Standard Model Fermion Masses and Mixing Angles generated in a 3HDM

Ana Solaguren-Beascoa Negre

in collaboration with Alejandro Ibarra

Technische Universität München & Max-Planck-Institut für Physik

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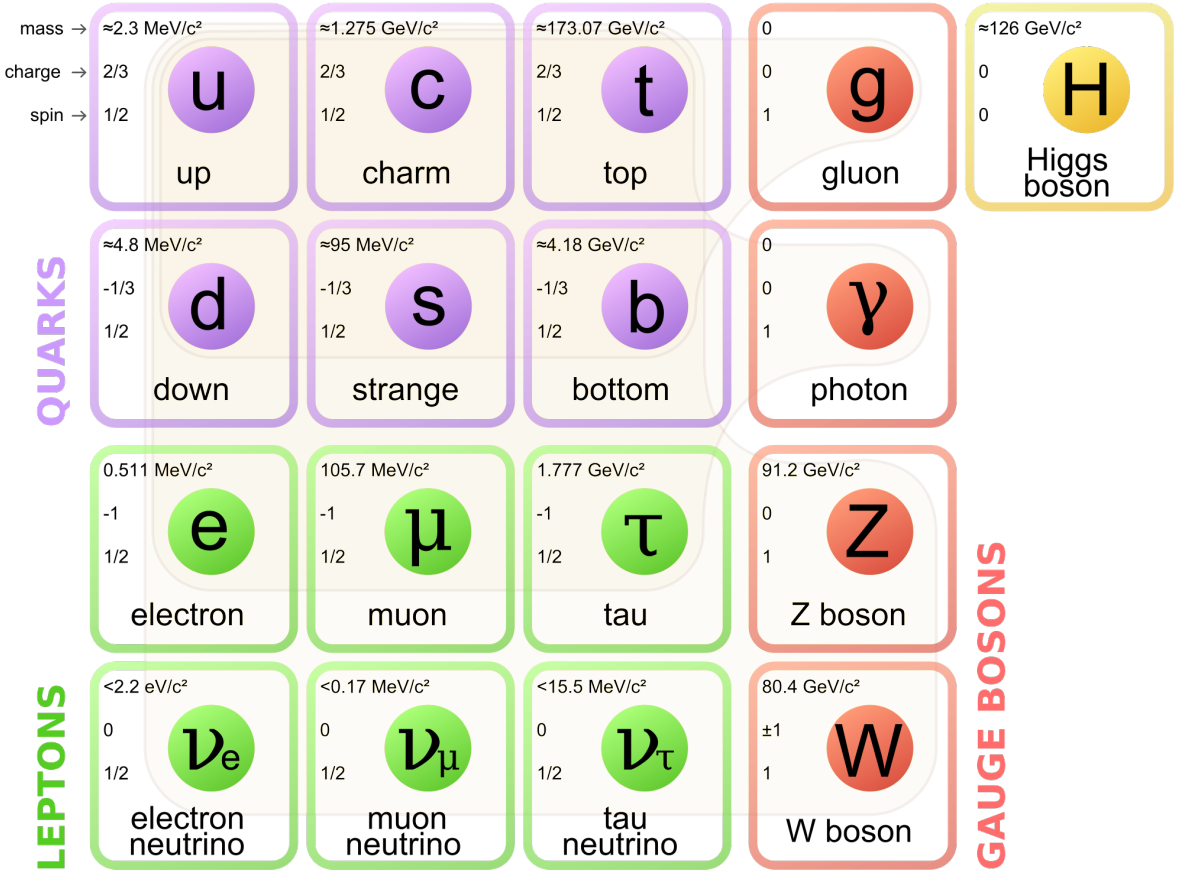
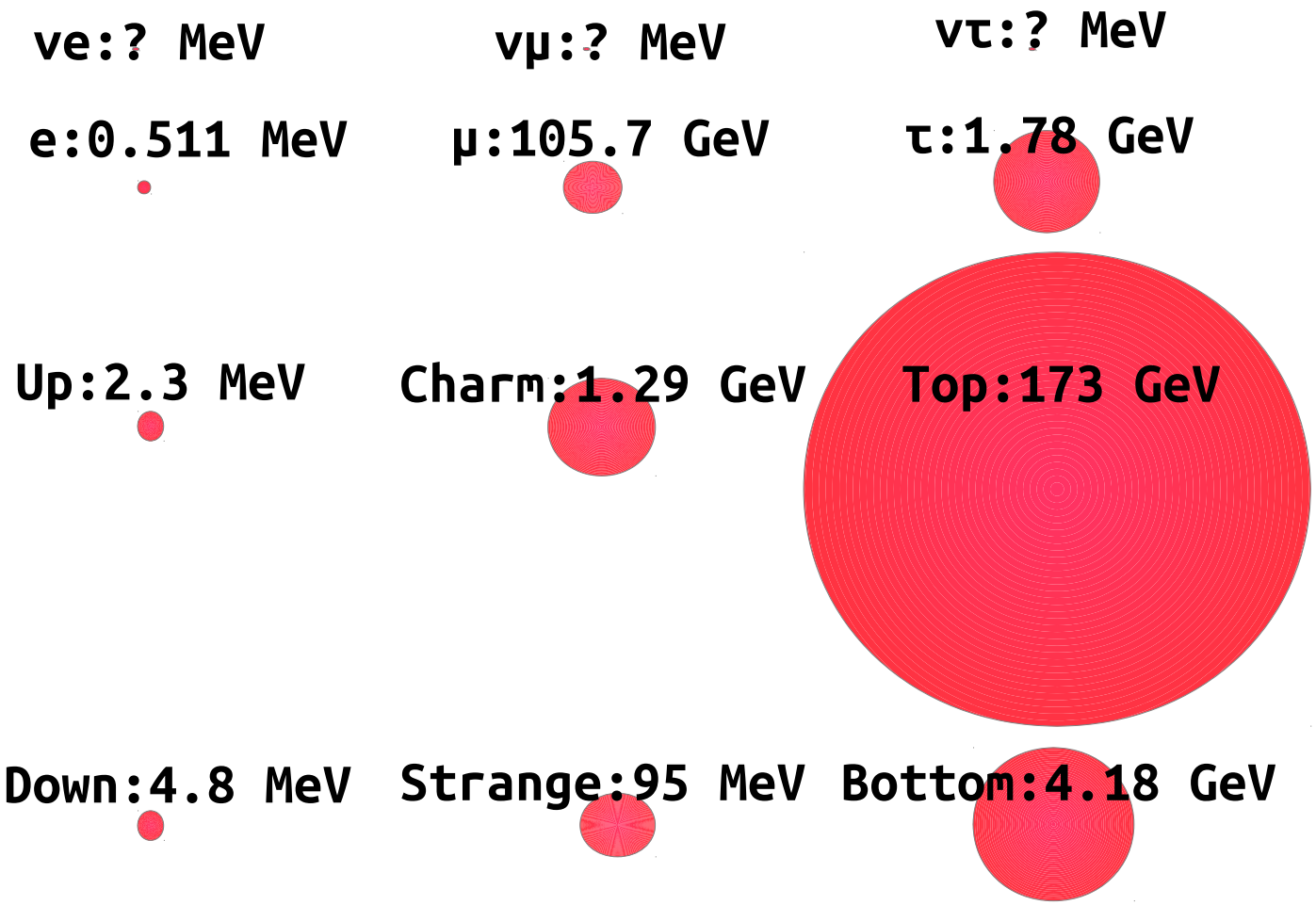
Introduction



Pattern of masses



Pattern of mixing angles

$$V_{CKM} = \begin{pmatrix} 0.974 & 0.225 & 0.0035 \\ 0.225 & 0.973 & 0.041 \\ 0.0087 & 0.04 & 0.999 \end{pmatrix}$$

$$U_{PMNS} = \begin{pmatrix} 0.822 & 0.574 & 0.156 \\ 0.355 & 0.704 & 0.614 \\ 0.443 & 0.452 & 0.774 \end{pmatrix}$$

Our goal:

Reproduce masses and mixing angles.
(~~extra symmetries~~)

The Two-Higgs Doublet Model

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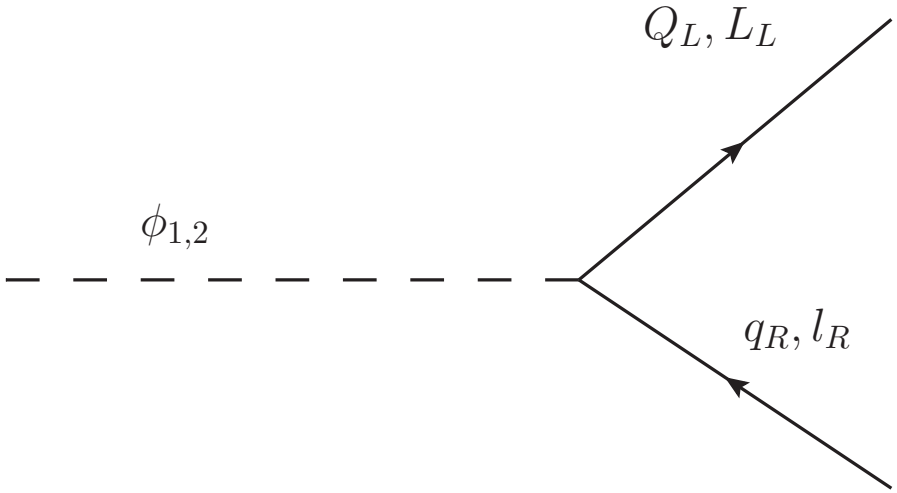

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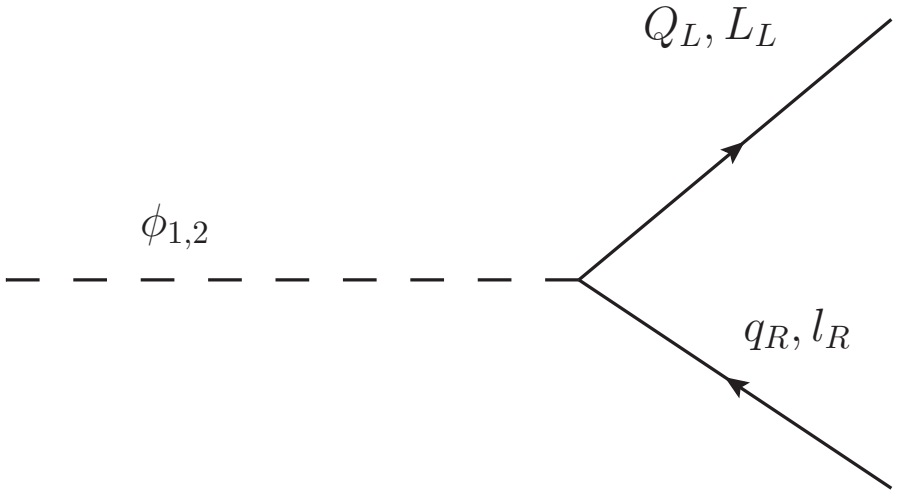

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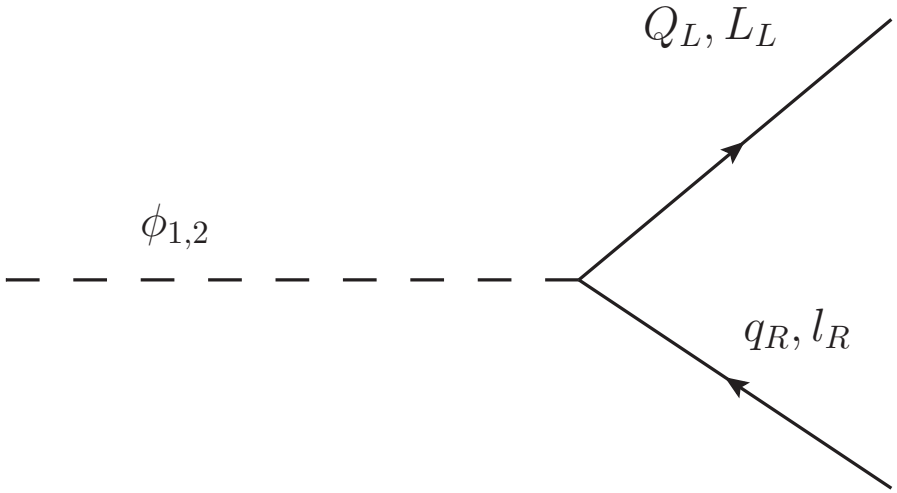

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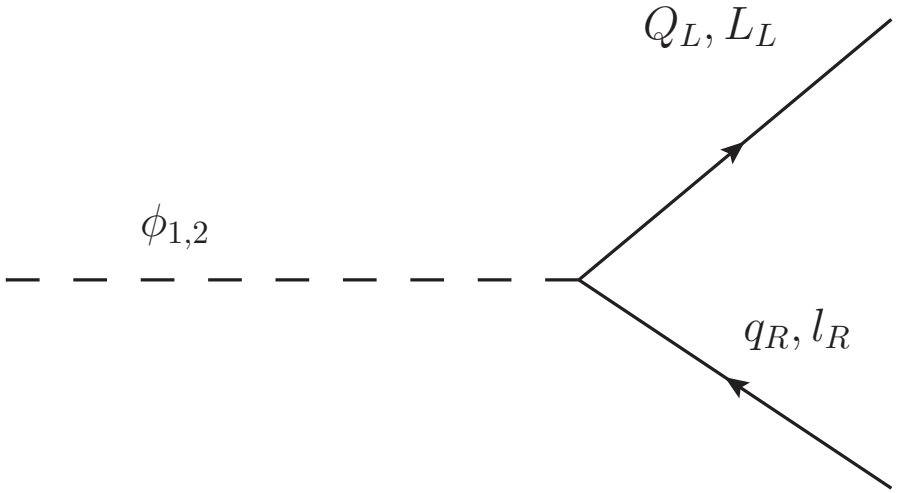

- Interactions with SM Higgs
- Self-interactions
- Decoupling limit ($M_{\phi 1} \sim 126$ GeV, $M_{\phi 2} >> M_{\phi 1} \rightarrow$ ~~FCNC~~, ~~LFV processes~~)

The Mass Sector

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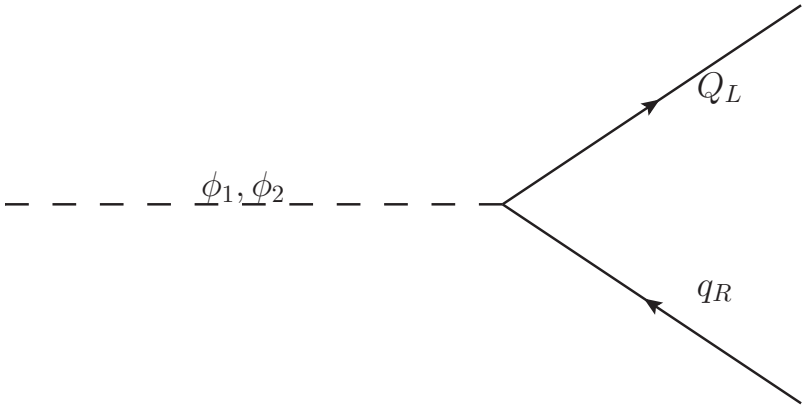
- Basis $\langle \Phi_1^0 \rangle = v/\sqrt{2}$, $\langle \Phi_2^0 \rangle = 0$
- Yukawa interaction (rank-1):



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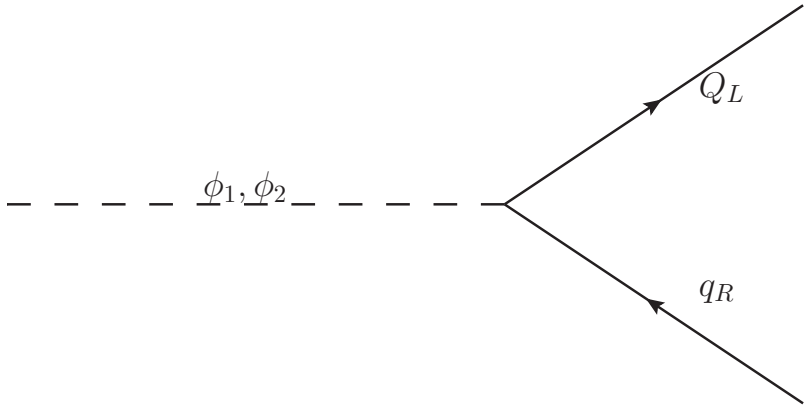

$$Y_x^{(1,2)}|_{\text{tree}} = |y_{x_L}^{(1,2)}\rangle \langle y_{x_R}^{(1,2)}|$$

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Tree Level: $m_t,\ m_b$

$$Y_u^{(2)}|_{\text{tree}} = |y_{u_L}^{(2)}\rangle\langle y_{u_R}^{(2)}|, \qquad Y_d^{(2)}|_{\text{tree}} = |y_{d_L}^{(2)}\rangle\langle y_{d_R}^{(2)}|$$

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- Parametrize:

$$|y_{u_L}^{(2)}\rangle = \sqrt{y_u^{(2)}} \begin{pmatrix} e^{i\rho_{u_L}} \sin\theta_{u_L} \sin\omega_{u_L} \\ e^{i\xi_{u_L}} \sin\theta_{u_L} \cos\omega_{u_L} \\ \cos\theta_{u_L} \end{pmatrix}$$

- Neglect phases.

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$$\boxed{\textbf{Tree Level}: \ m_\tau, \ m_{\nu_3}}$$

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- 1-loop from β function:

$$Y_x^{(1)}|_{1-\text{loop}} \simeq Y_x^{(1)}|_{\text{tree}} + \frac{1}{16\pi^2}\beta_x^{(1)} \log \frac{\Lambda}{M_{\phi 2}}$$

Radiative Masses

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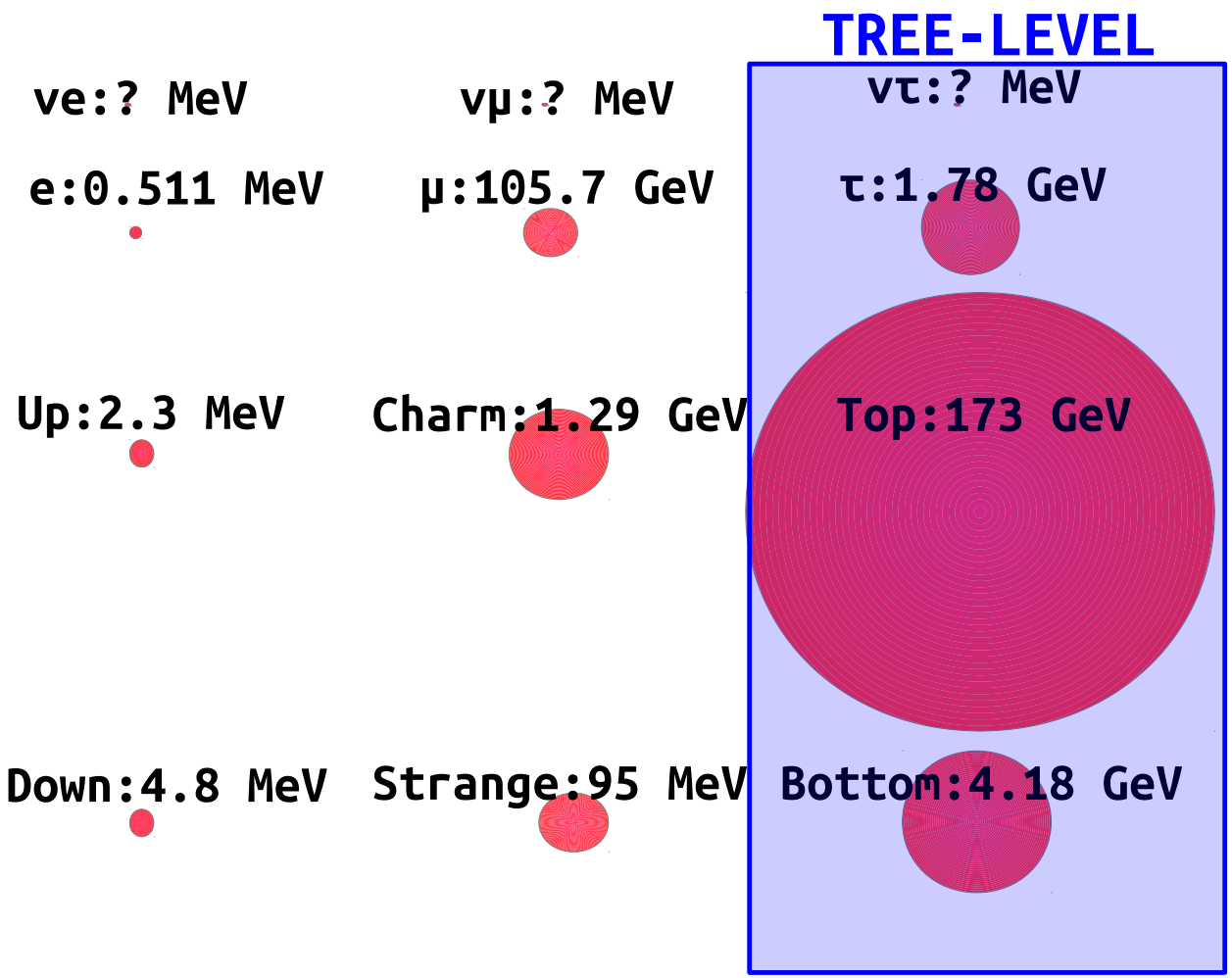
- First generation massless.

Radiative Masses



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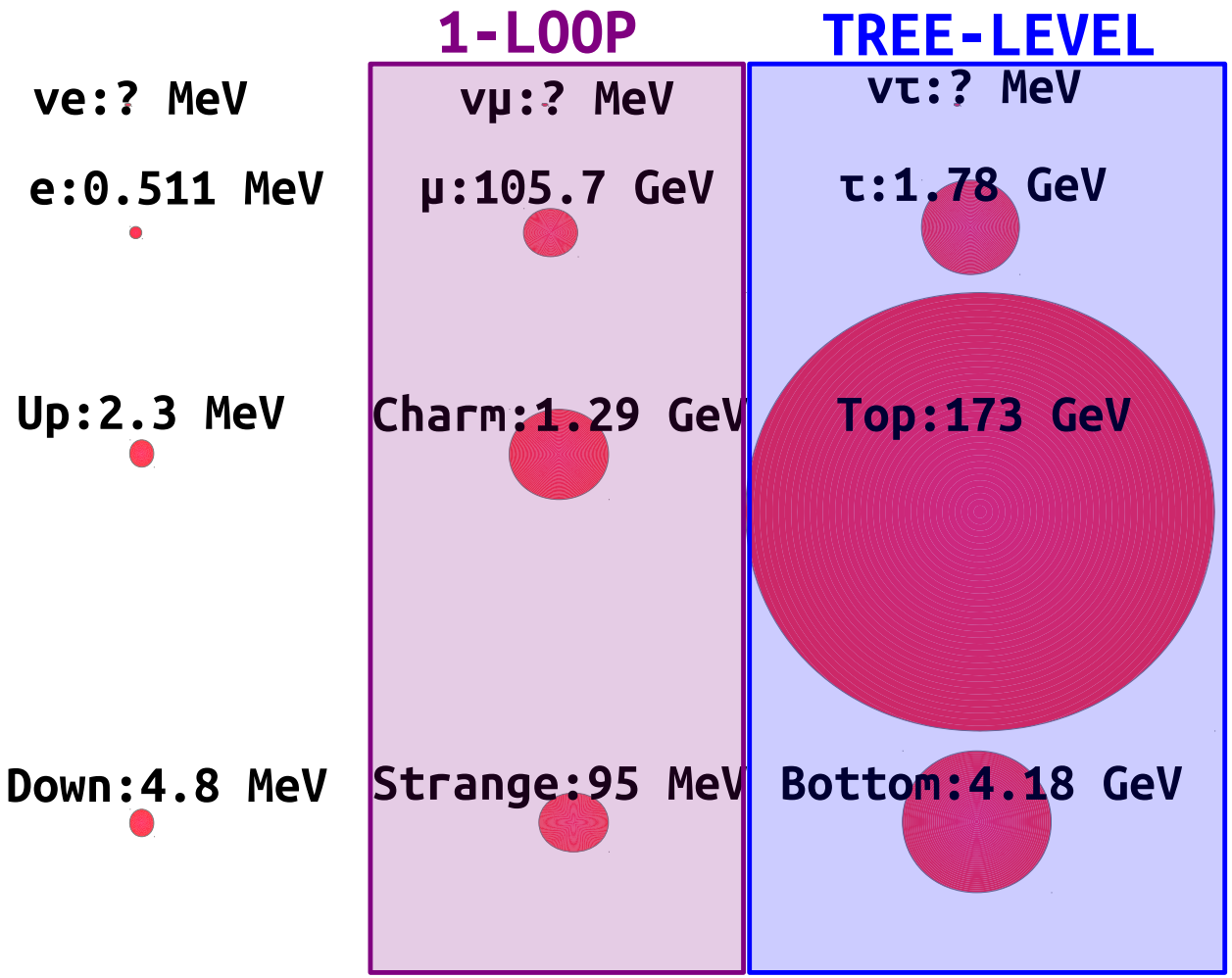

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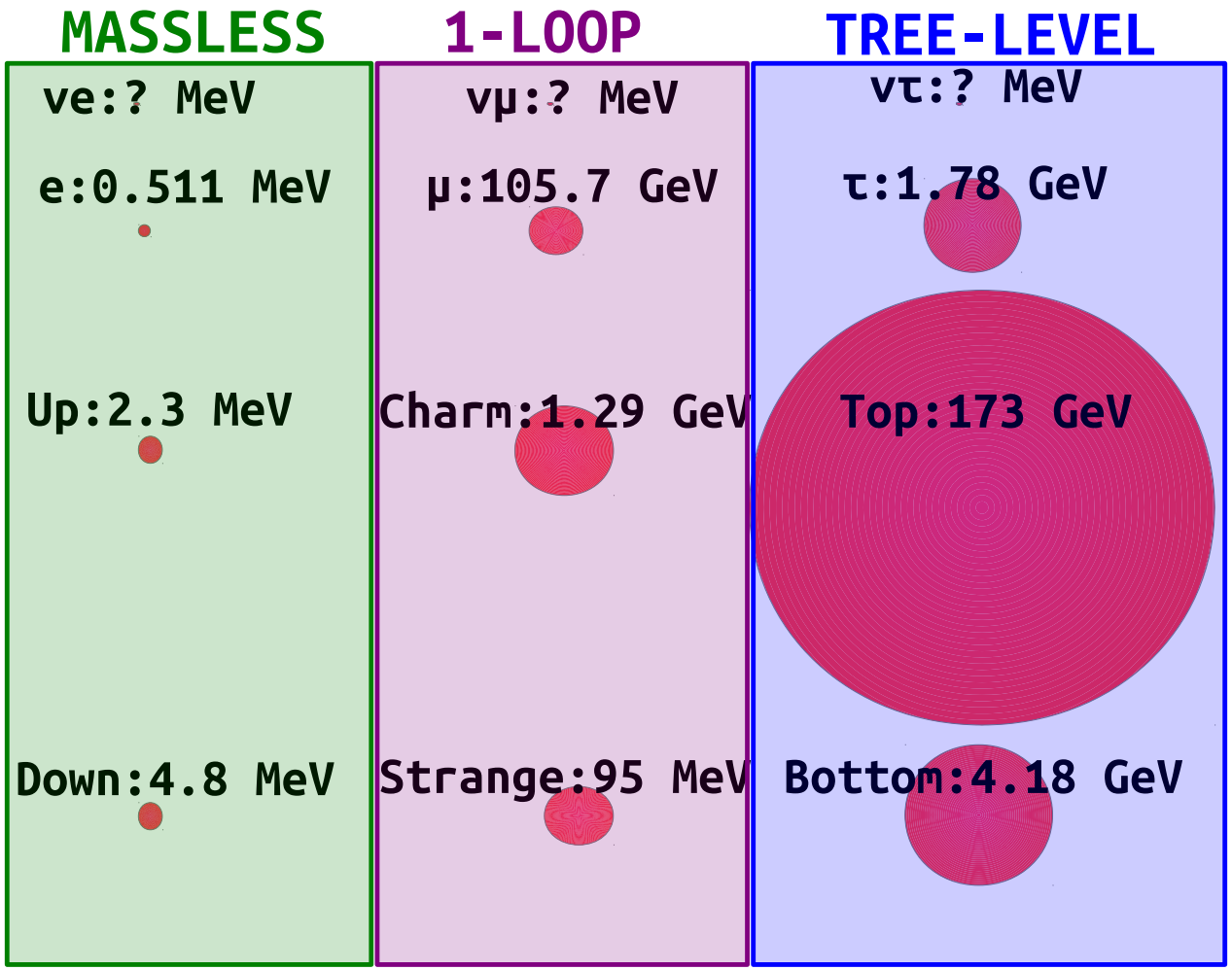

The Mixing Sector

$$\begin{pmatrix} y_1 & 0 & 0 \\ 0 & y_2 & 0 \\ 0 & 0 & y_3 \end{pmatrix} = V_{L_x}^{\dagger} Y_x^{(1)} V_{R_x}$$

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$$V_{CKM} = V^\dagger_{u_L} V_{d_L} = \begin{pmatrix} 0.974 & 0.225 & 0.0035 \\ 0.225 & 0.973 & 0.041 \\ 0.0087 & 0.04 & 0.999 \end{pmatrix}$$

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$$V_u = \begin{array}{c} \begin{array}{ccc} u_L & c_L & t_L \end{array} \\ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{array}, \quad V_d = \begin{array}{c} \begin{array}{ccc} d_L & s_L & b_L \end{array} \\ \begin{pmatrix} 0.974 & 0.225 & 0.0035 \\ 0.225 & 0.973 & 0.041 \\ 0.0087 & 0.04 & 0.999 \end{pmatrix} \end{array}$$

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The Quark Sector

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$$|V_{ub}|^2 + |V_{cb}|^2 \ll 1 \Rightarrow \ \boldsymbol{\epsilon \to 0}$$

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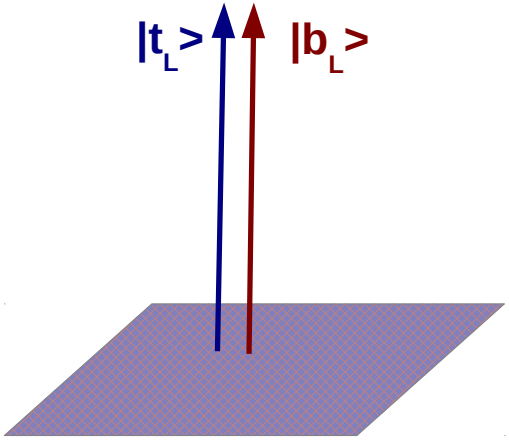
Tree Level:

$$\Rightarrow V_{CKM} = \begin{pmatrix} ? & ? & 0 \\ ? & ? & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



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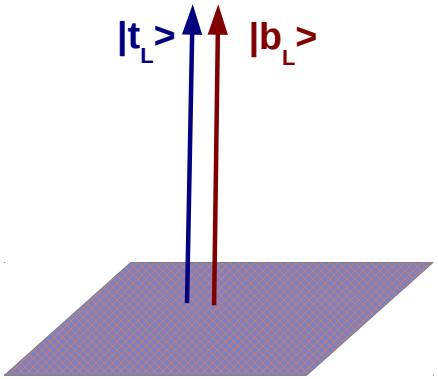

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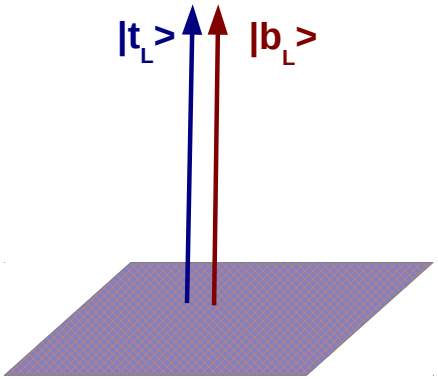

- Cabibbo angle @ 0-order in perturbation theory:

$$V_{us} \simeq -V_{cd} \simeq \frac{3 \sin\theta_{d_L} \cos\theta_{u_L} \sin(\omega_{d_L} - \omega_{u_L})}{N_d}$$

$$N_d = \left[9 \sin^2\theta_{d_L} \cos^2\theta_{u_L} + 4\cos^2\theta_{d_L} \sin^2\theta_{u_L} - 3\sin 2\theta_{d_L} \sin 2\theta_{u_L} \cos(\omega_{d_L} - \omega_{u_L})\right]^{1/2}$$

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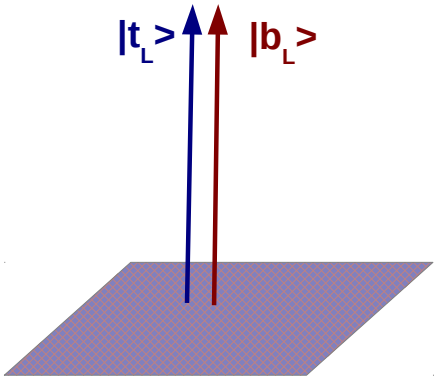

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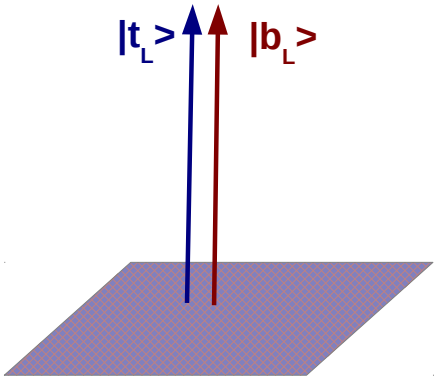

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- @ 1st order: $V_{ub} \simeq \left(\frac{1}{16\pi^2}\log\frac{\Lambda}{M_H}\right)\frac{3y_u^{(1)}y_u^{(2)}y_d^{(2)}}{y_d^{(1)}} \times$ mixing angles

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1st Order **Tree-level**

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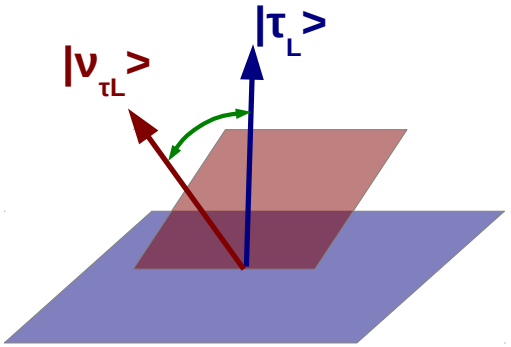

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0-Order **Tree-Level**

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- 1 mass @ tree level for each family

- $\frac{m_{x_2}}{m_{x_3}} \sim \left(\frac{1}{16\pi^2} \log \frac{\Lambda}{M_H} \right) \times$ model parameters

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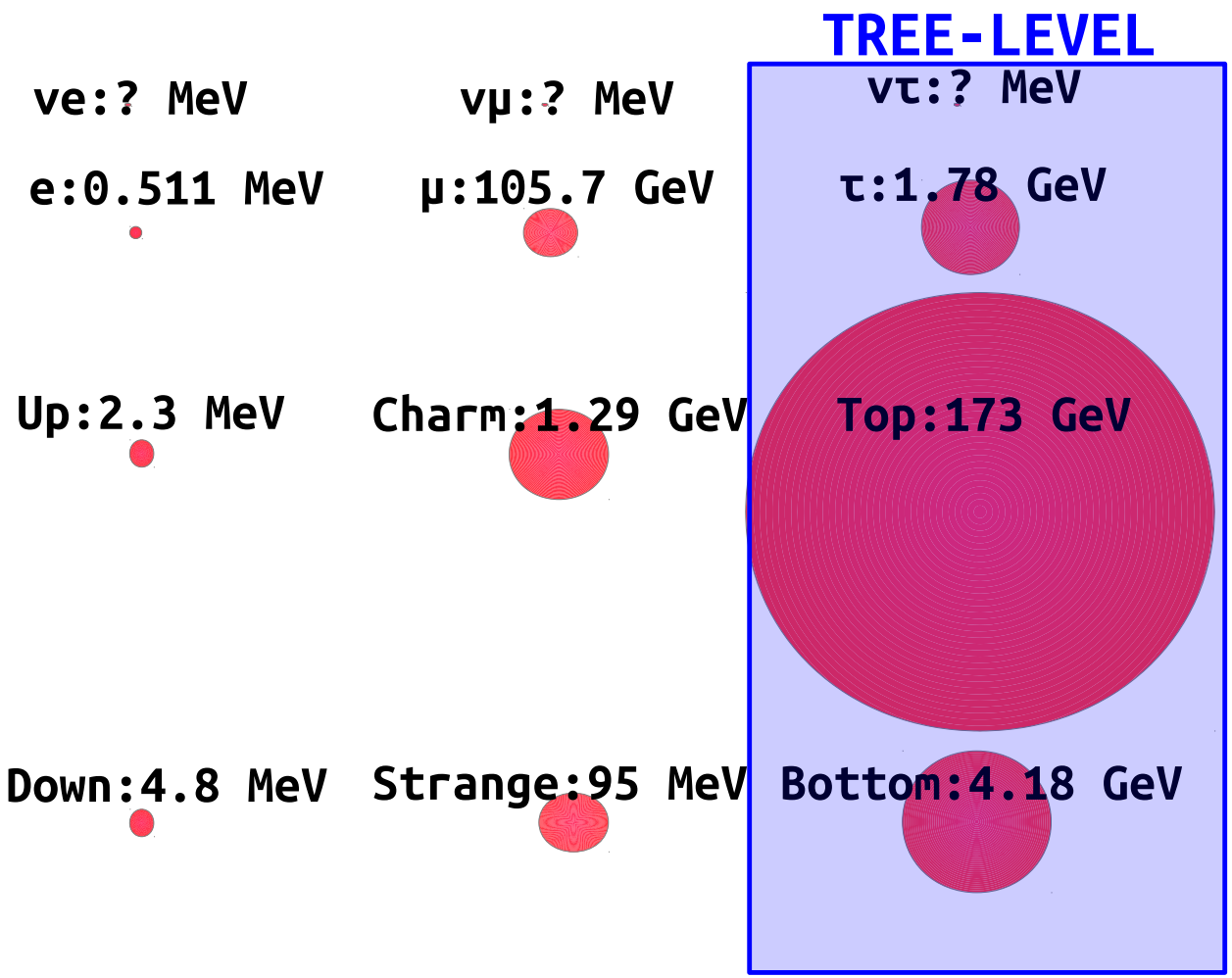
- Mixing already in a 2HDM $\Rightarrow$ no extra conditions needed

Radiative Masses – The 3HDM



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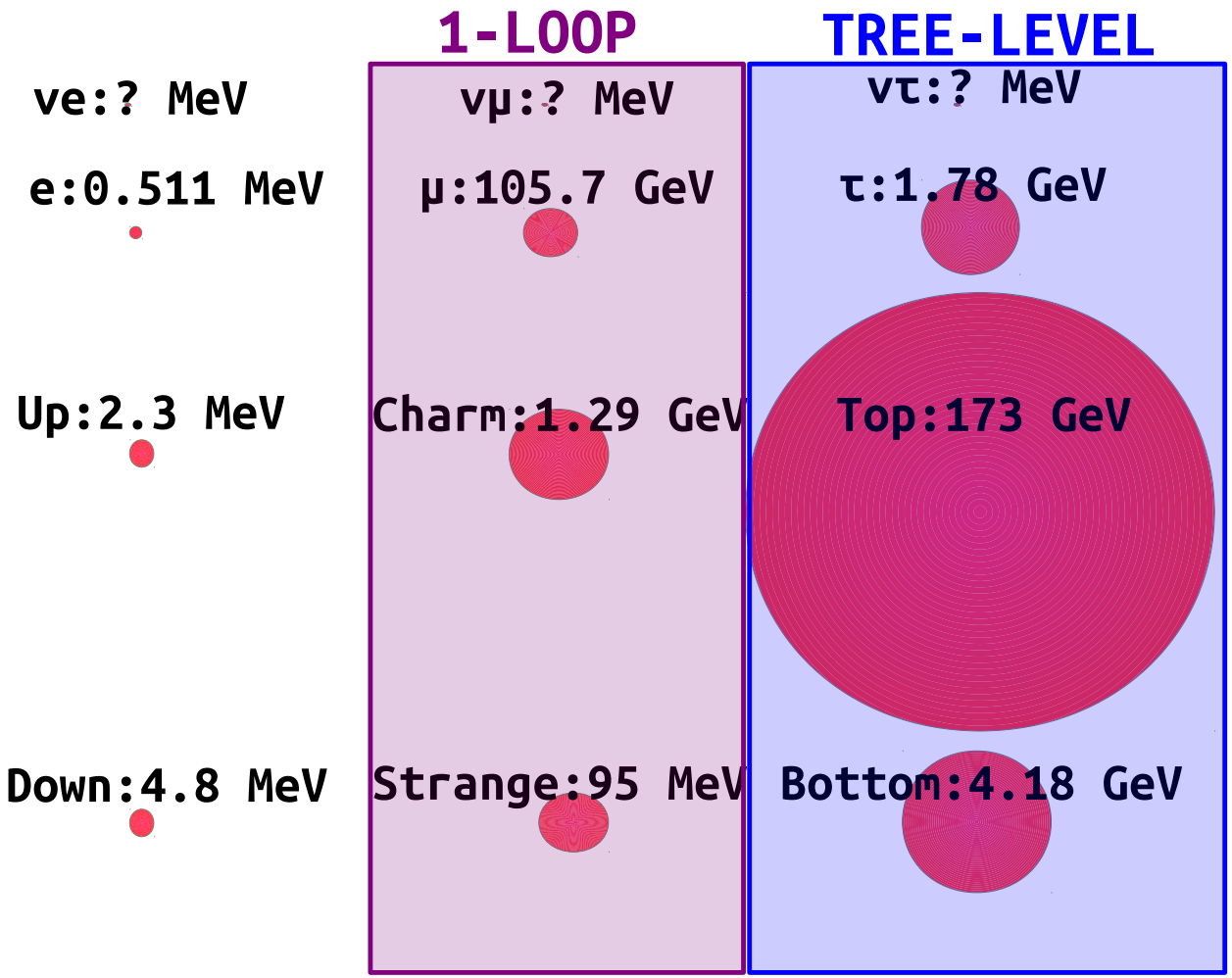

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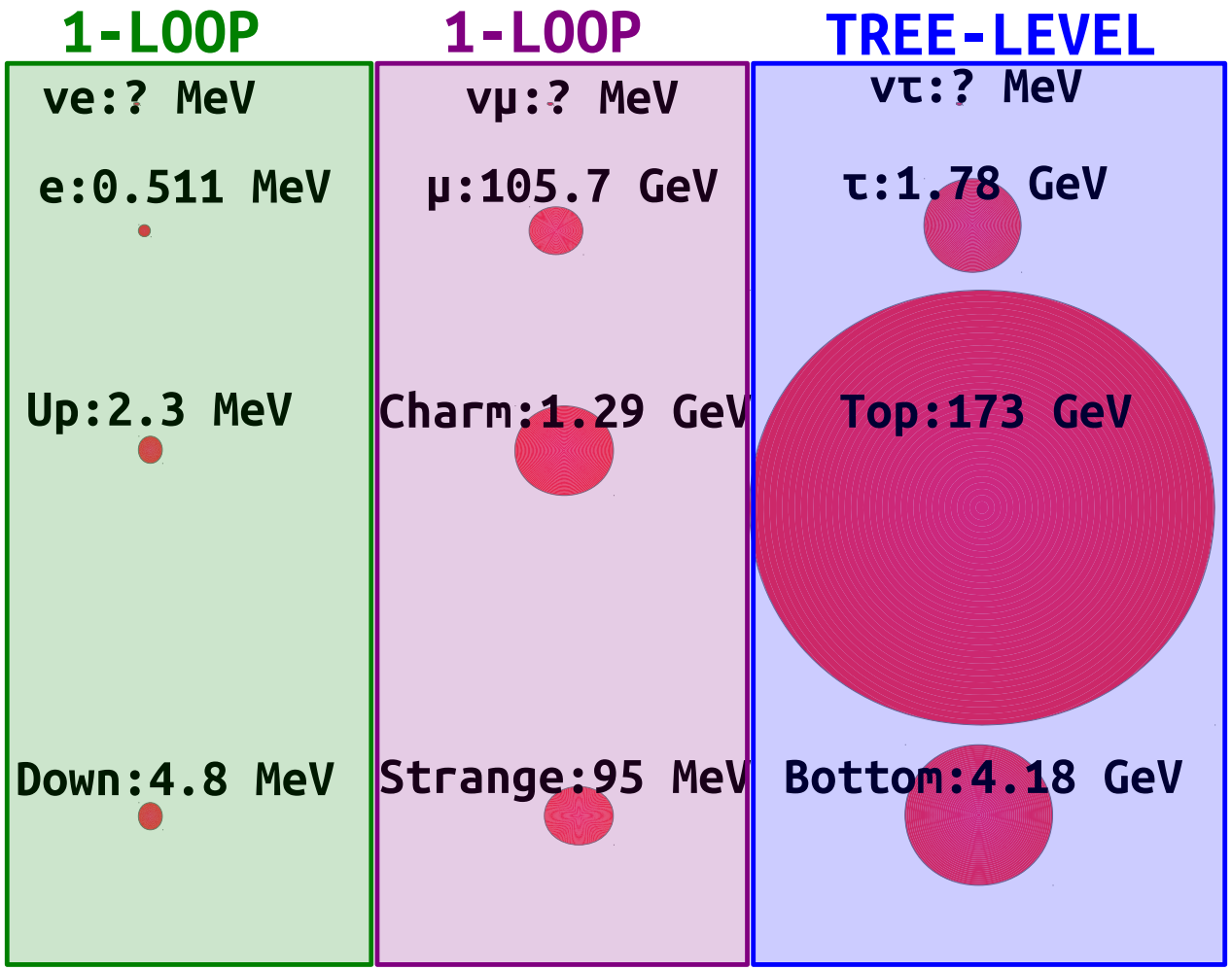
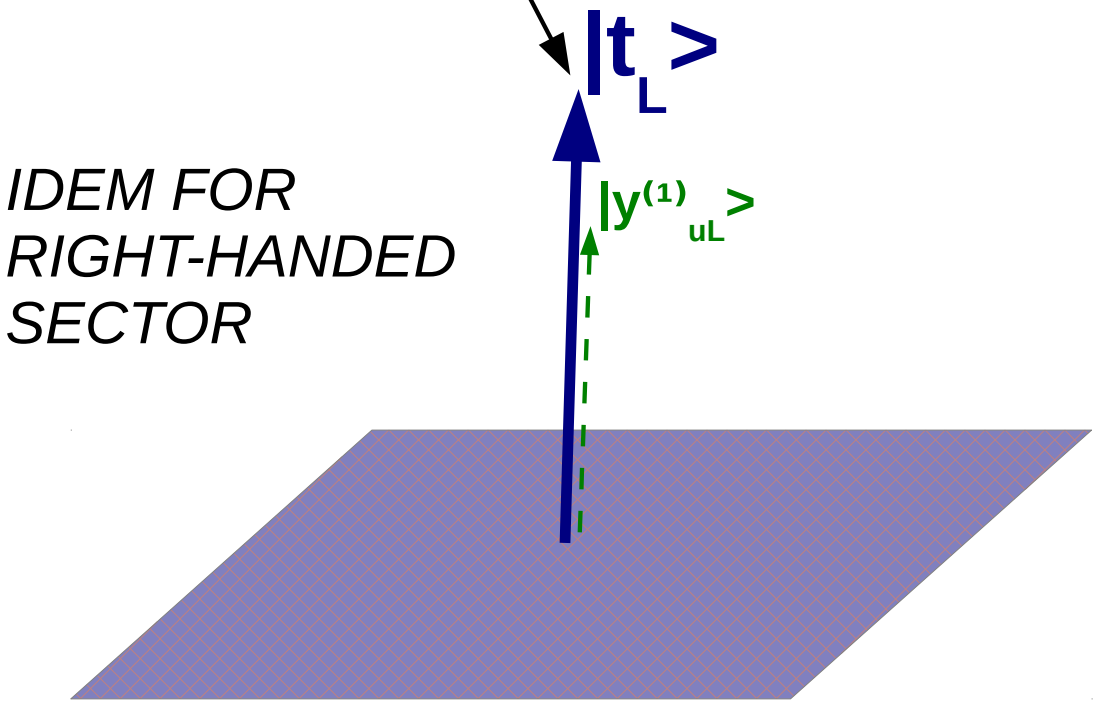


2HDM

$$Y^{(1)}_{u,d}|_{1-\text{loop}} \simeq Y^{(1)}_{u,d}|_{\text{tree}} + \frac{1}{16\pi^2}\beta^{(1)}_{u,d}\log\frac{\Lambda}{M_H}$$



IDEM FOR RIGHT-HANDED SECTOR

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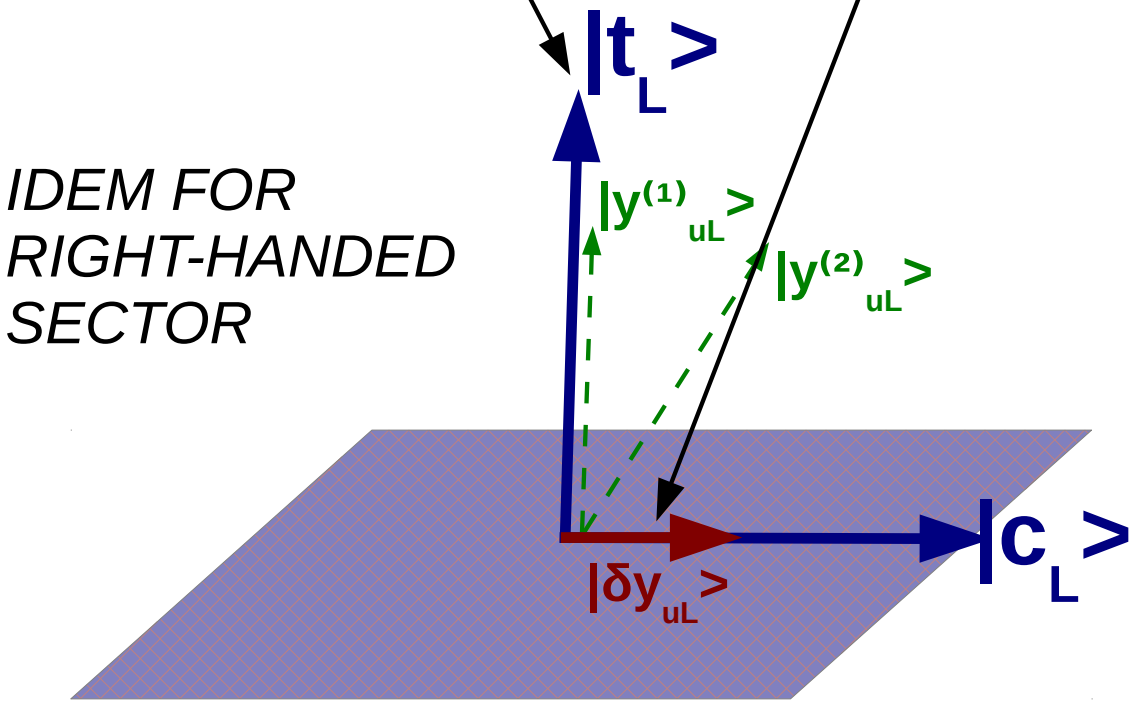

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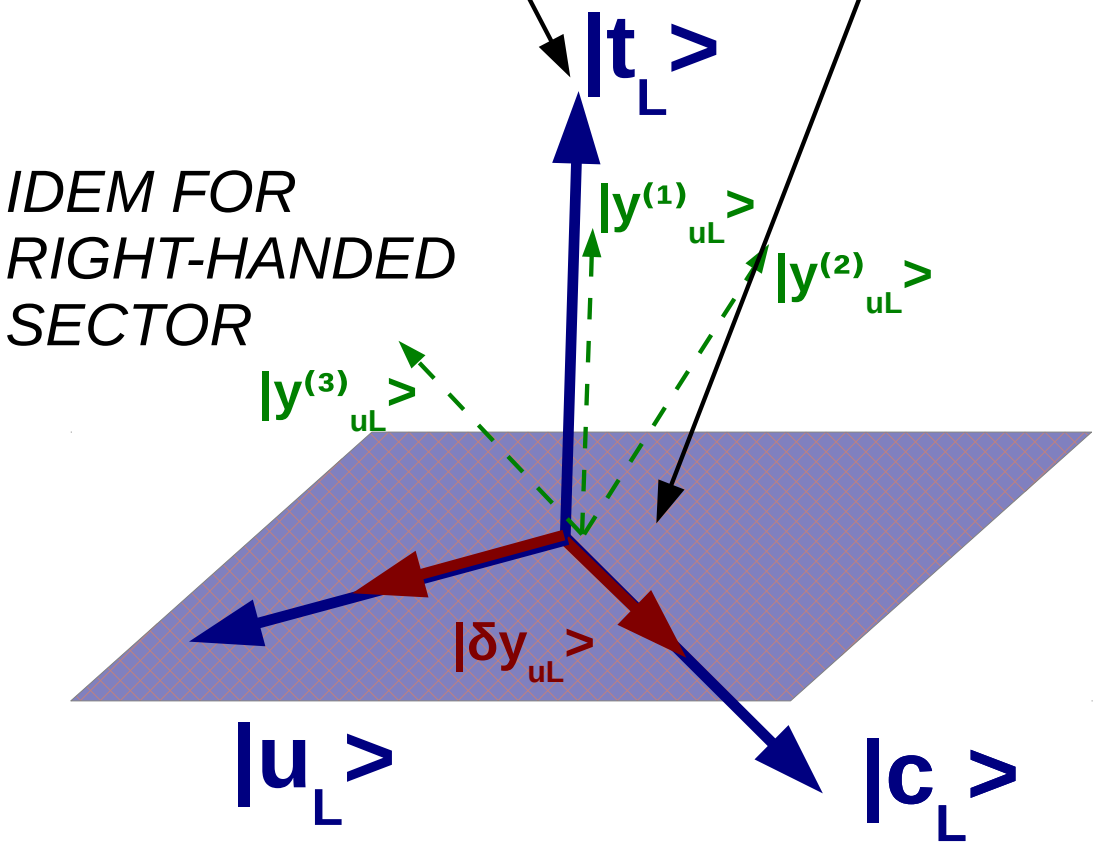

Hierarchy 1st and 2nd generation

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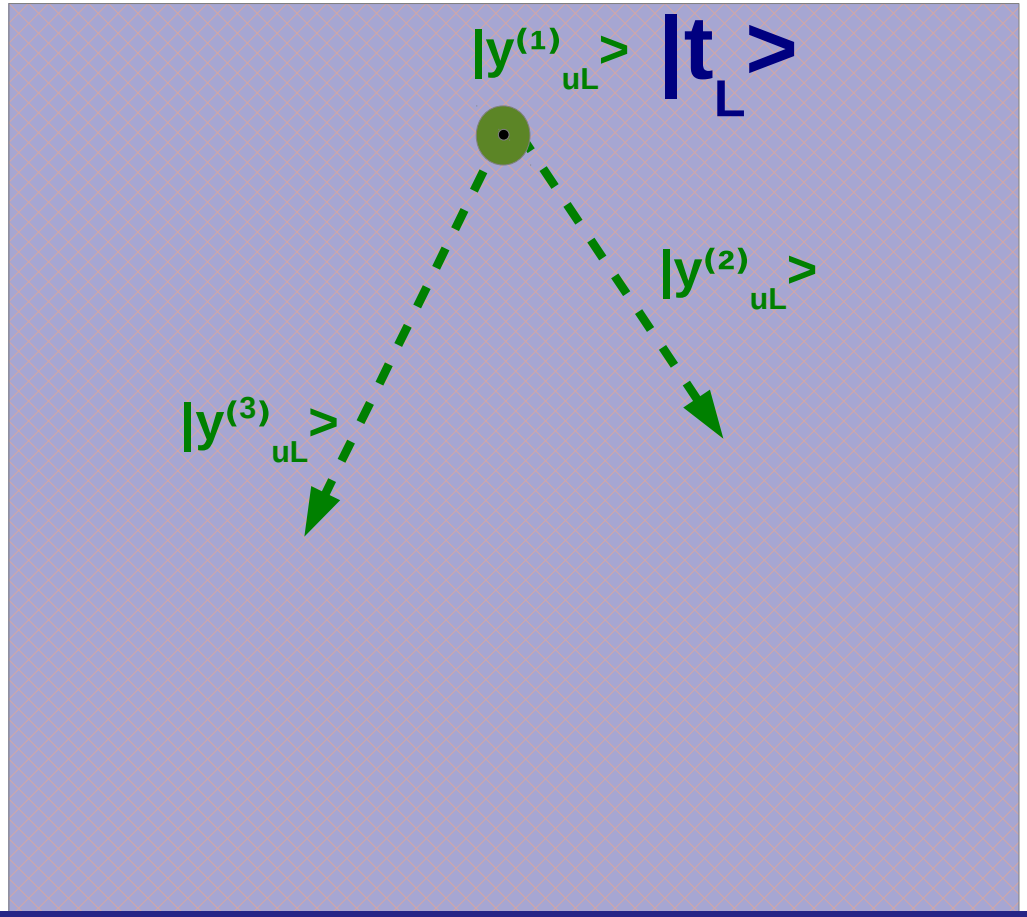

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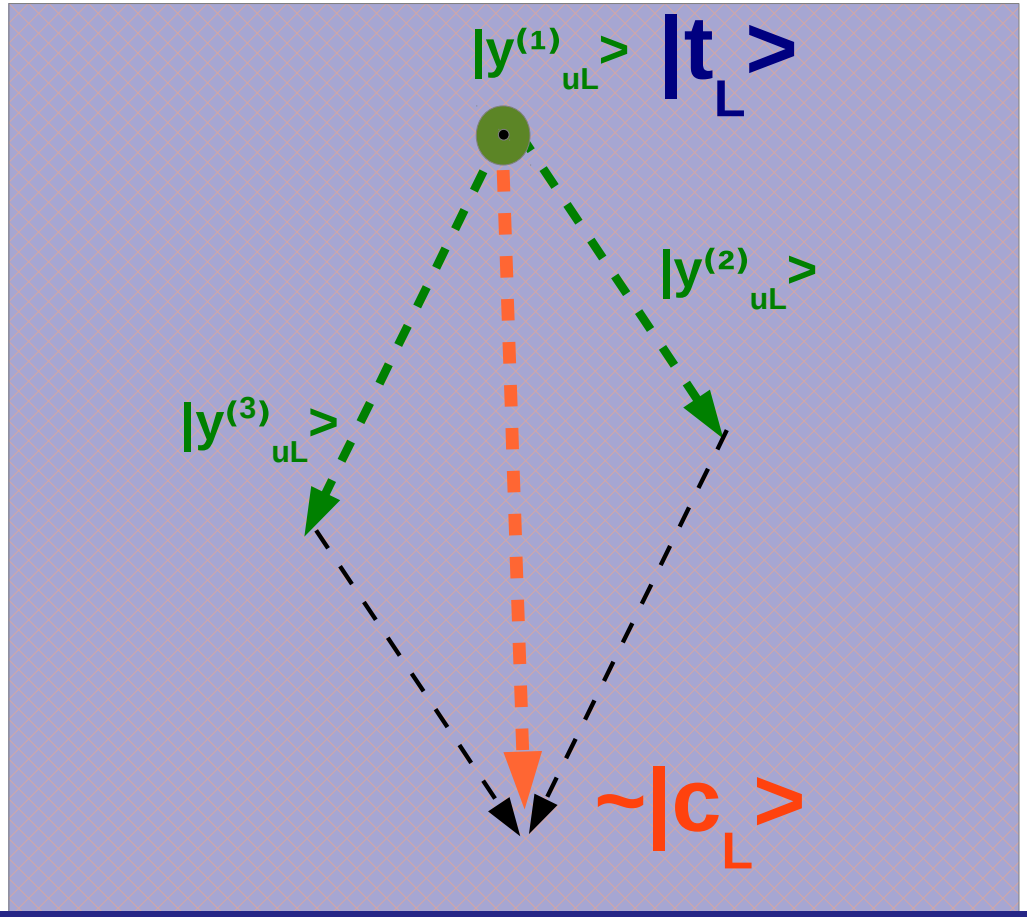

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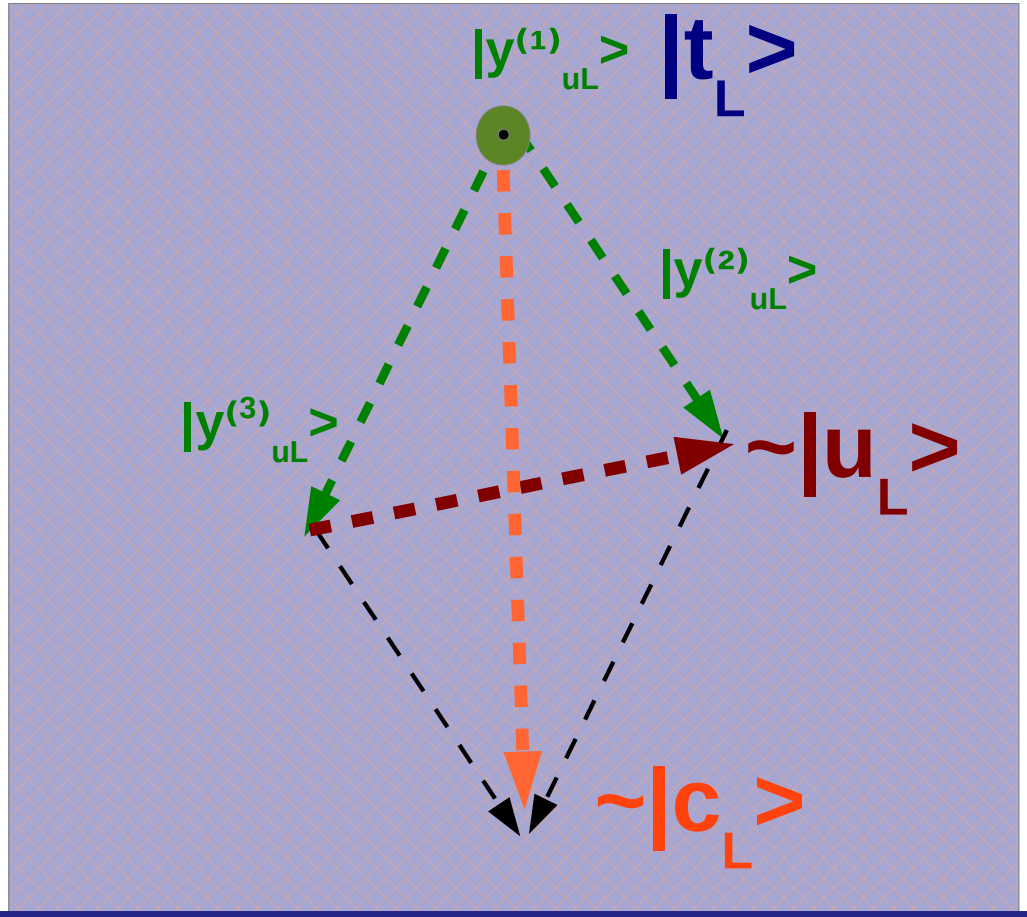

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