA(different versions); 2) ShM; 3) PHFB; 4) IBM-2; 5) GCM

Differences between NME values computed with these methods are still significant (a factor 2-3)

:| due to assumptions/approximations which are specific of each of these methods
|: due to the different nuclear ingredients/parameters used in calculations

ShM – is well suited to do calculations for DBD. It incorporates all types of correlations and uses effective NN interactions which are checked with other spectroscopic calculations for the nuclei from the same mass region; the obtained states have the correct no. of p and n

However, for most nuclei the valence spaces do not include enough states; it faces the problem with large model spaces and associated computational resources

Issues: i) GT operator needs to be quenched in the case of $2\nu\beta\beta$ calculations.

Question: should it be quenched as well, for $0\nu\beta\beta$ calculations?

ii) contribution of higher order nuclear currents: a reduction of 20-30%

Question: how much is this contribution for larger model spaces? Is this reduction amplified by the equivalent effective operator ?

Investigations of these issues imply calculations in increasingly larger model spaces (for example using 8 major HO shells or more) → improved (fast, efficient) ShM codes which reduce substantially the computing time of calculation of the TBME of the transition operators for $0\nu\beta\beta$ decay

ShM calculations

Fast numerical code for computing the TBME

Horoi, Stoica, PRC81(2010); Neacsu, Stoica, Horoi, PRC 86(2012), Neacsu, Stoica, JPG 41(2014)

$$M_\alpha^{0\nu} = \sum_{j_p j_{p'} j_n j_{n'} J_\pi} \mathrm{TBTD}(j_p j_{p'}, j_n j_{n'}; J_\pi) \langle j_p j_{p'}; J_\pi \| \tau_{-1}\tau_{-2} O_{12}^{\alpha} \| j_n j_{n'}; S_\alpha J_\pi \rangle$$

$$M^{0\nu} = M_{\mathrm{GT}}^{0\nu} - \left(\frac{g_V}{g_A}\right)^2 M_F^{0\nu} + M_T^{0\nu}$$

The most difficult is the computation of radial part of $M^{0\nu}$ which contains ν potentials

$$\langle nl|H_\alpha|n'l'\rangle \qquad H_\alpha(r) = \frac{2R}{\pi}\int_0^\infty j_i(qr)\frac{h_\alpha(q)}{\omega}\frac{1}{\omega + \langle E\rangle} q^2\,\mathrm{d}q$$

Ingredients, which may differ from one computation to another:

SRC $\psi_{nl}(r) \to [1 + f(r)]\psi_{nl}(r)$ $\qquad f(r) = -c \cdot \mathrm{e}^{-ar^2}(1 - br^2)$

FNS $G_A(q^2) = g_A\left(\frac{\Lambda_A^2}{\Lambda_A^2 + q^2}\right)^2$ $\qquad G_V(q^2) = g_V\left(\frac{\Lambda_V^2}{\Lambda_V^2 + q^2}\right)^2$

HC $\mathbf{H_T^{0\nu}}$, $\mathbf{M_T^{0\nu}}$

$$\langle nl|H_\alpha(r)|n'l'\rangle = \int_0^\infty r^2\,\mathrm{d}r\psi_{nl}(r)\psi_{n'l'}(r)[1+f(r)]^2 \times \int_0^\infty q^2\,\mathrm{d}qV_\alpha(q)j_n(qr)$$

$$\langle nl|H_\alpha(r)|n'l'\rangle = \sum_{s=0}^{n+n'} A_{l+l'+2s}(nl,n'l')\mathcal{K}_\alpha(m)$$

This procedure reduces substantially the CPU time: ~ with a factor of 30 as compared with our older ShM code from ref. PRC81 (2010)

Other ingredients: the effective NN interaction(GXPF1A, KB3, GN28, GN50, etc.)

Input parameters: $R = r_0A^{1/3}$ (r_0 = 1.1, or 1.2 fm), $\langle E_N \rangle$ = closure energy, g_A = 1.0, 1.25, 1.264, 1.272

Effects of these ingredients on the $M^{0\nu}$ values

Table 1 . The NMEs obtained with inclusion of different nuclear effects. "b" denotes the value obtained without any effect included, while "F", H" "S" and "total" indices denote the $M^{0\nu}$ values obtained when FNS, HOC, SRC and all effects, are, respectively, included. The set of the three values from the columns with SRC effects included refers to the particular prescriptions: (a)=Jastrow with MS parameterization, (b)=CCM-AV18 and (c)=CCM-CD-Bonn type. The calculations are performed with g_A=1.25, $r_0 = 1.2fm$, $\Lambda_V = 850MeV$, $\Lambda_A = 1086MeV$.

	M_b	M_{b+F}	M_{b+H}	M_{b+F+H}	M_{b+S}	M_{b+S+F}	M_{b+S+H}	$M^{0\nu}_{total}$
					(a)-0.731	-0.680	-0.542	-0.508
^{48}Ca	-1.166	-0.959	-0.923	-0.773	(b)-1.023	-0.930	-0.800	-0.733
					(c)-1.153	-1.008	-0.914	-0.809
					(a) 0.856	0.798	0.670	0.628
$^{48}Ca^*$	1.351	1.116	1.102	0.928	(b) 1.188	1.082	0.962	0.884
					(c) 1.337	1.171	1.092	0.969
					(a) 3.025	2.889	2.499	2.378
^{76}Ge	4.168	3.615	3.497	3.066	(b) 3.807	3.557	3.187	2.979
					(c) 4.153	3.762	3.489	3.177
					(a)-2.779	-2.665	-2.275	-2.176
^{82}Se	-3.779	-3.305	-3.140	-2.780	(b)-3.467	-3.256	-2.876	-2.703
					(c)-3.770	-3.438	-3.137	-2.878

Effects of the nuclear ingredients/input parameters

-their overall effect is to decrease the NME values

- SRC inclusion: J-MS prescription decreases significantly the NME value as compared with softer CCM prescriptions.

- however, NME values calculated with inclusion of only SRC by J-MS prescription, are close (within 10%) to the values calculated with SRC by CCM prescriptions and with the inclusion of other nuclear ingredients (FNS+HOC)→ a kind a compensation effect

- inclusion of HOC → correction up to 15 ~ 20%

- tensor component: contribution of (4-9)% (has to be taken with correct sign)

- dependence of NN interactions: up to 17%

- dependence on input nuclear parameters:

g_A quenched/unquenched – (10-14)%; $R = r_0 A^{1/3}$ (r_0=1.1fm or 1.2fm) ~ 7%

(Λ_A, Λ_V) ~ 8%;

- average energy used in closer approx. <E> - negligible

Phase Space Factors

i) Non-relativistic treatment: Primakov,Rosen, Rep.Prog.Phys. (1959); Haxton, Stephenson, Prog.Part.Nucl.Phys.(1984).

ii) Relativistic (simplest) treatment : approx. Dirac (s) electron w.f.
Suhonen&Civitarese, Phys. Rep.(1998)

iii) Relativistic more accurate treatment: electron w.f.: superposition of s and p distorted spherical waves, solutions of the Dirac eq. with a central (Coulomb) potential.
Doi et al., Prog.Theor.Phys. (1983, 1985, 1992,1993); T. Tomoda, Rep. Prog.(1991)

$$V(Z,r) = \begin{cases} -\frac{Z\alpha\hbar c}{r}, & r \geq R_A \\ -Z(\alpha\hbar c)\left(\frac{3-(r/R_A)^2}{2R_A}\right), & r < R_A \end{cases}$$

iv) **Further improvement**: take into account the electron screening effect. This was done by multiplying the expression of the potential with a function Φ(r), solution of the Thomas Fermi equation: Kotila&Iachello:PRC(2012, 2013)

v) Present work: follows the K&I's approach but with the following improvements: i) V(r) derived from a realistic proton density distribution in the nucleus; ii) new (own) numerical routines for solving the Dirac equations and integrating the PSF expressions; iii) a procedure to identify unambiguously the electron/positron bound states; iv) use of up-dated Q values.
Stoica,Mirea, PRC88(2013); Pahomi et al., Rom.Rep.Phys.67,272(2015).

a) 2νβ-β-

$$G_{2\nu}^{\beta\beta}(0^+ \to 0^+) = \frac{2\tilde{A}^2}{3\ln 2 g_A^4 (m_e c^2)^2} \int_{m_e c^2}^{Q^{\beta\beta}+m_e c^2} d\epsilon_1 \int_{m_e c^2}^{Q^{\beta\beta}+2m_e c^2-\epsilon_1} d\epsilon_2 \int_0^{Q^{\beta\beta}+2mc_e^2-\epsilon_1-\epsilon_2} d\omega_1$$
$$\times f_{11}^{(0)} w_{2\nu} (\langle K_N \rangle^2 + \langle L_N \rangle^2 + \langle K_N \rangle \langle L_N \rangle)$$

$$Q^{\beta\beta} = M(A, Z_0) - M(A, Z_0 - 2) - 4m_e c^2$$

$$f_{11}^{(0)} = | f^{-1-1} |^2 + | f_{11} |^2 + | f^{-1}{}_1 |^2 + | f_1{}^{-1} |^2$$

$$f^{-1-1} = g_{-1}(\epsilon_1) g_{-1}(\epsilon_2) \; ; \; f_{11} = f_1(\epsilon_1) f_1(\epsilon_2),$$
$$f^{-1}{}_1 = g_{-1}(\epsilon_1) f_1(\epsilon_2) \; ; \; f_1{}^{-1} = f_1(\epsilon_1) g_1(\epsilon_2)$$

$$\langle K_N \rangle = \frac{1}{\epsilon_1 + \omega_1 + \langle E_N \rangle - E_I} + \frac{1}{\epsilon_2 + \omega_2 + \langle E_N \rangle - E_I} \qquad \langle L_N \rangle = \frac{1}{\epsilon_1 + \omega_2 + \langle E_N \rangle - E_I} + \frac{1}{\epsilon_2 + \omega_1 + \langle E_N \rangle - E_I}$$

b) 0νβ-β-

$$G_{0\nu}^{\beta\beta}(0^+ \to 0^+) = \frac{2}{4 g_A^4 R_A^2 \ln 2} \int_{m_e c^2}^{Q^{\beta\beta}+m_e c^2} f_{11}^{(0)} w_{0\nu} d\epsilon_1 \qquad w_{0\nu} = \frac{g_A^4 (G \cos\theta_C)^4}{16\pi^5} (m_e c^2)^2 (\hbar c^2)(p_1 c)(p_2 c)\epsilon_1 \epsilon_2$$

- g_A , R_A , $\langle E_N \rangle$ input parameters which enter in the computation of G and M Hence, their use should be done in a consistent way in the computation of both quantities

Table 1: PSF for $\beta^-\beta^-$ decays to final g.s.

Nucleus	$Q^{\beta^-\beta^-}_{g.s.}$	$G^{\beta^-\beta^-}_{2\nu}(g.s.)$ $(10^{-21}$ yr$^{-1})$				$G^{\beta^-\beta^-}_{0\nu}(g.s.)$ $(10^{-15}$ yr$^{-1})$			
	(MeV)	This work	[27]	[23, 24]	[26]	This work	[27]	[23, 24]	[26]
^{48}Ca	4.267	15536	15550	16200	16200	24.65	24.81	26.1	26.0
^{76}Ge	2.039	46.47	48.17	53.8	52.6	2.372	2.363	2.62	2.55
^{82}Se	2.996	1573	1596	1830	1740	10.14	10.16	11.4	11.1
^{96}Zr	3.349	6744	6816		7280	20.48	20.58		23.1
^{100}Mo	3.034	3231	3308	3860	3600	15.84	15.92	18.7	45.6
^{110}Pd	2.017	132.5	137.7			4.915	4.815		
^{116}Cd	2.813	2688	2764		2990	16.62	16.70		18.9
^{128}Te	0.8665	0.2149	0.2688	0.35	0.344	0.5783	0.5878	0.748	0.671
^{130}Te	2.528	1442	1529	1970	1940	14.24	14.22	19.4	16.7
^{136}Xe	2.458	1332	1433	2030	1980	14.54	14.58	19.4	17.7
^{150}Nd	3.371	35397	36430	48700	48500	61.94	63.03	85.9	78.4
^{238}U	1.144	98.51	14.57			32.53	33.61		

[23] M. Doi, T. Kotani and E. Takasugi, Prog. Theor. Phys. Suppl. **83**, 1 (1985).

[24] M. Doi and T. Kotani, Prog. Theor. Phys. **87**, 1207 (1992); ibidem **89**, 139 (1993).

[26] J. Suhonen and O. Civitarese, Phys. Rep. **300**, 123 (1998).

[27] J. Kotila and F. Iachello, Phys. Rev. C **85**, 034316 (2012).

Observations

– very good agreement with [27] both for $G^{2\nu}$ and $G^{0\nu}$ for the majority of nuclei
exceptions: 128Te(~20%) and 238U(factor of 7)

- in comparison with older calculations there are some notable differences

Table 2: PSF for $\beta^-\beta^-$ decays to final excited 0_1^+ states

Nucleus	$Q_{0_1^+}^{\beta^-\beta^-}$	$G_{2\nu}^{\beta^-\beta^-}(0_1^+)$ $(10^{-21}\ \mathrm{yr}^{-1})$			$G_{0\nu}^{\beta^-\beta^-}(0_1^+)$ $(10^{-15}\ \mathrm{yr}^{-1})$	
	(MeV)	This work	[27]	[26]	This work	[27]
^{48}Ca	1.270	0.3518	0.3627	0.376	0.3041	0.2989
^{76}Ge	0.9171	0.06129	0.06978	0.0769	0.1932	0.1776
^{82}Se	1.508	4.170		4.80	0.9440	
^{96}Zr	2.201	169.4	175.4	190	4.594	4.566
^{100}Mo	1.904	57.08	60.55	101	3.168	3.162
^{110}Pd	0.5472	3.3×10^{-3}	4.8×10^{-3}		0.1223	0.08844
^{116}Cd	1.056	0.7590	0.8737	0.89	0.7585	0.7163
^{130}Te	0.7335	0.05460	0.07566	18.6	0.3651	0.3086
^{136}Xe	0.8790	0.2823	0.3622	0.485	0.6746	0.6127
^{150}Nd	2.631	4116	4329	4850	26.96	27.27
^{238}U	0.2032	1.5×10^{-4}	4.6×10^{-4}		0.8229	0.7534

[26] J. Suhonen and O. Civitarese, Phys. Rep. **300**, 123 (1998).

[27] J. Kotila and F. Iachello, Phys. Rev. C **85**, 034316 (2012).

Observations

- several cases, especially for heavier nuclei, where the differences are of (10-40)%; again, for ^{238}U our $G^{2\nu}$ value is 3 times smaller than KI result
- notable differences with older results

PSF for $\beta^-\beta^-$ decays to final excited 2_1^+ states

Nucleus	$Q_{2_1^+}^{\beta^-\beta^-}$ (MeV)	$G_{2\nu}^{\beta^-\beta^-}(2_1^+)$ (10^{-21} yr^{-1})			$G_{0\nu}^{\beta^-\beta^-}(2_1^+)$ (10^{-15} yr^{-1})	
		This work	[27]	[32]	This work	[27]
^{48}Ca	3.284	4074	4410	4400	57.09	60.4
^{76}Ge	1.480	0.384	0.48	0.49	1.66	1.84
^{82}Se	2.219	69.6	90.6	85	12.13	13.8
^{96}Zr	2.571	745.5		850	33.87	
^{100}Mo	2.494	569.0		690	32.1	
^{110}Pd	1.359	0.46			2.41	
^{116}Cd	1.520	1.88		2.3	4.28	
^{128}Te	0.4255	6.8×10^{-7}	1.36×10^{-6}	1.3×10^{-6}	0.049	0.067
^{130}Te	1.990	79.6	116	120	18.34	22.8
^{136}Xe	1.640	7.68		15	8.31	
^{150}Nd	3.037	30308	45600	49000	223	301
^{238}U	1.099	2.66			26.3	

27. M. Doi, T. Kotani, H. Nishiura, and E. Takasugi, Prog. Theor. Phys. **69**, 602 (1983).

32. J. Suhonen and O. Civitarese, Phys. Rep. **300**, 123 (1998).

Observations

- there are only older results
- there are many notable differences

Table 4: PSF for $\beta^+\beta^+$ decay mode

Nucleus	$Q^{\beta^+\beta^+}$ (MeV)	$G_{2\nu}^{\beta^+\beta^+}$ $(10^{-29}\ \mathrm{yr}^{-1})$			$G_{0\nu}^{\beta^+\beta^+}$ $(10^{-20}\ \mathrm{yr}^{-1})$		
		This work	[28]	[24]	This work	[28]	[24]
^{78}Kr	0.8023	9159	9770	13600	243.2	250	293
^{96}Ru	0.6706	942.3	1040	1080	80.98	84.5	90.7
^{106}Cd	0.7314	1794	2000	1970	91.75	92.6	102
^{124}Xe	0.8203	4261	4850	4770	107.8	114	123
^{130}Ba	0.5748	91.54	110	47.9	23.82	25.7	21
^{136}Ce	0.3345	0.205	0.267	0.559	2.13	2.42	3.55

[24] M. Doi and T. Kotani, Prog. Theor. Phys. **87**, 1207 (1992); ibidem **89**, 139 (1993).

[28] J. Kotila and F. Iachello, Phys. Rev. C **87**, 024313 (2013).

Observations

- differences of (11-30)% from KI results for $G^{2\nu}$, while for $G^{0\nu}$ there only one notable difference, for ^{136}Ce
- relevant differences in comparison with other results

Nucleus	$G_{2\nu}$ $[yr]^{-1}$	$M_{2\nu}$(th)	$T^{2\nu}_{1/2}$(th) $[yr]$	$T^{2\nu}_{1/2}$(exp) $[yr]^{-1}$	$G_{0\nu}$ $[yr]^{-1}$	$M_{0\nu}$	$T^{0\nu}_{1/2}$ $[yr]$
^{48}Ca	1.554×10^{-17}	0.0276^{1}	3.46×10^{19}	4.4×10^{19}	2.465×10^{-14}	0.82^{1} 2.2^{2}	$> 1.3\times10^{22}$NE3
^{76}Ge	4.647×10^{-20}	0.0613^{1}	2.34×10^{21}	1.65×10^{21}	2.372×10^{-15}	3.19^{1} 5.39^{3}	$> 2.1\times10^{25}$GE
^{82}Se	1.573×10^{-18}	0.0634^{1}	6.48×10^{19}	0.92×10^{20}	1.014×10^{-14}	2.99^{1} 4.34^{3}	$> 2.1\times10^{23}$NE3
^{96}Zr	6.744×10^{-18}			2.3×10^{19}	2.048×10^{-14}	2.37^{4}	$> 8.6\times10^{21}$NE3
^{100}Mo	3.231×10^{-18}			7.1×10^{18}	1.584×10^{-14}	4.42^{4}	$> 1.1\times10^{24}$NE3
^{128}Te	2.149×10^{-22}	0.0245^{1}	3.17×10^{24}	2.0×10^{24}	5.783×10^{-16}	1.73^{1} 4.47^{3}	
^{130}Te	1.442×10^{-18}	0.0179^{1}	8.88×10^{20}	6.9×10^{20}	1.424×10^{-14}	2.02^{1} 4.04^{4}	$> 2.8\times10^{24}$CU
^{136}Xe	1.332×10^{-18}	0.0118^{1}	2.04×10^{21}	2.19×10^{21}	1.454×10^{-14}	1.85^{1} 3.39^{3}	$> 1.1\times10^{25}$EX
^{150}Nd	3.540×10^{-17}			8.2×10^{18}	6.194×10^{-14}	2.59^{4}	$> 1.8\times10^{22}$NE3

1.ShM: Caurier et al., PLB2012; Menendez et al., NPA2009, Horoi et al., PRC2007-2014, Neacsu, Stoica, JPG2014**. 2. IBM**:Iachello et al., PRC2009, **GMC**:Rodriguez et al., PRL2010. **3**. **QRPA**: Simkovic et al., PRC2009, Suhonen et al. JPconf2009, **IBM, GMC. 4. QRPA; IBM; PHFB**: Rath et al., PRC2010.
G: Mirea, Stoica, RRP**67**, 872, 2015; **N3**=NEMO3; **GE**=GERDA; **CU**=CUORICINO+CUORE; **EX**=EXO2000
$T^{2\nu}$: Barabash**,** NPA2015

Conclusions

- Double beta decay is still the most sensitive process able to provide us info on the LNC and the v character: Dirac or Majorana, but, also, it has a broader potential to investigate BSM processes

- NME and PSF are the key quantities to predict DBD lifetimes, extract neutrino properties and learn about other BSM processes, hence their accurate computation is of great importance.

- NME values are calculated with several methods (QRPA, ShM, IBM-2, PHFB, GCM, etc.) but discrepancies of a factor of 2-3 between the reported values still persists. These are due both to the specific features of these methods and of using different ingredients/nuclear imput parameters. Further efforts are making to understand and reconcile the NME results, the main issues being: value of g_A, use of appropriate SRC, occupation no. of the orbits close to the Fermi level, closure approximation, etc

- PSF computed with improved methods led to a more reliable values. NME and PSF should be computed consistently with the same values of the common parameters.

- Info on LNC, v properties and other BSM processes provided by the DBD come now, more and more, also from the LHC experiments. Particularly, at LHCb data analysis of several rare B and D decays got branching rations which are close to the th. predictions