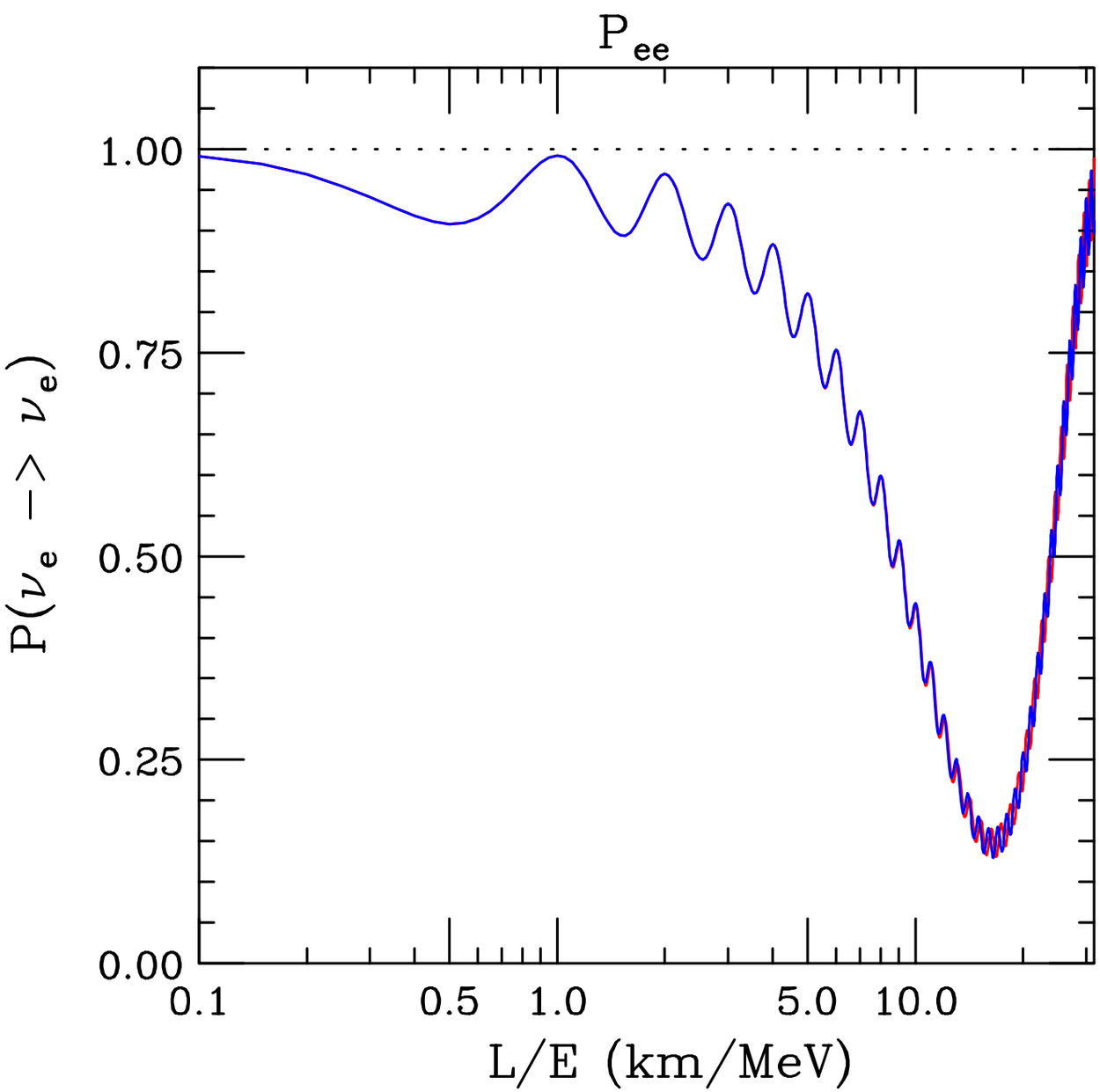
What is δm^2_{ee} ?

Stephen Parke
Fermilab

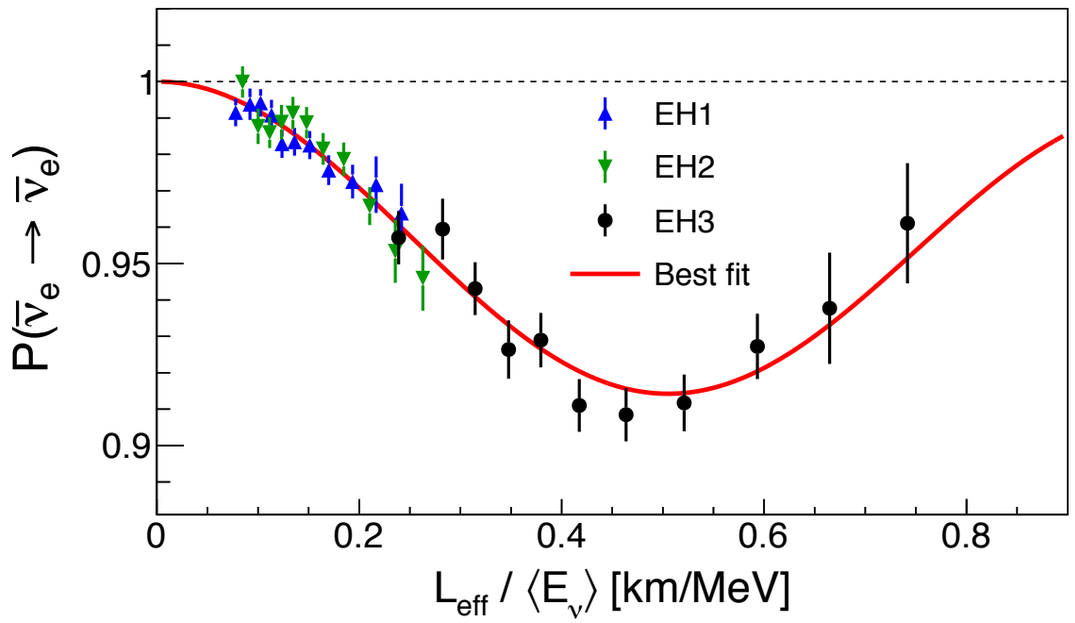
What is δm^2_{ee} ?



Stephen Parke
Fermilab

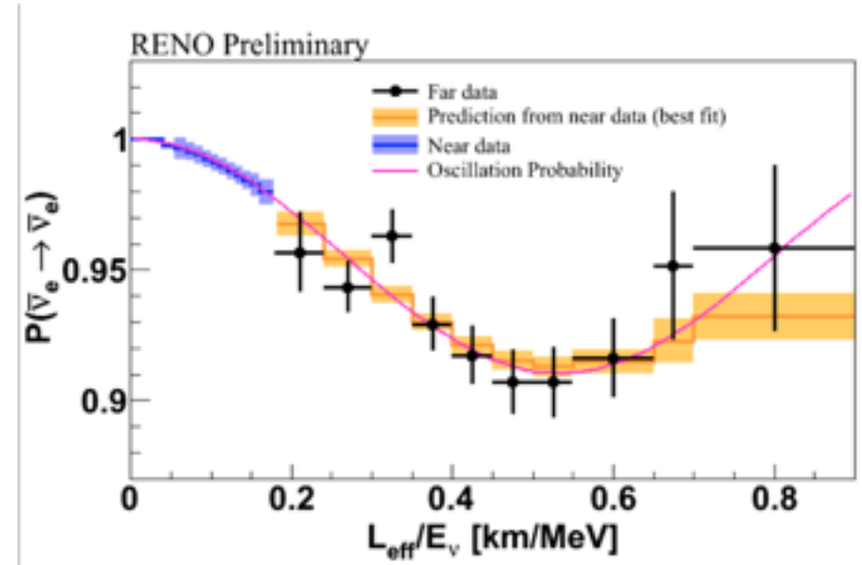
What is δm^2_{ee} ?

from Daya Bay: arXiv:1505.03456



RENO

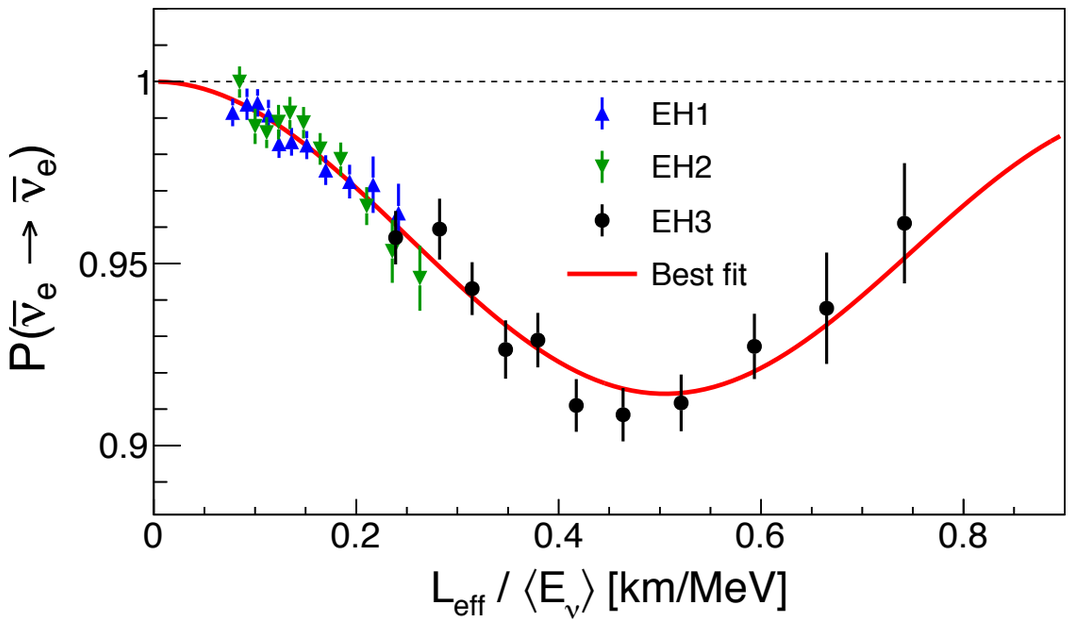
NeuTel 2015



NDM 2015

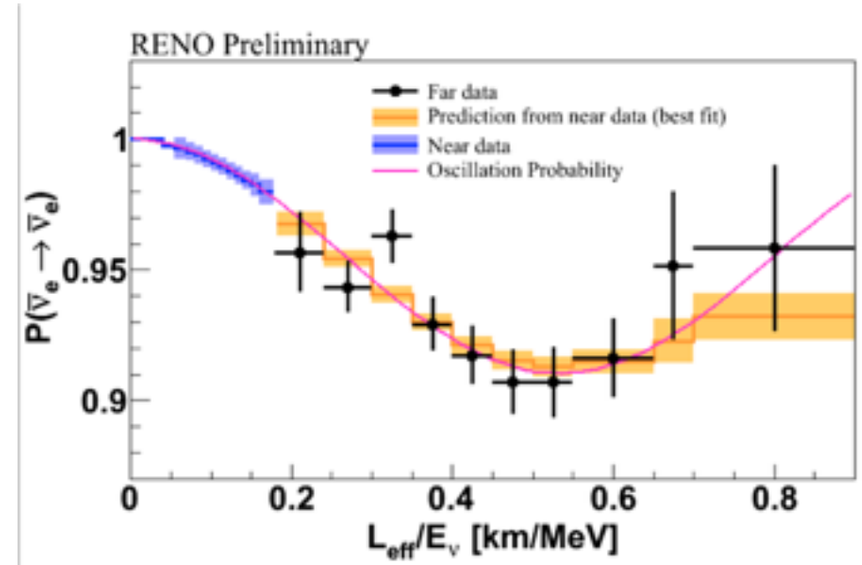
What is δm^2_{ee} ?

from Daya Bay: arXiv:1505.03456



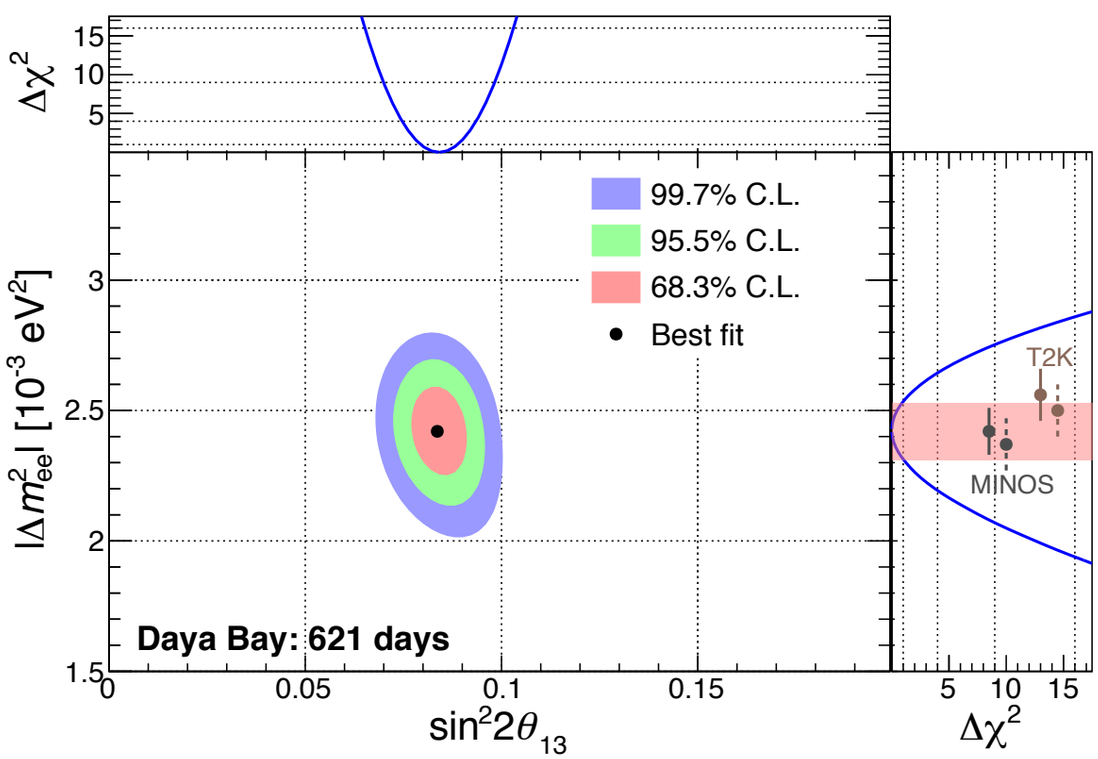
RENO

NeuTel 2015

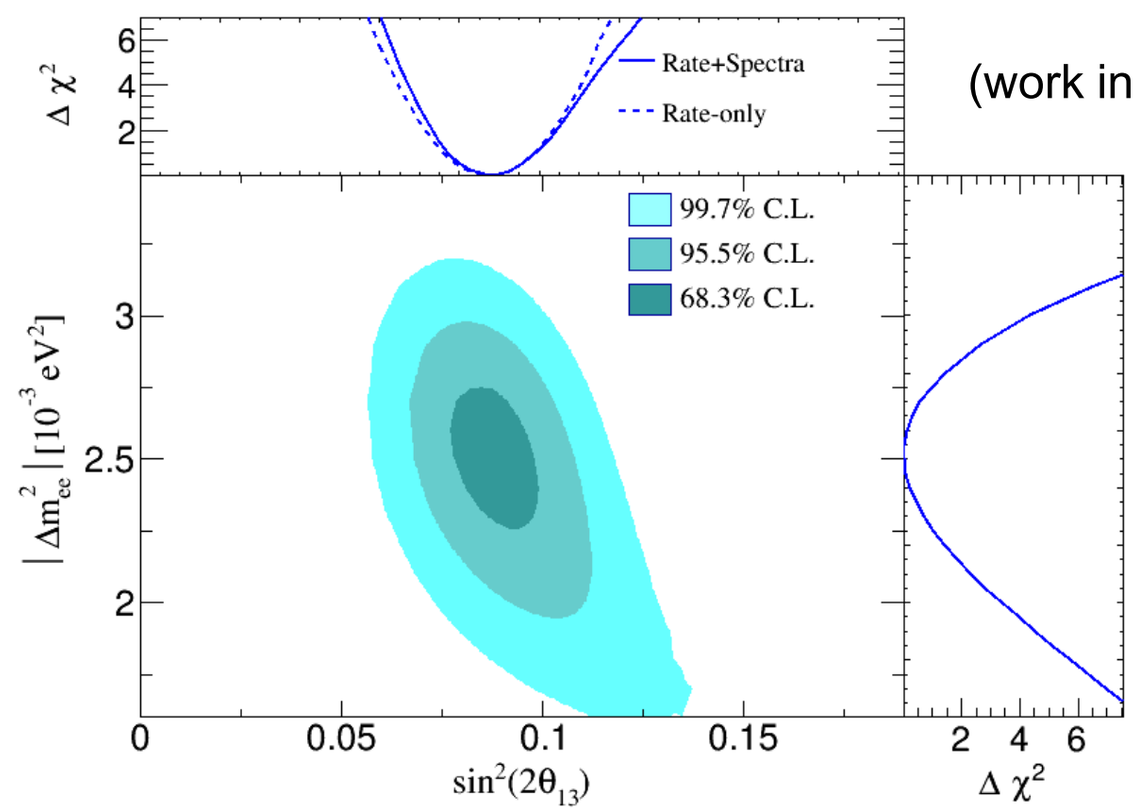


possible in Double Chooz also!

NDM 2015

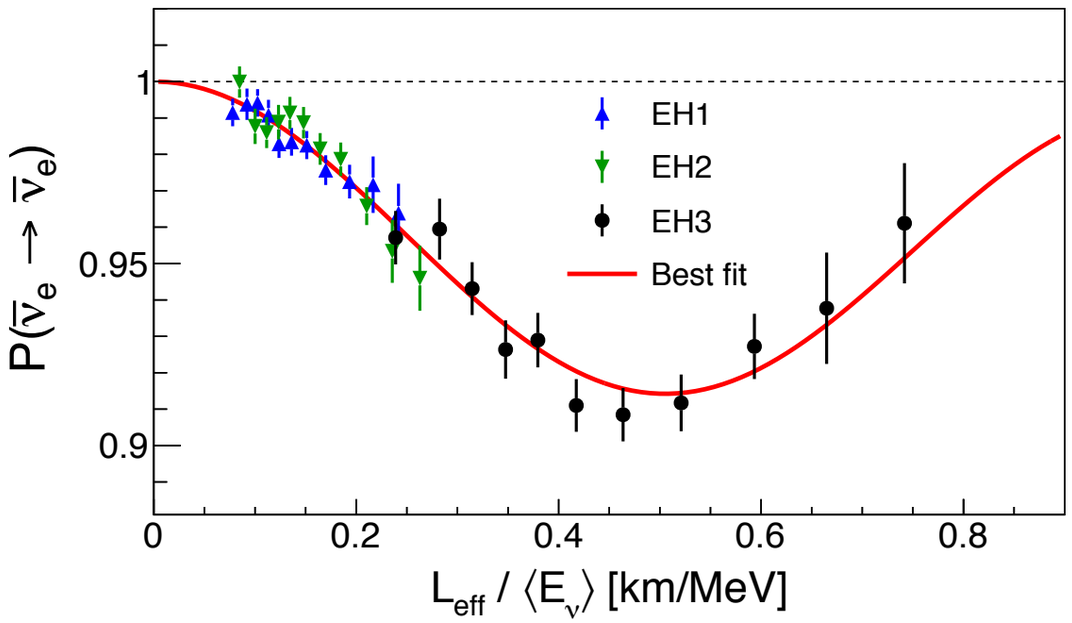


(work in prog



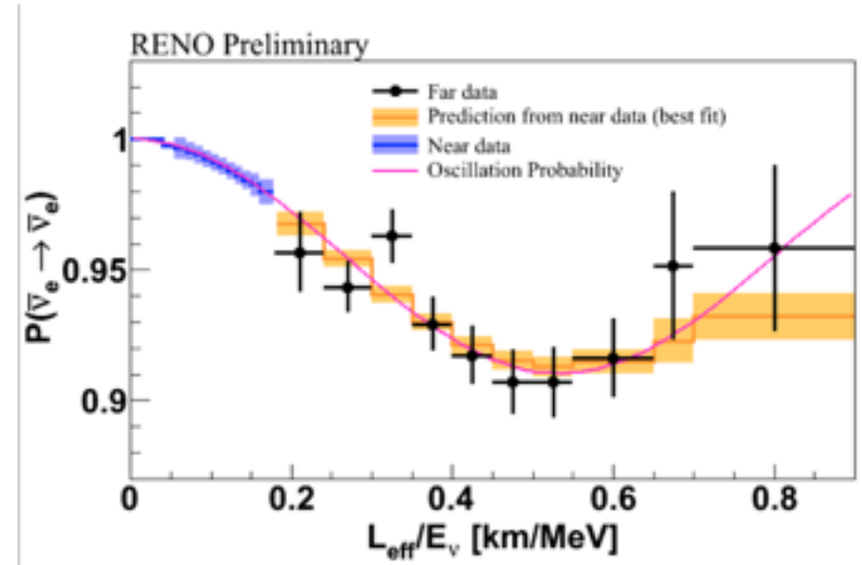
What is δm^2_{ee} ?

from Daya Bay: arXiv:1505.03456



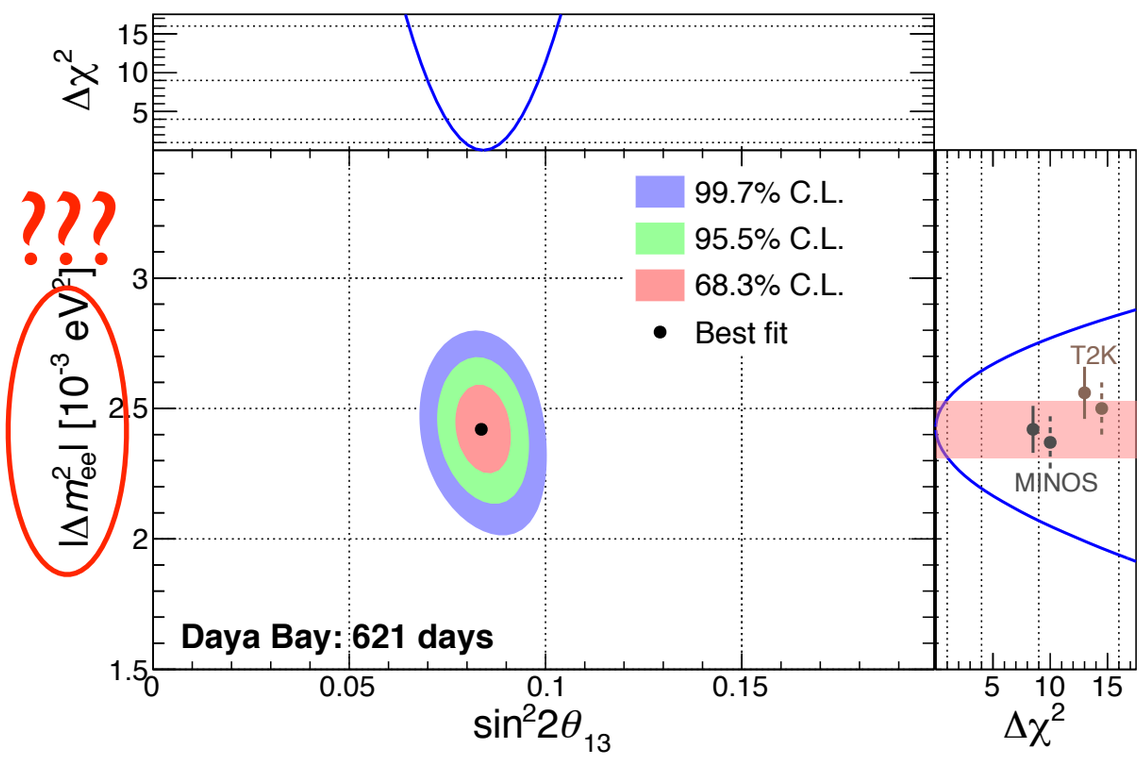
RENO

NeuTel 2015

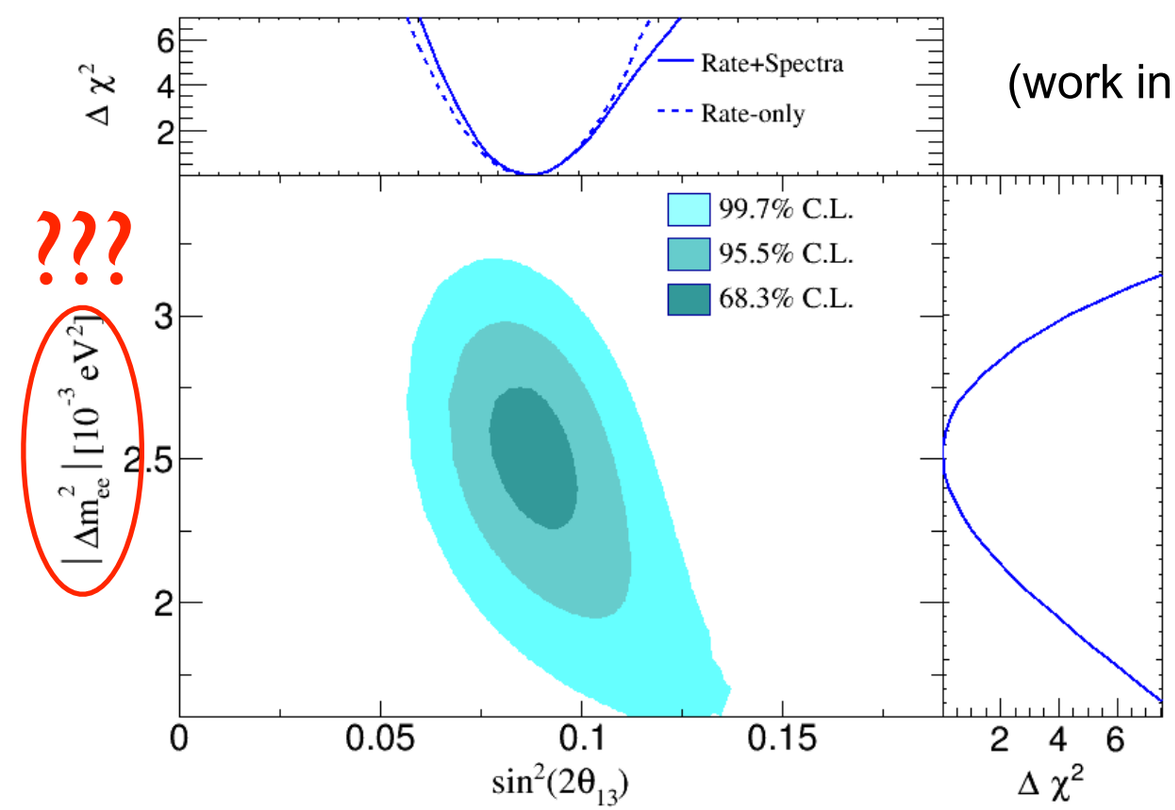


possible in Double Chooz also!

NDM 2015



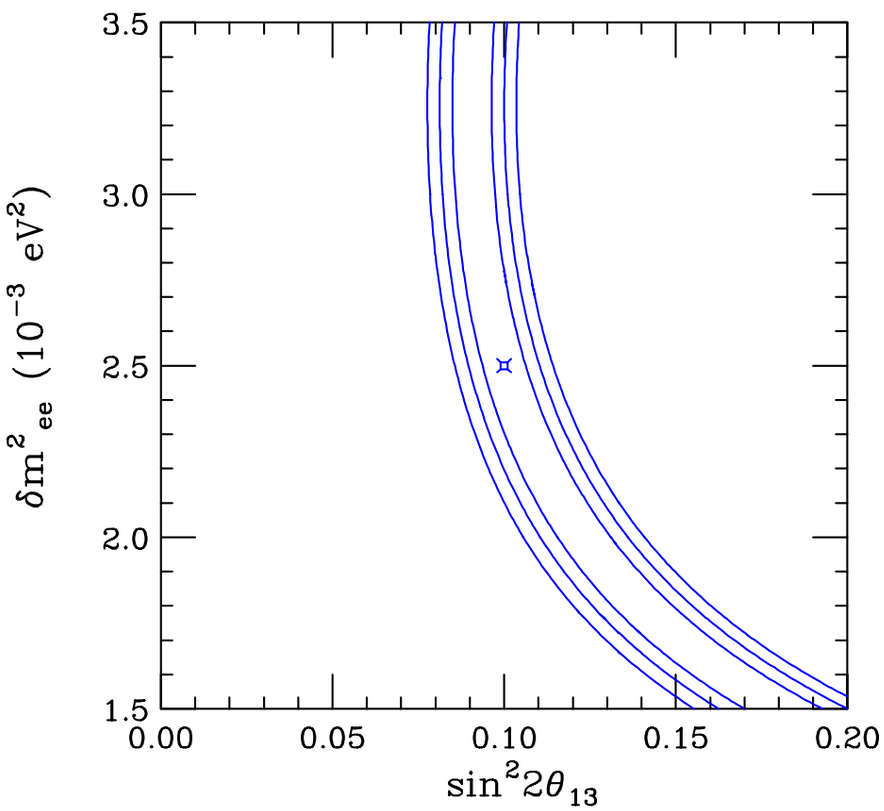
(work in prog



Correlation between Δm^2_{ee} and $\sin^2 2\theta_{13}$

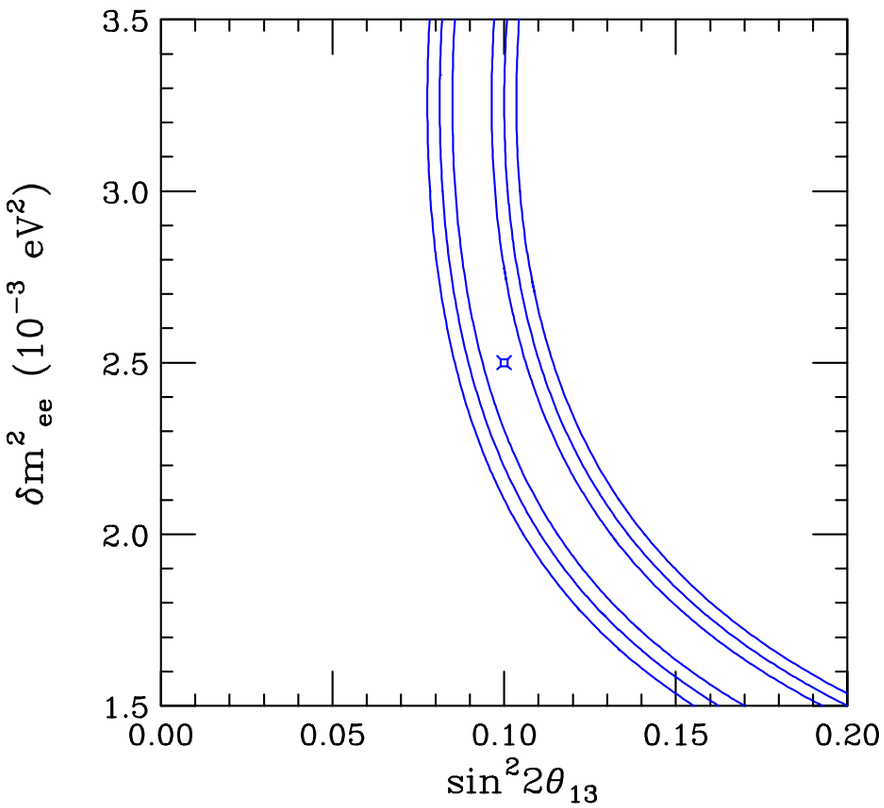
Correlation between Δm^2_{ee} and $\sin^2 2\theta_{13}$

Total Rate Only

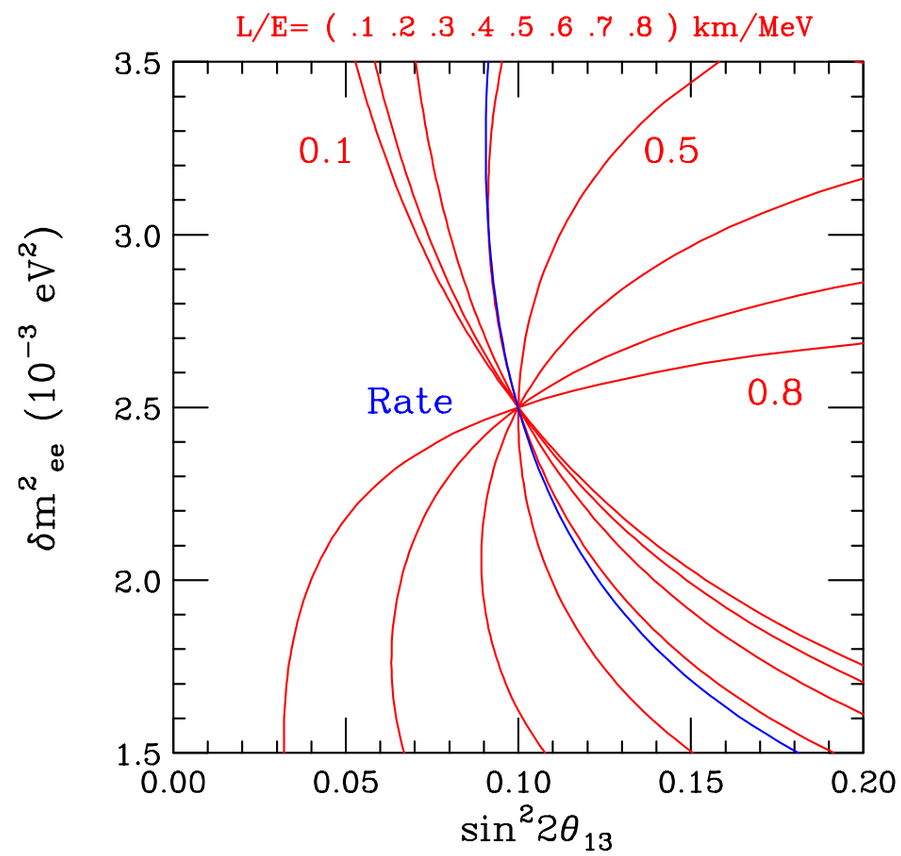


Correlation between Δm^2_{ee} and $\sin^2 2\theta_{13}$

Total Rate Only

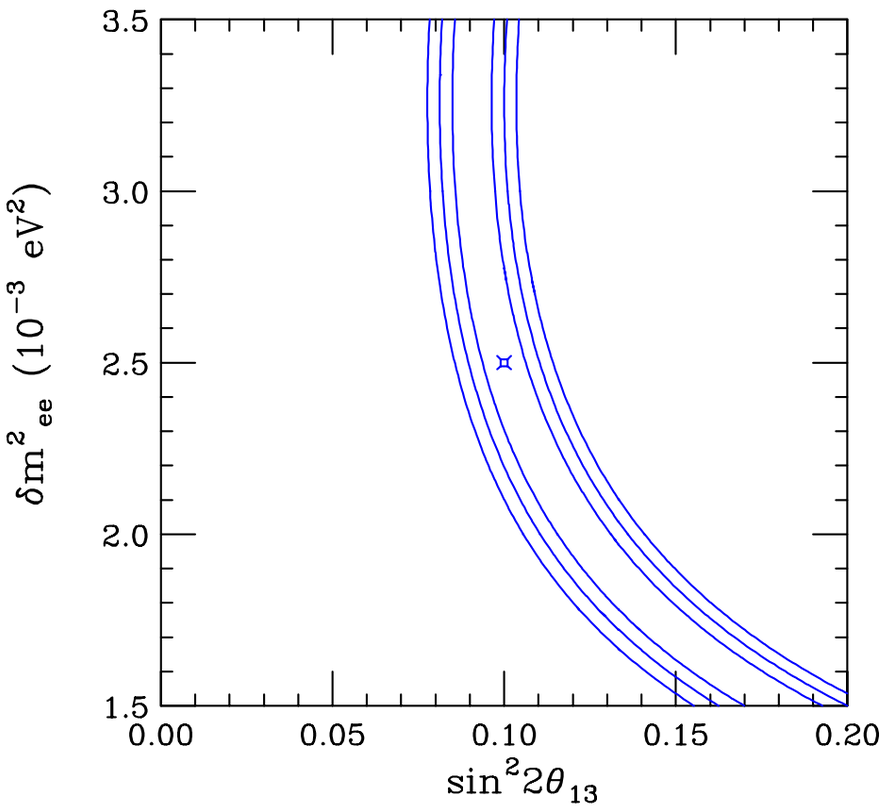


Rate per L/E bin

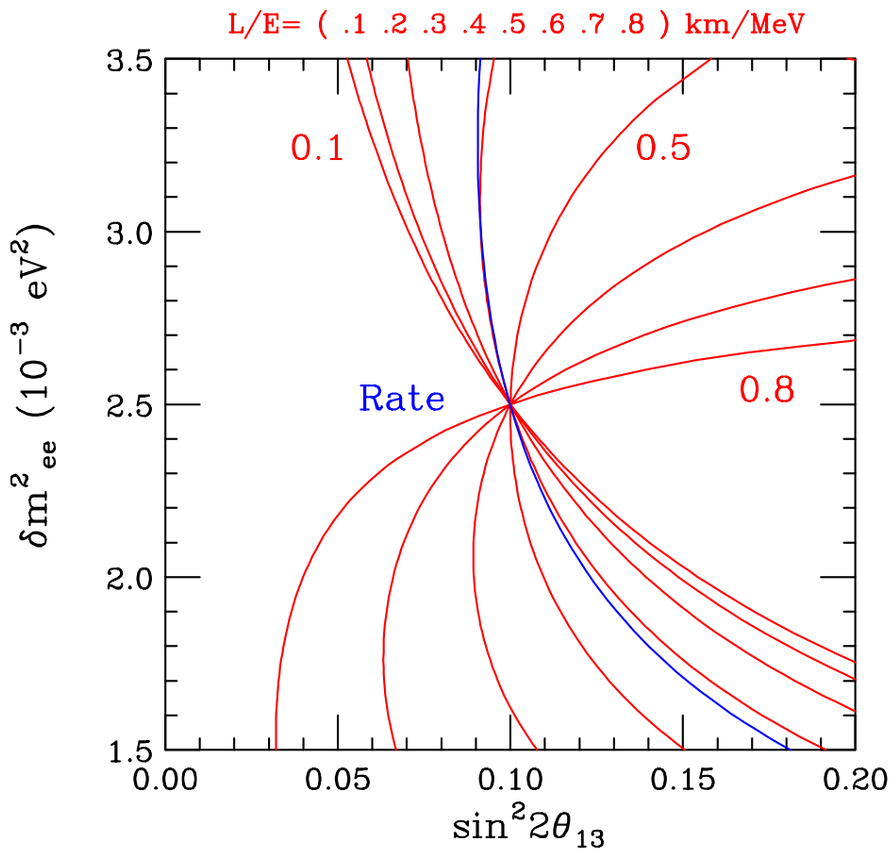


Correlation between Δm^2_{ee} and $\sin^2 2\theta_{13}$

Total Rate Only



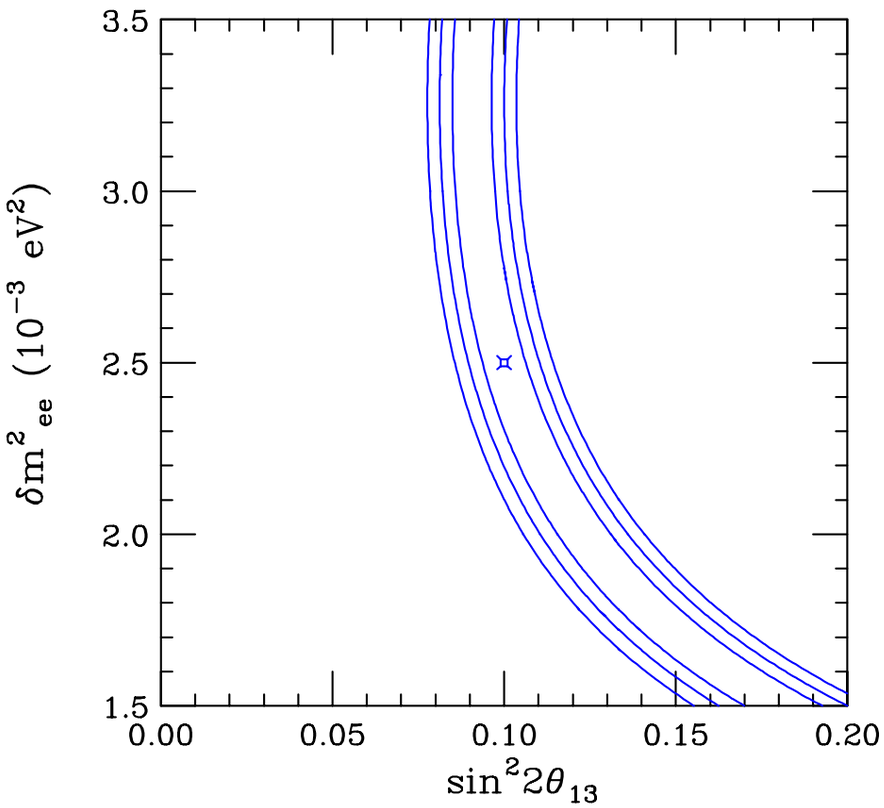
Rate per L/E bin



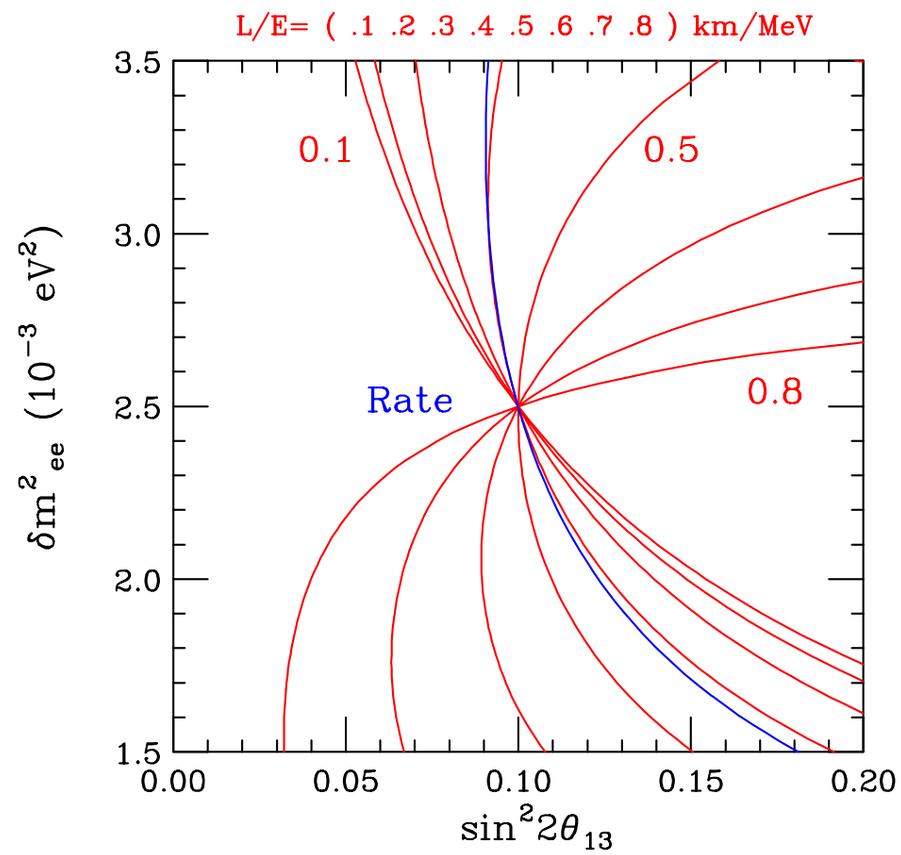
Total Rate
approx
0.4 km/MeV bin

Correlation between Δm^2_{ee} and $\sin^2 2\theta_{13}$

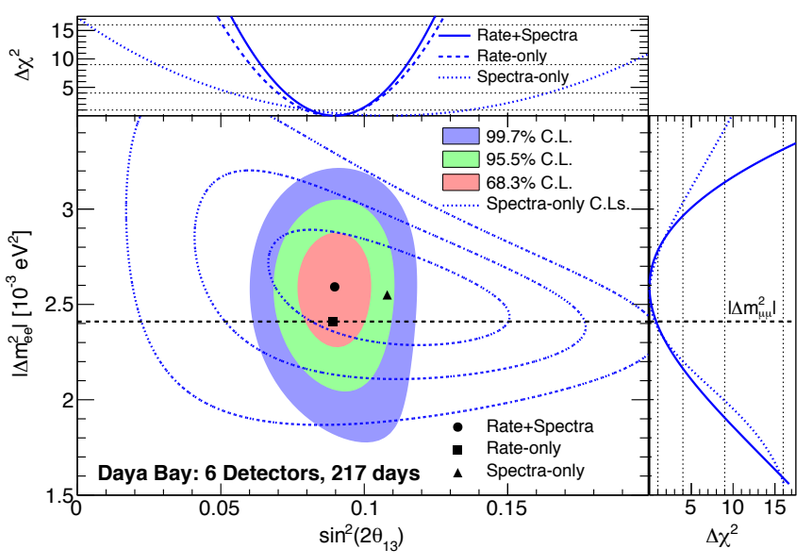
Total Rate Only

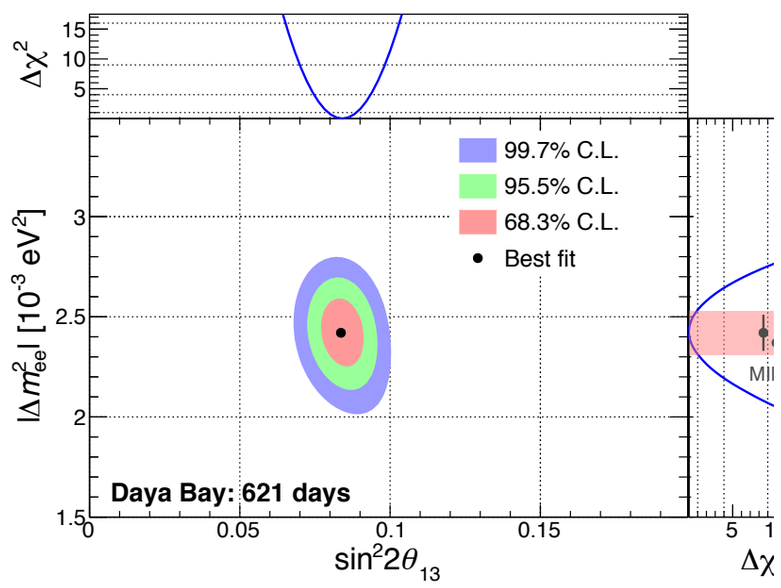


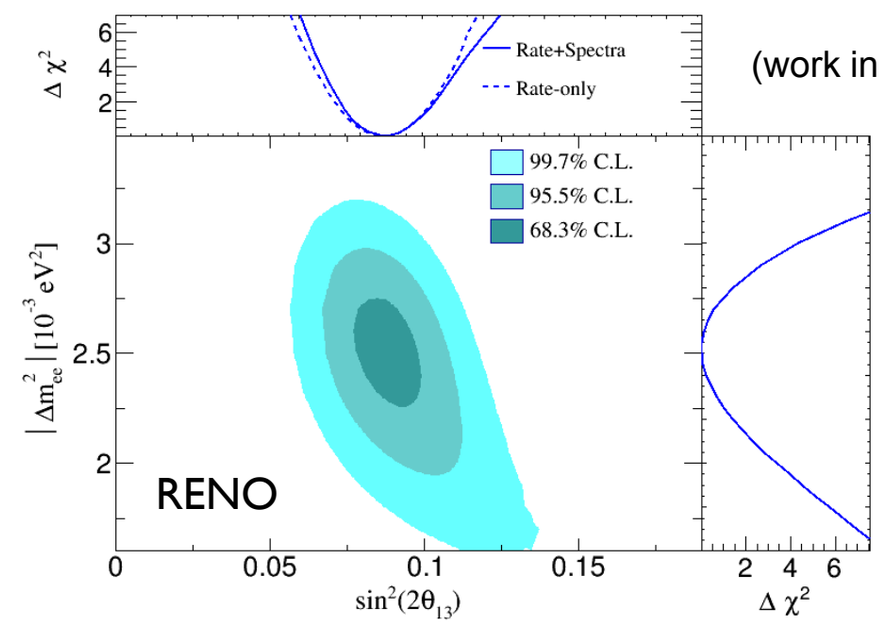
Rate per L/E bin



Total Rate
approx
0.4 km/MeV bin







(work in pr

What is δm^2_{ee} ?

Daya Bay:

$$|\Delta m^2_{ee}| = (2.42 \pm 0.11) \times 10^{-3}\ \mathrm{eV}^2$$

5% measurement !

RENO:

$$\Delta m_{ee}^{\ 2} = [2.52 \pm 0.19(\mathrm{stat}) \pm 0.17(\mathrm{syst})]\ \mathrm{x}10^{-3}\ \mathrm{eV}^2$$

What is δm^2_{ee} ?

Daya Bay:

$$|\Delta m^2_{ee}| = (2.42 \pm 0.11) \times 10^{-3}\ \mathrm{eV}^2$$

5% measurement !

RENO:

$$\Delta m_{ee}^{\ 2} = [2.52 \pm 0.19(\mathrm{stat}) \pm 0.17(\mathrm{syst})]\ \mathrm{x}10^{-3}\ \mathrm{eV}^2$$

Δm^2_{ee} is, obviously, some combination of Δm^2_{31} and Δm^2_{32} !

What is δm^2_{ee} ?

Daya Bay:

$$|\Delta m^2_{ee}| = (2.42 \pm 0.11) \times 10^{-3}\ \mathrm{eV}^2$$

5% measurement !

RENO:

$$\Delta m_{ee}^{\ 2} = [2.52 \pm 0.19(\mathrm{stat}) \pm 0.17(\mathrm{syst})]\ \mathrm{x}10^{-3}\ \mathrm{eV}^2$$

Δm^2_{ee} is, obviously, some combination of Δm^2_{31} and Δm^2_{32} !

But What Combination ?

What is δm^2_{ee} ?

Daya Bay:

$$|\Delta m^2_{ee}| = (2.42 \pm 0.11) \times 10^{-3}\ \mathrm{eV}^2$$

5% measurement !

RENO:

$$\Delta m_{ee}^{\ 2} = [2.52 \pm 0.19(\mathrm{stat}) \pm 0.17(\mathrm{syst})]\ \mathrm{x}10^{-3}\ \mathrm{eV}^2$$

Δm^2_{ee} is, obviously, some combination of Δm^2_{31} and Δm^2_{32} !

But What Combination ?

ν_e average ???????

In vacuum the electron neutrino disappearance is

$$\Delta_{ij} \equiv \frac{\Delta m^2_{ij} L}{4E}$$

$$\begin{aligned} P &= 1 - 4|U_{e2}|^2|U_{e1}|^2 \sin^2 \Delta_{21} \\ &\quad -4|U_{e3}|^2|U_{e1}|^2 \sin^2 \Delta_{31} - 4|U_{e3}|^2|U_{e2}|^2 \sin^2 \Delta_{32} \\ &= 1 - \cos^4 \theta_{13} \sin^2 2\theta_{12} \sin^2 \Delta_{21} \\ &\quad - \sin^2 2\theta_{13}(\cos^2 \theta_{12} \sin^2 \Delta_{31} + \sin^2 \theta_{12} \sin^2 \Delta_{32}) \end{aligned}$$

(the mass ordering is built into the signs of Δm^2_{3i} and Δm^2_{ee})

In vacuum the electron neutrino disappearance is

$$\Delta_{ij} \equiv \frac{\Delta m^2_{ij} L}{4E}$$

$$\begin{aligned} P &= 1 - 4|U_{e2}|^2|U_{e1}|^2 \sin^2 \Delta_{21} \\ &\quad -4|U_{e3}|^2|U_{e1}|^2 \sin^2 \Delta_{31} - 4|U_{e3}|^2|U_{e2}|^2 \sin^2 \Delta_{32} \\ &= 1 - \cos^4 \theta_{13} \sin^2 2\theta_{12} \sin^2 \Delta_{21} \\ &\quad - \sin^2 2\theta_{13}(\cos^2 \theta_{12} \sin^2 \Delta_{31} + \sin^2 \theta_{12} \sin^2 \Delta_{32}) \end{aligned}$$

(the mass ordering is built into the signs of Δm^2_{3i} and Δm^2_{ee})

$$\approx 1 - \cos^4 \theta_{13} \sin^2 2\theta_{12} \sin^2 \Delta_{21} - \sin^2 2\theta_{13} \sin^2 \Delta_{ee}$$

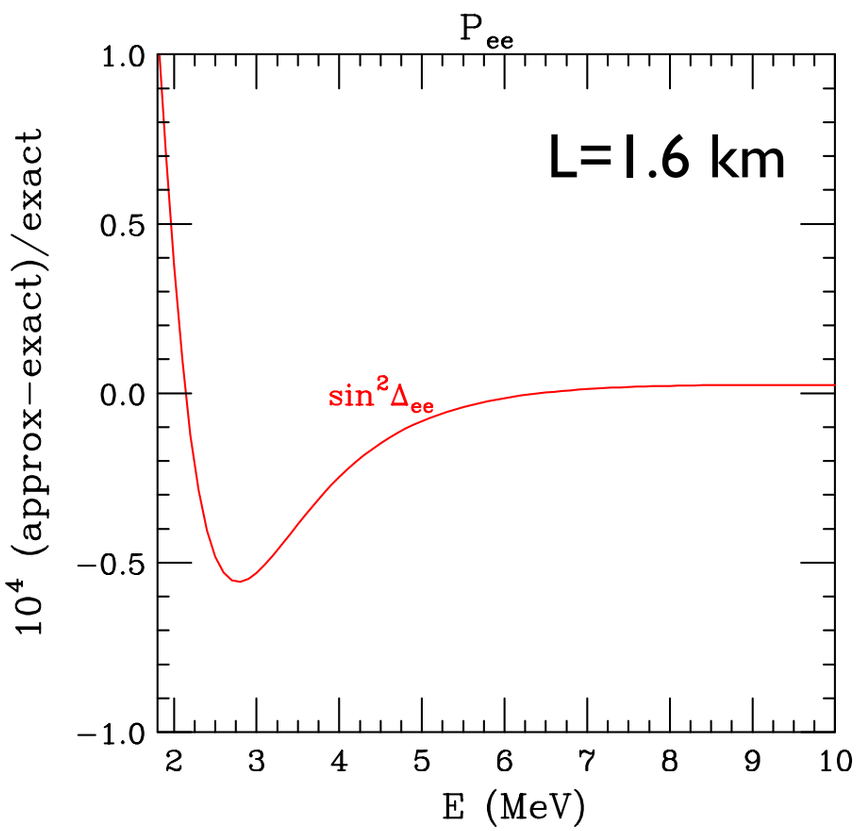
In vacuum the electron neutrino disappearance is

$$\Delta_{ij} \equiv \frac{\Delta m^2_{ij} L}{4E}$$

$$\begin{aligned} P &= 1 - 4|U_{e2}|^2|U_{e1}|^2 \sin^2 \Delta_{21} \\ &\quad -4|U_{e3}|^2|U_{e1}|^2 \sin^2 \Delta_{31} - 4|U_{e3}|^2|U_{e2}|^2 \sin^2 \Delta_{32} \\ &= 1 - \cos^4 \theta_{13} \sin^2 2\theta_{12} \sin^2 \Delta_{21} \\ &\quad - \sin^2 2\theta_{13}(\cos^2 \theta_{12} \sin^2 \Delta_{31} + \sin^2 \theta_{12} \sin^2 \Delta_{32}) \end{aligned}$$

(the mass ordering is built into the signs of Δm^2_{3i} and Δm^2_{ee})

$$\approx 1 - \cos^4 \theta_{13} \sin^2 2\theta_{12} \sin^2 \Delta_{21} - \sin^2 2\theta_{13} \sin^2 \Delta_{ee}$$



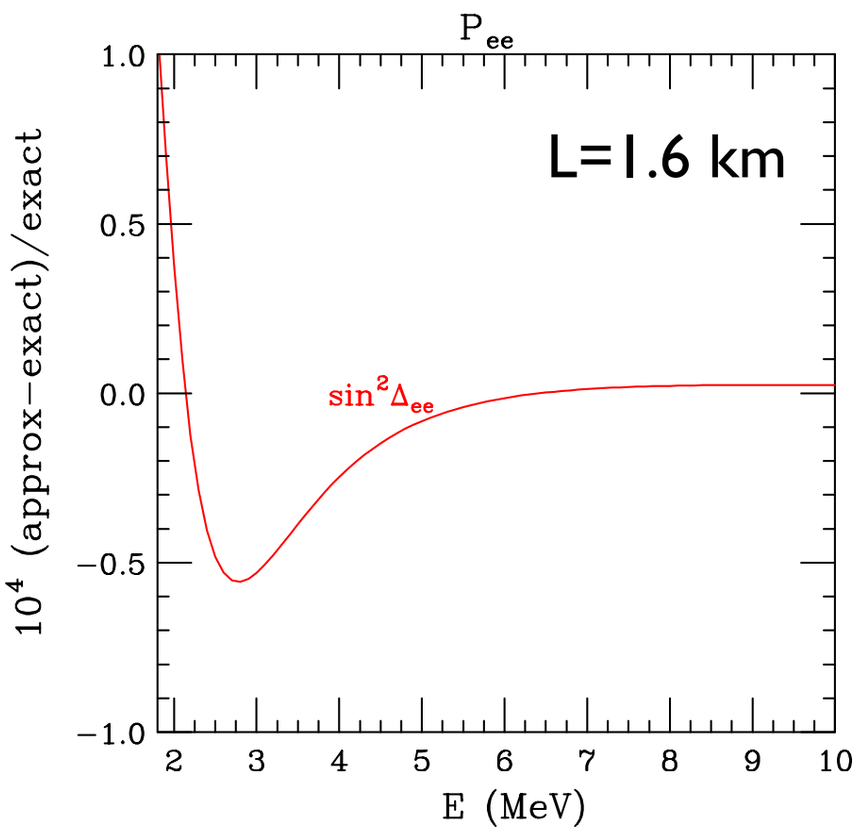
In vacuum the electron neutrino disappearance is

$$\Delta_{ij} \equiv \frac{\Delta m^2_{ij} L}{4E}$$

$$\begin{aligned} P &= 1 - 4|U_{e2}|^2|U_{e1}|^2 \sin^2\Delta_{21} \\ &\quad -4|U_{e3}|^2|U_{e1}|^2 \sin^2\Delta_{31} - 4|U_{e3}|^2|U_{e2}|^2 \sin^2\Delta_{32} \\ &= 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} \\ &\quad - \sin^2 2\theta_{13}(\cos^2\theta_{12}\sin^2\Delta_{31} + \sin^2\theta_{12}\sin^2\Delta_{32}) \end{aligned}$$

(the mass ordering is built into the signs of Δm^2_{3i} and Δm^2_{ee})

$$\approx 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} - \sin^2 2\theta_{13}\sin^2\Delta_{ee}$$



- $\delta m^2_{ee} = \cos^2\theta_{12}\,\Delta m^2_{31} + \sin^2\theta_{12}\,\Delta m^2_{32}$
  (>0 for NO and <0 for IO)
- Clearly this definition is L/E independent
- ν_e weighted average of Δm^2_{31} and Δm^2_{32} !

H. Nunokawa, S. J. Parke and R. Zukanovich Funchal, "Another possible way to determine the neutrino mass hierarchy," Phys. Rev. D **72**, 013009 (2005), hep-ph/0503283

Suppose we use some other linear combination:

$$\Delta m^2_{rr} = (1-r)\ \Delta m^2_{31} + r\ \Delta m^2_{32}$$

and $r = \sin^2\theta_{12}$ gives us δm^2_{ee}.

L=1.6 km

r=0, 0.05, 0.1 …1.0

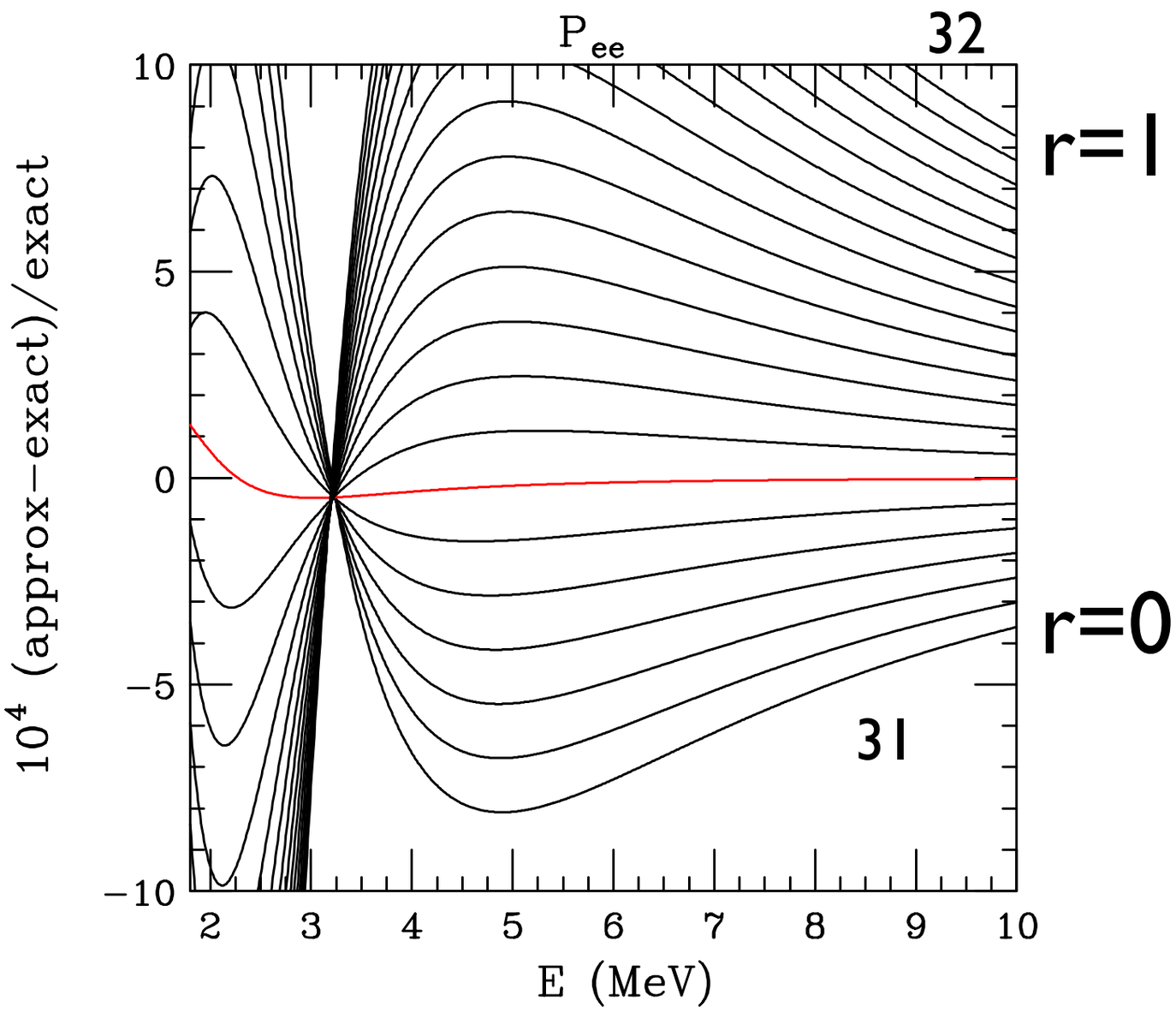
Suppose we use some other linear combination:

$$\Delta m^2_{rr} = (1-r)\ \Delta m^2_{31} + r\ \Delta m^2_{32}$$

and $r = \sin^2\theta_{12}$ gives us δm^2_{ee}.

L=1.6 km

r=0, 0.05, 0.1 …1.0



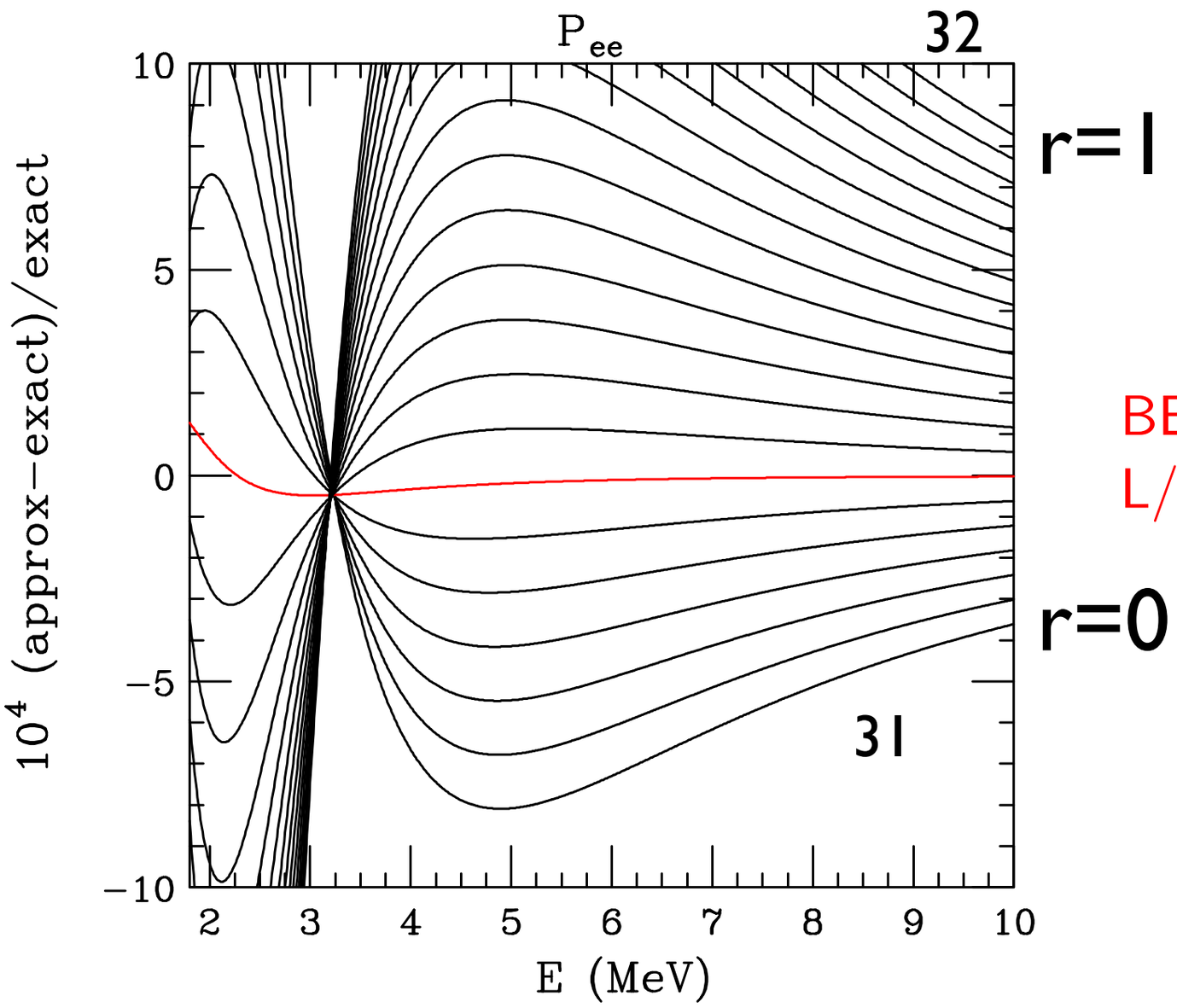
Suppose we use some other linear combination:

$$\Delta m^2_{rr} = (1-r)\ \Delta m^2_{31} + r\ \Delta m^2_{32}$$

and $r = \sin^2\theta_{12}$ gives us δm^2_{ee}.

L=1.6 km

r=0, 0.05, 0.1 …1.0



BEST you can do using L/E independent δm^2 ! ! !

Why Does this Work this Way?

Daya Bay, RENO and Double Chooz:

$\Delta_{31} \approx \Delta_{32} \leq \frac{3\pi}{4}$ and $\Delta_{21} \leq \frac{\pi}{40} \approx 0.1$

Why Does this Work this Way?

Daya Bay, RENO and Double Chooz:

$\Delta_{31} \approx \Delta_{32} \leq \frac{3\pi}{4}$ and $\Delta_{21} \leq \frac{\pi}{40} \approx 0.1$

Taylor Series Expansion:

$\Delta_{31} = \Delta_{ee} + s_{12}^2\ \Delta_{21}$ and $\Delta_{32} = \Delta_{ee} - c_{12}^2\ \Delta_{21}$

Why Does this Work this Way?

Daya Bay, RENO and Double Chooz:

$\Delta_{31} \approx \Delta_{32} \leq \frac{3\pi}{4}$ and $\Delta_{21} \leq \frac{\pi}{40} \approx 0.1$

Note, the Δ_{21} terms vanish !

Taylor Series Expansion:

$\Delta_{31} = \Delta_{ee} + s_{12}^2\ \Delta_{21}$ and $\Delta_{32} = \Delta_{ee} - c_{12}^2\ \Delta_{21}$

$$
\begin{aligned}
c_{12}^2 \sin^2 \Delta_{31} + s_{12}^2 \sin^2 \Delta_{31} &= \sin^2 \Delta_{ee} + 0 + s_{12}^2 c_{12}^2 \Delta_{21}^2 \cos(2\Delta_{ee}) \\
&\quad -\frac{1}{6} \cos 2\theta_{12} \sin^2 2\theta_{12}\ \Delta_{21}^3\ \sin(2\Delta_{ee}) + \mathcal{O}(\Delta_{21}^4) \\
&= 1 + \mathcal{O}(10^{-3}) \pm \mathcal{O}(10^{-5}) \quad \text{at OM}
\end{aligned}
$$

Mass Ordering effects !

Why Does this Work this Way?

Daya Bay, RENO and Double Chooz:

$\Delta_{31} \approx \Delta_{32} \leq \frac{3\pi}{4}$ and $\Delta_{21} \leq \frac{\pi}{40} \approx 0.1$

Note, the Δ_{21} terms vanish !

Taylor Series Expansion:

$\Delta_{31} = \Delta_{ee} + s_{12}^2\ \Delta_{21}$ and $\Delta_{32} = \Delta_{ee} - c_{12}^2\ \Delta_{21}$

improvement

$$
\begin{aligned}
c_{12}^2 \sin^2 \Delta_{31} + s_{12}^2 \sin^2 \Delta_{31} &= \sin^2 \Delta_{ee} + 0 + s_{12}^2 c_{12}^2 \Delta_{21}^2 \cos(2\Delta_{ee}) \\
&\quad -\frac{1}{6} \cos 2\theta_{12} \sin^2 2\theta_{12}\ \Delta_{21}^3\ \sin(2\Delta_{ee}) + \mathcal{O}(\Delta_{21}^4) \\
&= 1 + \mathcal{O}(10^{-3}) \pm \mathcal{O}(10^{-5}) \quad \text{at OM}
\end{aligned}
$$

Mass Ordering effects !

(Aside:)

In general, if we use $\Delta m^2_{rr} = (1-r)\ \Delta m^2_{31} + r\ \Delta m^2_{32}$
i.e. $\Delta m^2_{31} = \Delta m^2_{rr} + r\Delta_{21}$ and $\Delta m^2_{32} = \Delta m^2_{rr} - (1-r)\Delta_{21}$
then

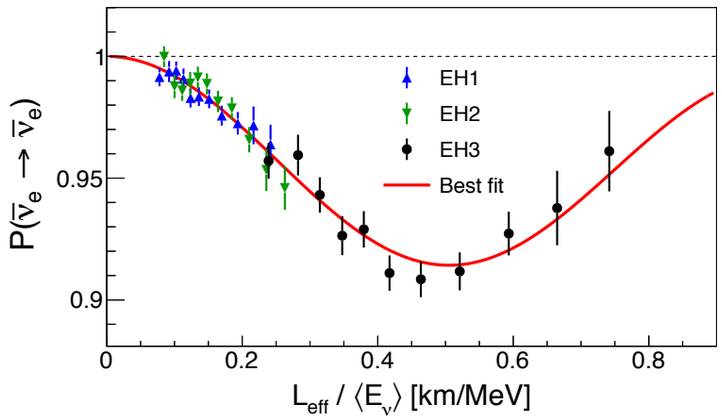
$$c^2_{12}\sin^2\Delta_{31} + s^2_{12}\sin^2\Delta_{32}$$

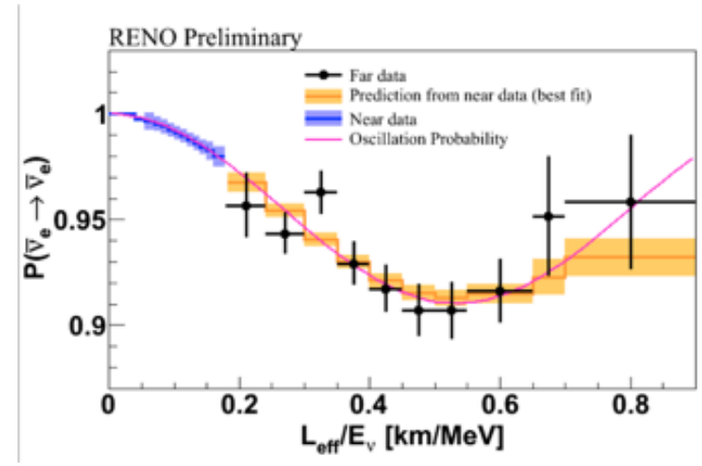
$$\begin{aligned} = \quad & \sin^2\Delta_{rr} + [c^2_{12}r - s^2_{12}(1-r)]\ \Delta_{21}\sin(2\Delta_{rr}) \\ & +[c^2_{12}r^2 + s^2_{12}(1-r)^2]\ \Delta^2_{21}\cos(2\Delta_{rr}) \\ & -\frac{2}{3}(c^2_{12}r^3 - s^2_{21}(1-r)^3)\Delta^3_{21}\sin(2\Delta_{rr}) + \mathcal{O}(\Delta^4_{21}) \end{aligned}$$

at $r = \sin^2\theta_{13}$ then coefficient of $\Delta_{21}\sin(2\Delta_{rr})$ is zero and the coefficient of $\Delta^2_{21}\cos(2\Delta_{rr})$ is a minimum ! Makes for an EXCELLENT expansion !

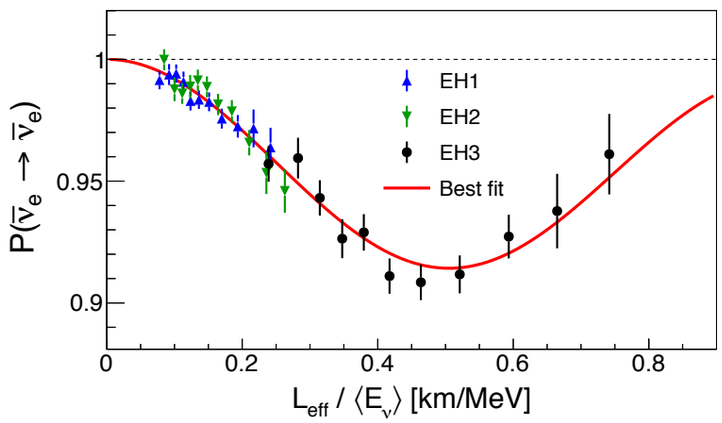
Especially since $\Delta_{21} < 0.1$!

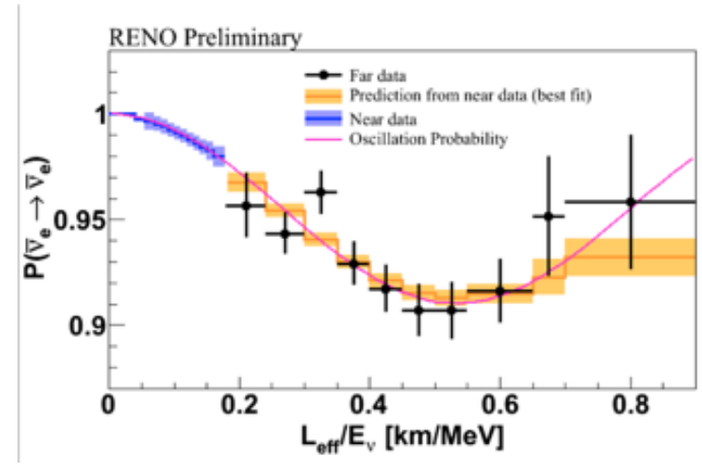
What Do the Experiments Do ?





What Do the Experiments Do ?

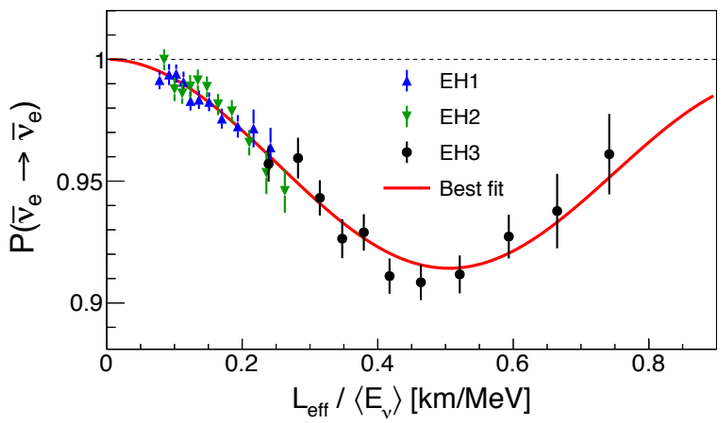


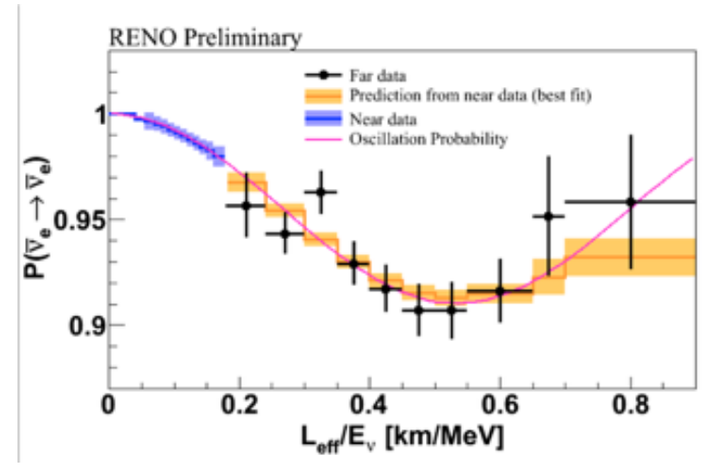


Daya Bay and RENO fit their L/E data to:

- $P_{ee} \approx 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} - \sin^2 2\theta_{13}\sin^2\Delta_{ee}$

What Do the Experiments Do ?

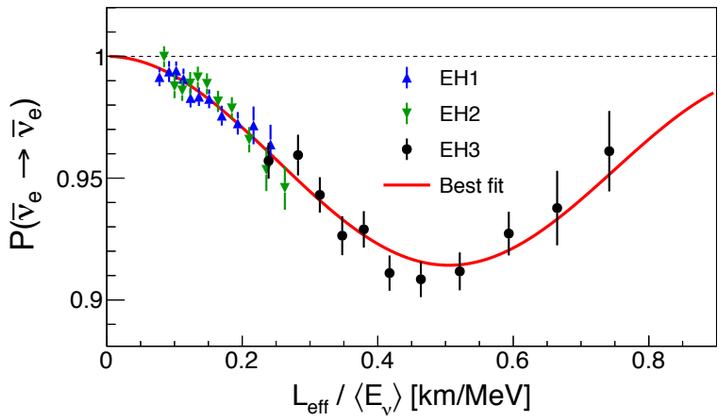


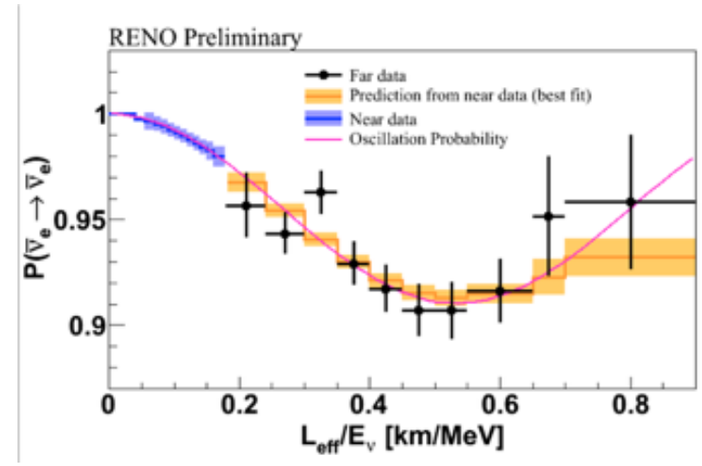


Daya Bay and RENO fit their L/E data to:

- $P_{ee} \approx 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} - \sin^2 2\theta_{13}\sin^2\Delta_{ee}$
- using L/E independent θ_{13} and $|\Delta m^2_{ee}|$. (θ_{12} and Δm^2_{21} obtained from other experiments.)

What Do the Experiments Do ?

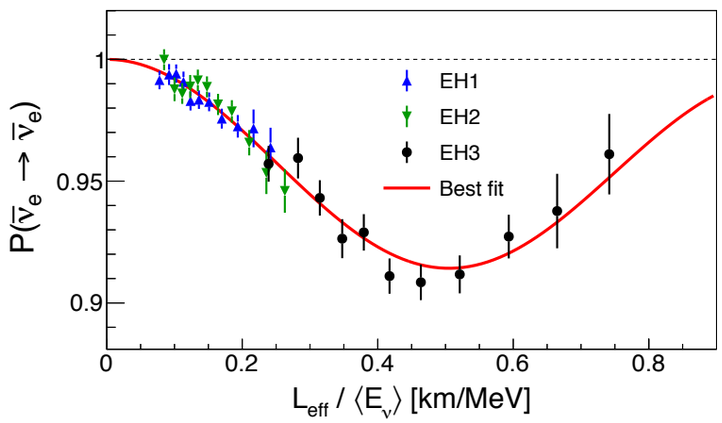


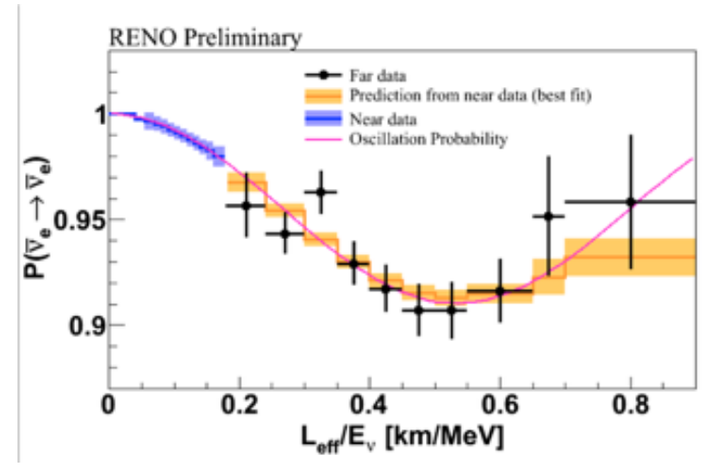


Daya Bay and RENO fit their L/E data to:

- $P_{ee} \approx 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} - \sin^2 2\theta_{13}\sin^2\Delta_{ee}$
- using L/E independent θ_{13} and $|\Delta m^2_{ee}|$. (θ_{12} and Δm^2_{21} obtained from other experiments.)
- θ_{13} and $|\Delta m^2_{ee}|$ are only weakly dependent on solar parameters and are INDEPENDENT of mass ordering.

What Do the Experiments Do ?





Daya Bay and RENO fit their L/E data to:

- $P_{ee} \approx 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} - \sin^2 2\theta_{13}\sin^2\Delta_{ee}$
- using L/E independent θ_{13} and $|\Delta m^2_{ee}|$. (θ_{12} and Δm^2_{21} obtained from other experiments.)
- θ_{13} and $|\Delta m^2_{ee}|$ are only weakly dependent on solar parameters and are INDEPENDENT of mass ordering.
- Trivial to calculate $|\Delta m^2_{31}| = |\Delta m^2_{ee}| \pm \sin^2\theta_{12}\Delta m^2_{21}$ and $|\Delta m^2_{32}| = |\Delta m^2_{ee}| \mp \cos^2\theta_{12}\Delta m^2_{21}$ using mass ordering and solar parameters. But $|\Delta m^2_{31}|$ and $|\Delta m^2_{32}|$ are more dependent on solar parameters than $|\Delta m^2_{ee}|$!

How Does Daya Bay Define Δm^2_{EE} ?

(arXiv:1310.6732 and 1505.03456v1)

How Does Daya Bay Define Δm^2_{EE} ?

(arXiv:1310.6732 and 1505.03456v1)

$$\sin^2\Delta_{EE} \equiv c_{12}^2\sin^2\Delta_{31} + s_{12}^2\sin^2\Delta_{32} \quad (1)$$

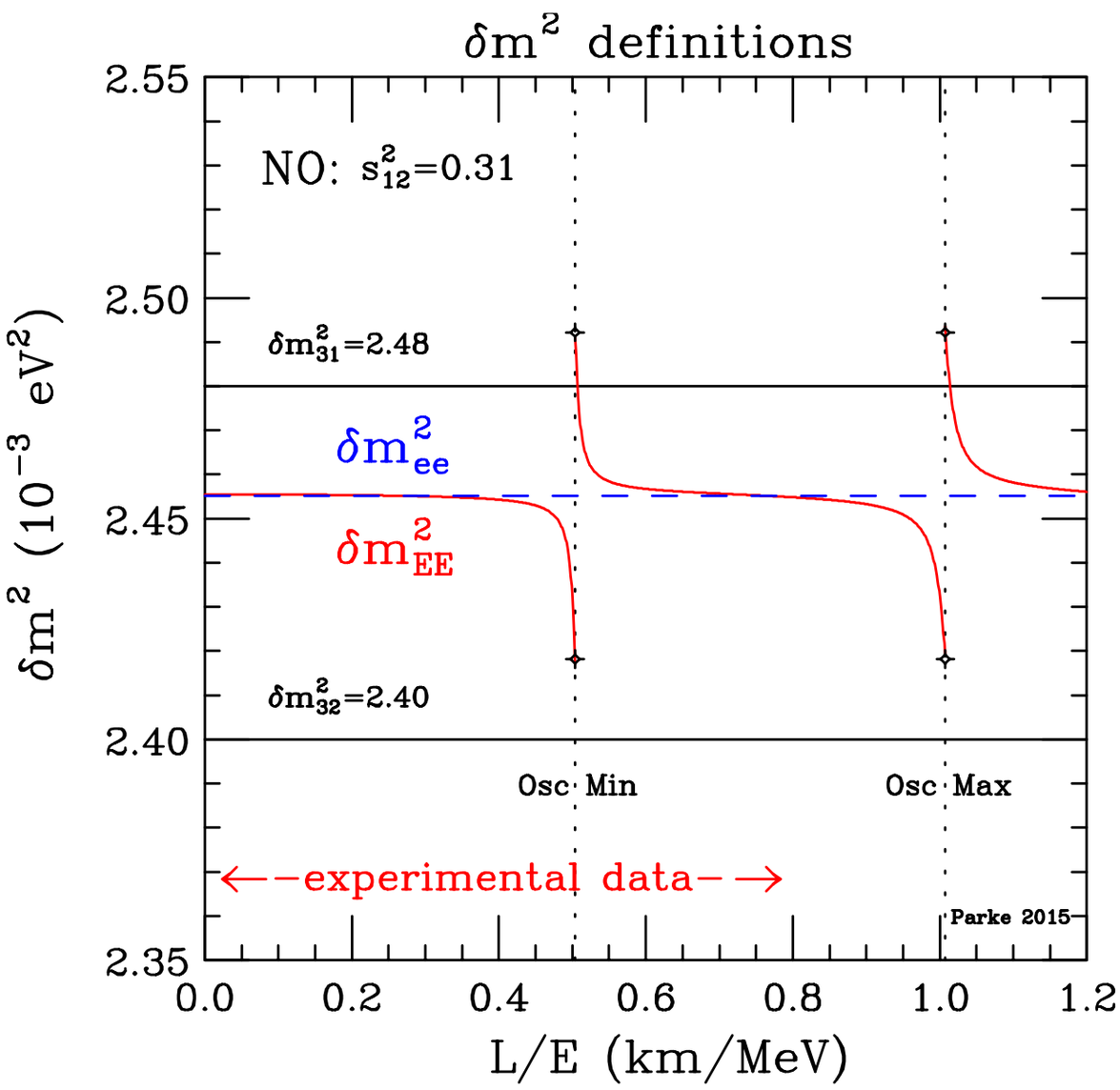
$$\Rightarrow \quad \Delta m^2_{EE} = \left(\frac{4E}{L}\right)\arcsin\left[\sqrt{(c_{12}^2\sin^2\Delta_{31} + s_{12}^2\sin^2\Delta_{32})}\right]$$

How Does Daya Bay Define Δm^2_{EE} ?

(arXiv:1310.6732 and 1505.03456v1)

$$\sin^2\Delta_{EE} \equiv c_{12}^2\sin^2\Delta_{31} + s_{12}^2\sin^2\Delta_{32} \quad (1)$$

$$\Rightarrow \quad \Delta m^2_{EE} = \left(\frac{4E}{L}\right)\arcsin\left[\sqrt{(c_{12}^2\sin^2\Delta_{31} + s_{12}^2\sin^2\Delta_{32})}\right]$$

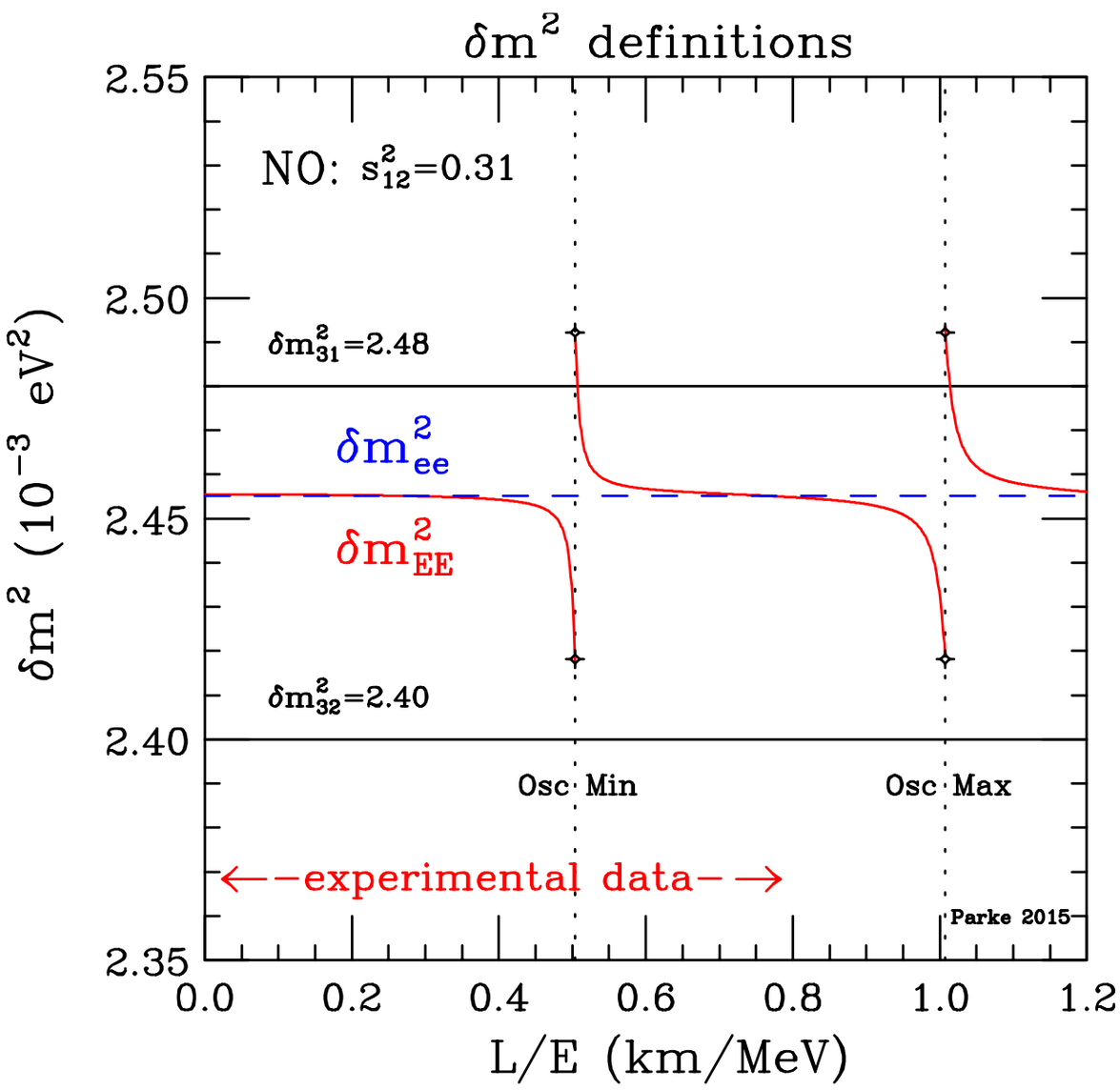


How Does Daya Bay Define Δm^2_{EE} ?

(arXiv:1310.6732 and 1505.03456v1)

$$\sin^2\Delta_{EE} \equiv c_{12}^2\sin^2\Delta_{31} + s_{12}^2\sin^2\Delta_{32} \quad (1)$$

$$\Rightarrow \quad \Delta m^2_{EE} = \left(\frac{4E}{L}\right)\arcsin\left[\sqrt{(c_{12}^2\sin^2\Delta_{31} + s_{12}^2\sin^2\Delta_{32})}\right]$$



Maladies:

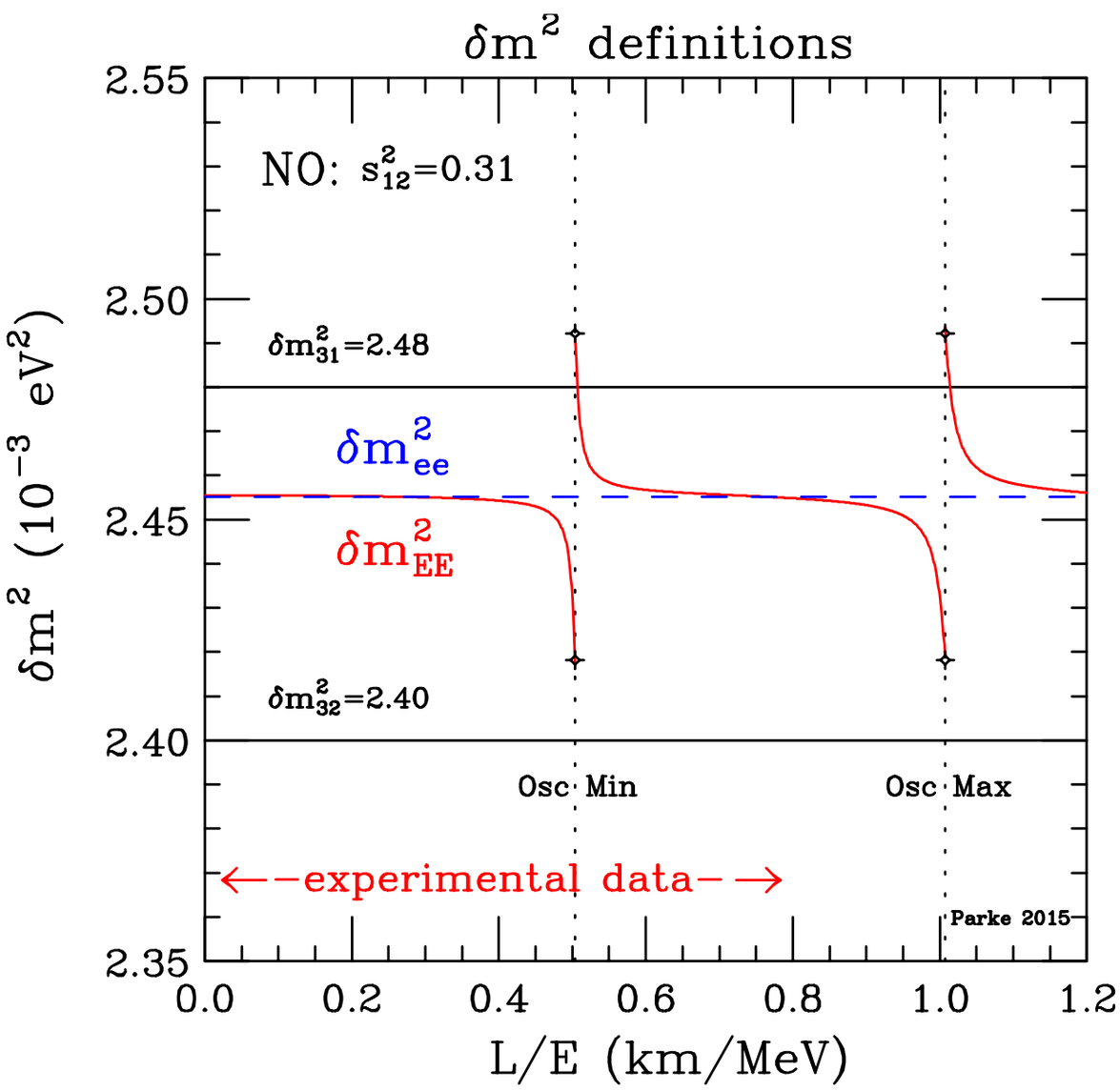
- L/E dependent
- Discontinuous at Osc. Max./Min. (L/E ≈ 0.5, 1.0, .. km/MeV) 3% jump
- no simple physical meaning !

How Does Daya Bay Define Δm^2_{EE} ?

(arXiv:1310.6732 and 1505.03456v1)

$$\sin^2\Delta_{EE} \equiv c_{12}^2\sin^2\Delta_{31} + s_{12}^2\sin^2\Delta_{32} \qquad (1)$$

$$\Rightarrow \quad \Delta m^2_{EE} = \left(\frac{4E}{L}\right)\arcsin\left[\sqrt{(c_{12}^2\sin^2\Delta_{31} + s_{12}^2\sin^2\Delta_{32})}\right]$$



Maladies:

- L/E dependent
- Discontinuous at Osc. Max./Min. (L/E $\approx$ 0.5, 1.0, .. km/MeV) 3% jump
- no simple physical meaning !

Why???

RHS (1) never gets exactly to 1, or back to 0 whereas LHS does !

eg $\sin^2(\frac{\pi}{2} \mp \epsilon) = 1 - \epsilon^2 + \mathcal{O}(\epsilon^4)$
with $\epsilon = s_{12}c_{12}\Delta_{21}$

Future Experiments:

Medium Baseline Experiments: JUNO and RENO-50

$$P_{ee} = 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} - \sin^2 2\theta_{13}(\cos^2\theta_{12}\sin^2\Delta_{31} + \sin^2\theta_{12}\sin^2\Delta_{32})$$

$$= 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} - \frac{1}{2}\sin^2 2\theta_{13}\left(1 - \sqrt{(1-\sin^2 2\theta_{12}\sin^2\Delta_{21})}.\cos\Omega\right)$$

where $\boxed{\Omega = 2|\Delta_{ee}| \pm \phi}$ (+/- mass ordering)

with $\phi = \{\arctan(\cos 2\theta_{12}\tan\Delta_{21}) - \Delta_{21}\cos 2\theta_{12}\}$

H. Minakata, H. Nunokawa, S. J. Parke and R. Zukanovich Funchal, Phys. Rev. D **74**, 053008 (2006) [hep-ph/0607284].

Medium Baseline Experiments: JUNO and RENO-50

$$P_{ee} = 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} - \sin^2 2\theta_{13}(\cos^2\theta_{12}\sin^2\Delta_{31} + \sin^2\theta_{12}\sin^2\Delta_{32})$$

$$= 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} - \frac{1}{2}\sin^2 2\theta_{13}\left(1 - \sqrt{(1-\sin^2 2\theta_{12}\sin^2\Delta_{21})}.\cos\Omega\right)$$

where $\boxed{\Omega = 2|\Delta_{ee}| \pm \phi}$ (+/- mass ordering)

with $\phi = \{\arctan(\cos 2\theta_{12}\tan\Delta_{21}) - \Delta_{21}\cos 2\theta_{12}\}$

Note:

- Separation $2\Delta_{ee}$ and ϕ is unique !
- $2\Delta_{ee}$ is linear in L/E and depends on an atm Δm^2.
- $\phi = \{\arctan(\cos 2\theta_{12}\tan\Delta_{21}) - \Delta_{21}\cos 2\theta_{12}\}$ starts at $\Delta_{21}^3 \sim (L/E)^3$ and depends on only solar parameters, staircase function.

H. Minakata, H. Nunokawa, S. J. Parke and R. Zukanovich Funchal, Phys. Rev. D **74**, 053008 (2006) [hep-ph/0607284].

Medium Baseline Experiments: JUNO and RENO-50

$$P_{ee} = 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} - \sin^2 2\theta_{13}(\cos^2\theta_{12}\sin^2\Delta_{31} + \sin^2\theta_{12}\sin^2\Delta_{32})$$

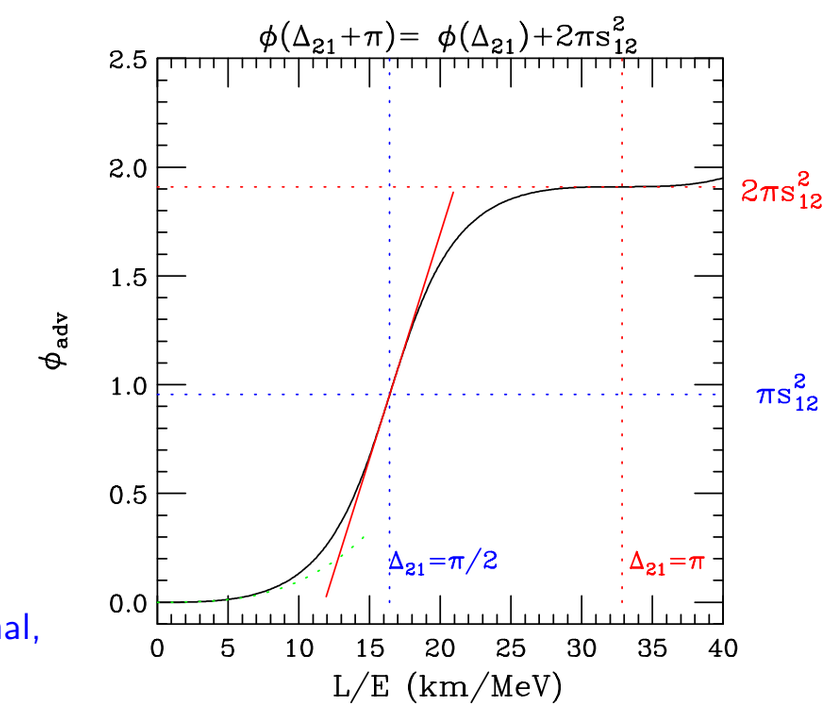
$$= 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} - \frac{1}{2}\sin^2 2\theta_{13}\left(1 - \sqrt{(1-\sin^2 2\theta_{12}\sin^2\Delta_{21})}.\cos\Omega\right)$$

where $\Omega = 2|\Delta_{ee}| \pm \phi$ (+/- mass ordering)

with $\phi = \{\arctan(\cos 2\theta_{12}\tan\Delta_{21}) - \Delta_{21}\cos 2\theta_{12}\}$

Note:

- Separation $2\Delta_{ee}$ and ϕ is unique !
- $2\Delta_{ee}$ is linear in L/E and depends on an atm Δm^2.
- $\phi = \{\arctan(\cos 2\theta_{12}\tan\Delta_{21}) - \Delta_{21}\cos 2\theta_{12}\}$ starts at $\Delta_{21}^3 \sim (L/E)^3$ and depends on only solar parameters, staircase function.



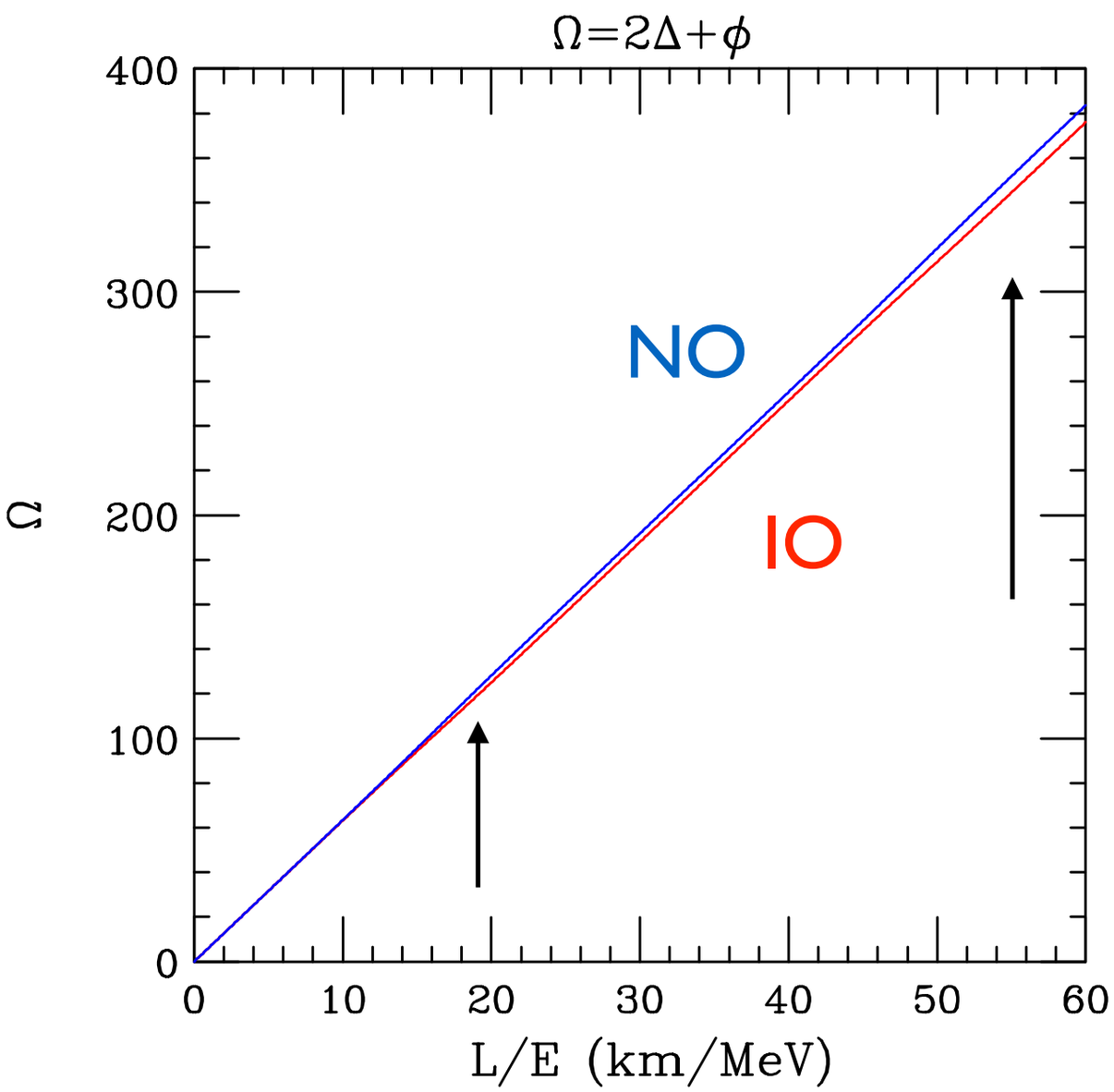
H. Minakata, H. Nunokawa, S. J. Parke and R. Zukanovich Funchal, Phys. Rev. D **74**, 053008 (2006) [hep-ph/0607284].

New DB definition: $\Delta m^2_{\epsilon\epsilon} \equiv \frac{2E}{L}\,\Omega$ (1505.03456 PRL version)

(Contrast: $\Delta m^2_{ee} = \frac{\partial}{\partial L/2E}\,\Omega|_{L/2E\to 0}$)

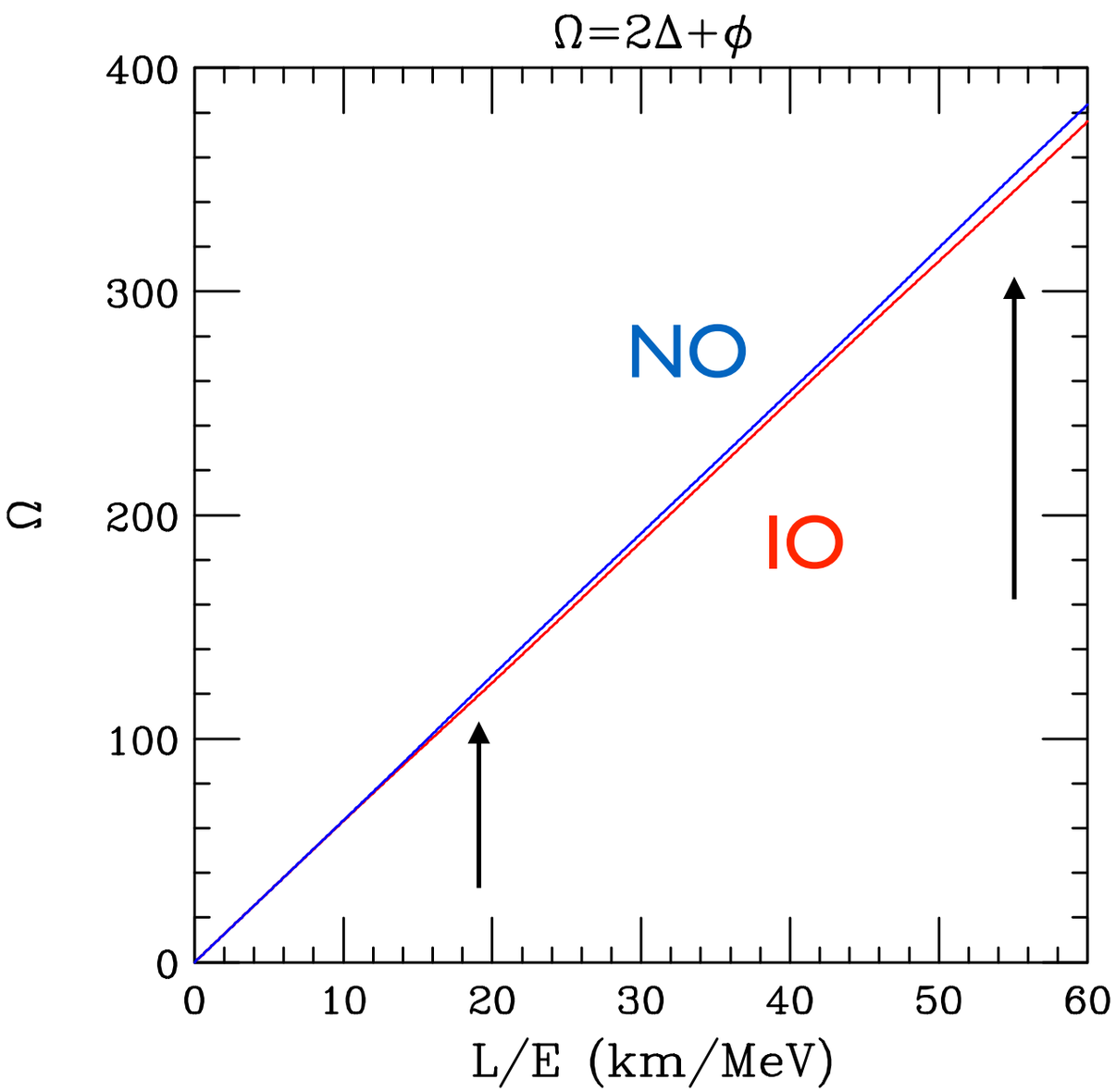
New DB definition: $\Delta m^2_{\epsilon\epsilon} \equiv \frac{2E}{L}\,\Omega$ (1505.03456 PRL version)

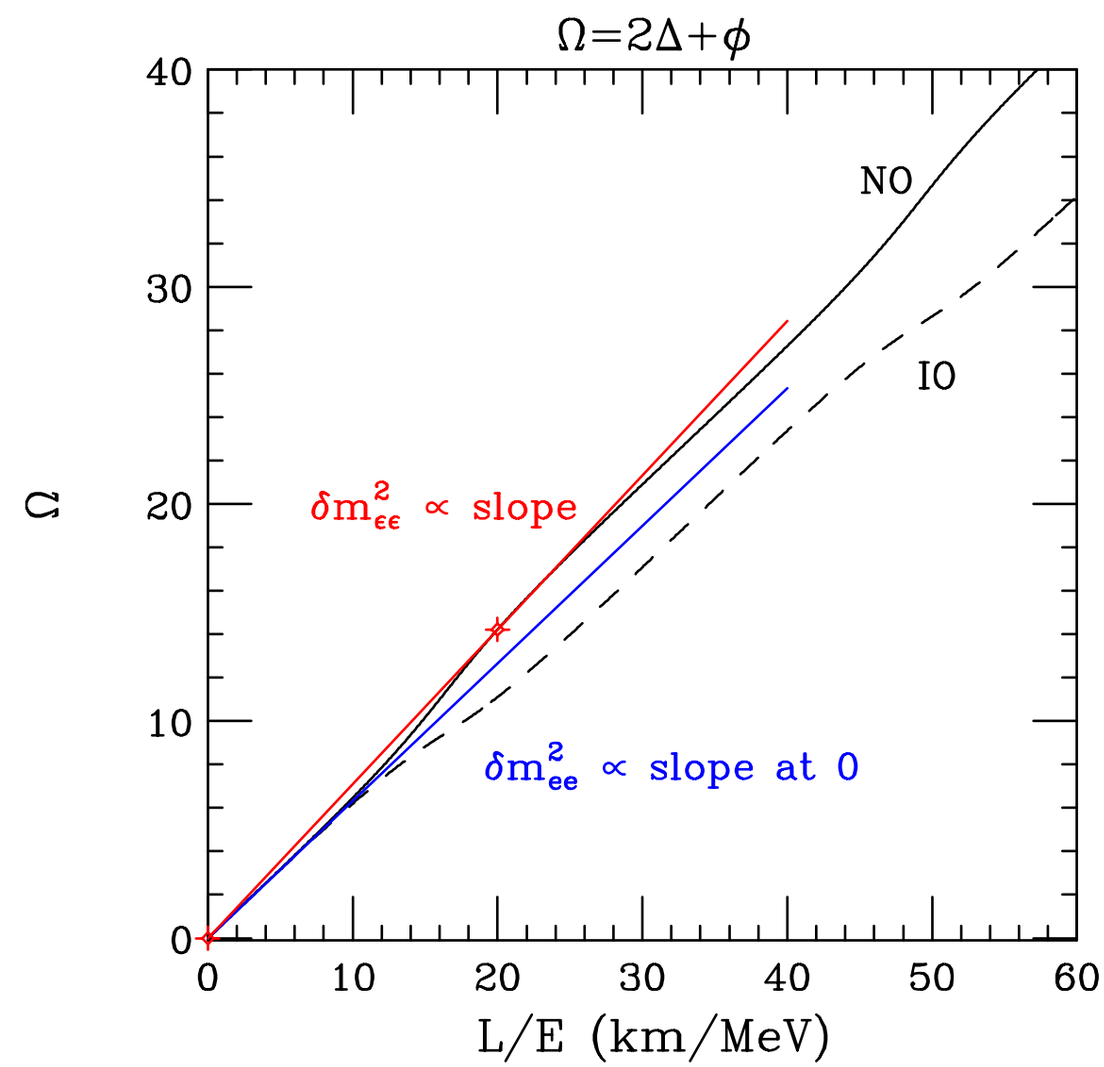
(Contrast: $\Delta m^2_{ee} = \frac{\partial}{\partial L/2E}\,\Omega|_{L/2E\to 0}$)



New DB definition: $\Delta m^2_{\epsilon\epsilon} \equiv \frac{2E}{L}\,\Omega$ (1505.03456 PRL version)

(Contrast: $\Delta m^2_{ee} = \frac{\partial}{\partial L/2E}\,\Omega|_{L/2E\to 0}$)

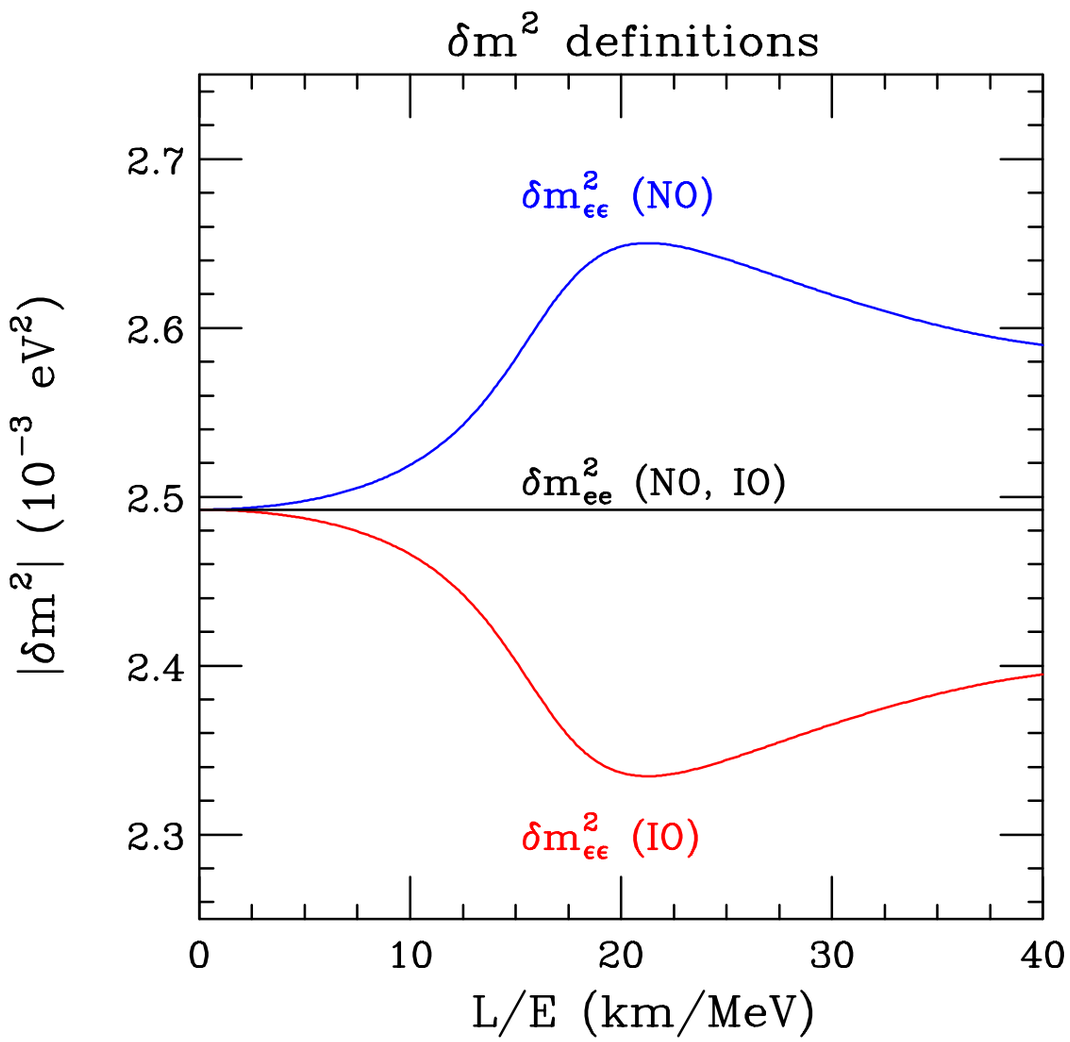




(For this figure, to make the effect visible, the factor 2Δ is reduced by factor 10.)

New DB definition: $\Delta m^2_{\epsilon\epsilon} \equiv \frac{2E}{L}\,\Omega$ (1505.03456 PRL version)

(Contrast: $\Delta m^2_{ee} = \frac{\partial}{\partial L/2E}\,\Omega|_{L/2E\to 0}$)

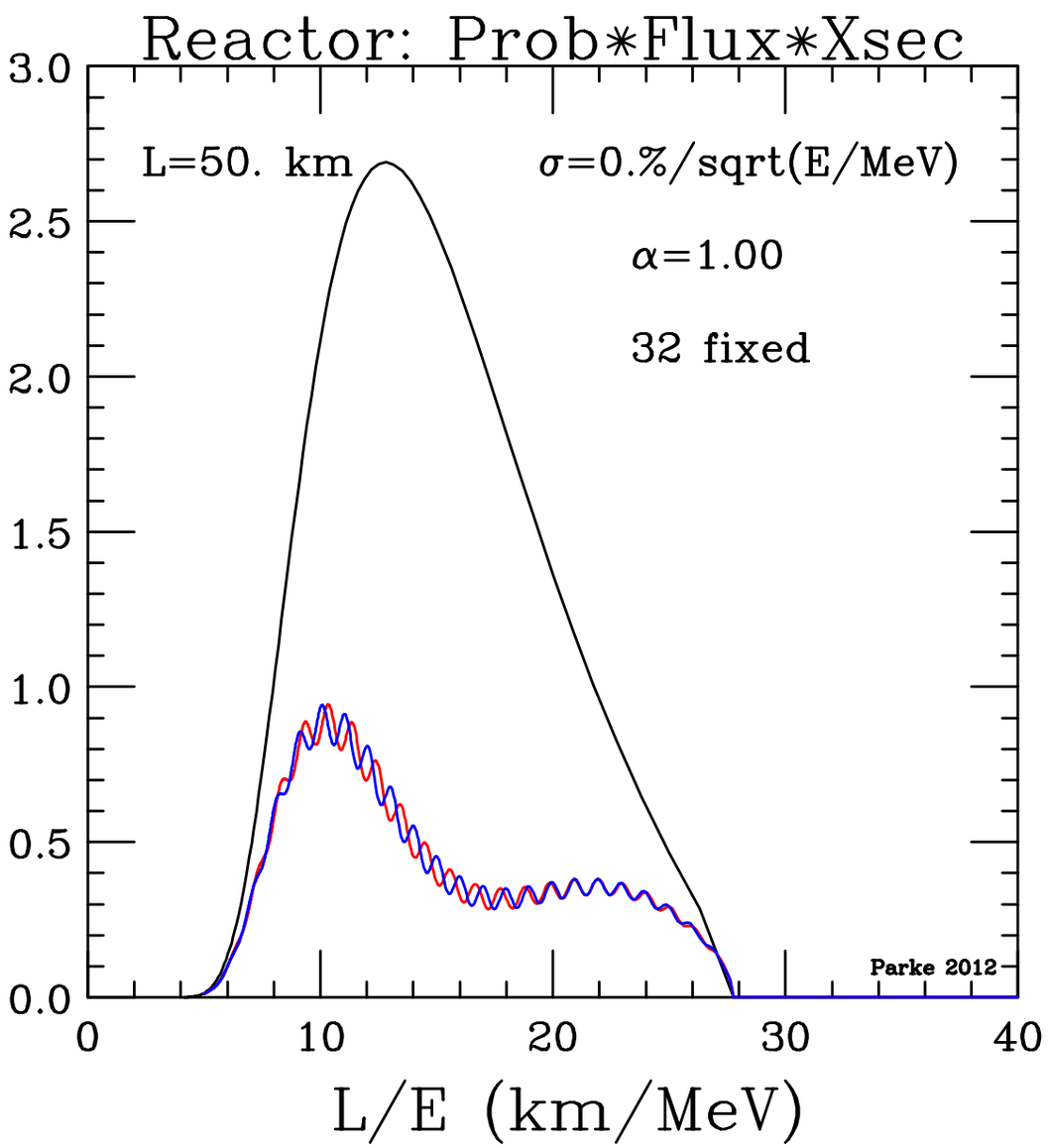


Identical for L/E $<$ 5 -10 km/MeV !

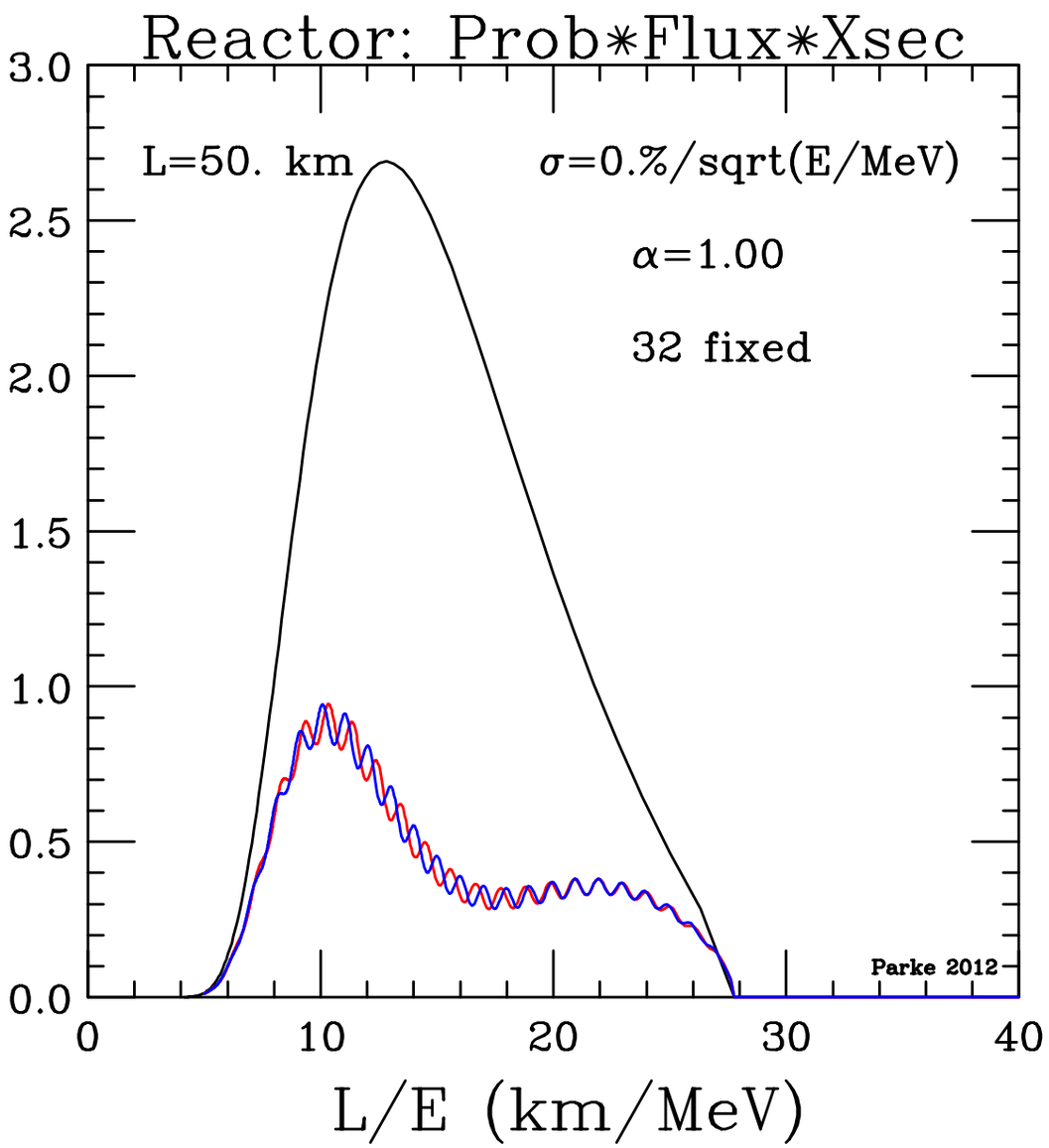
Significant L/E and
Mass Ordering Dependence!

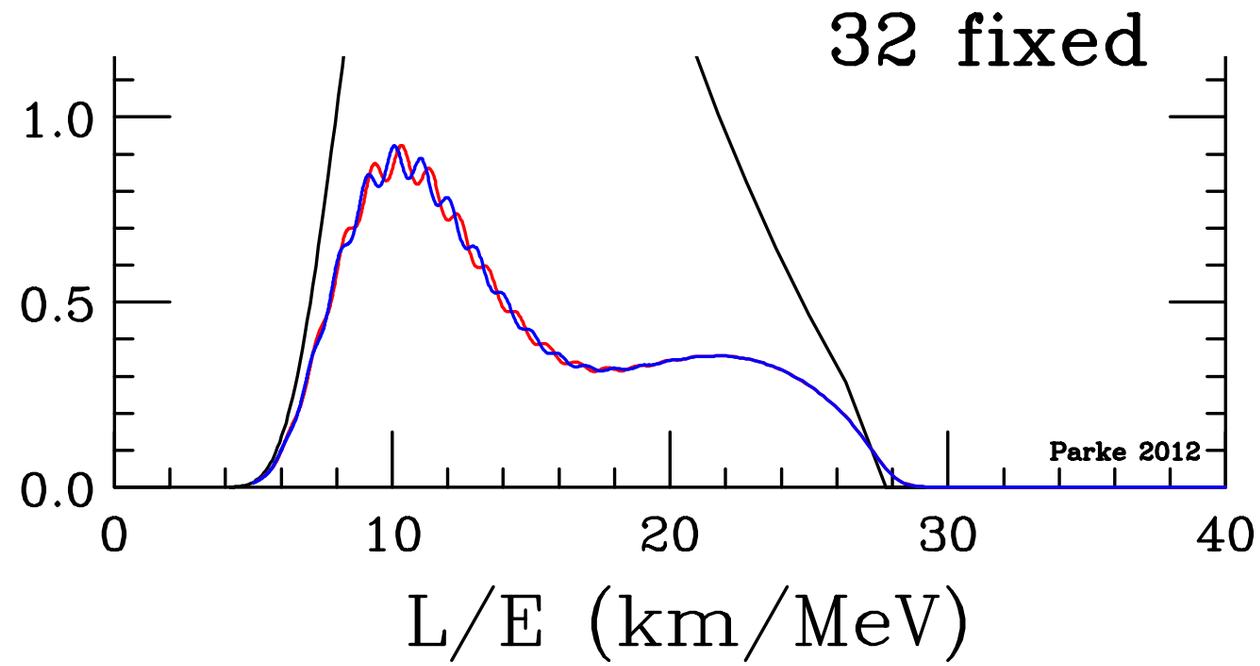
Mass Ordering:

Reactors & the Mass Ordering:

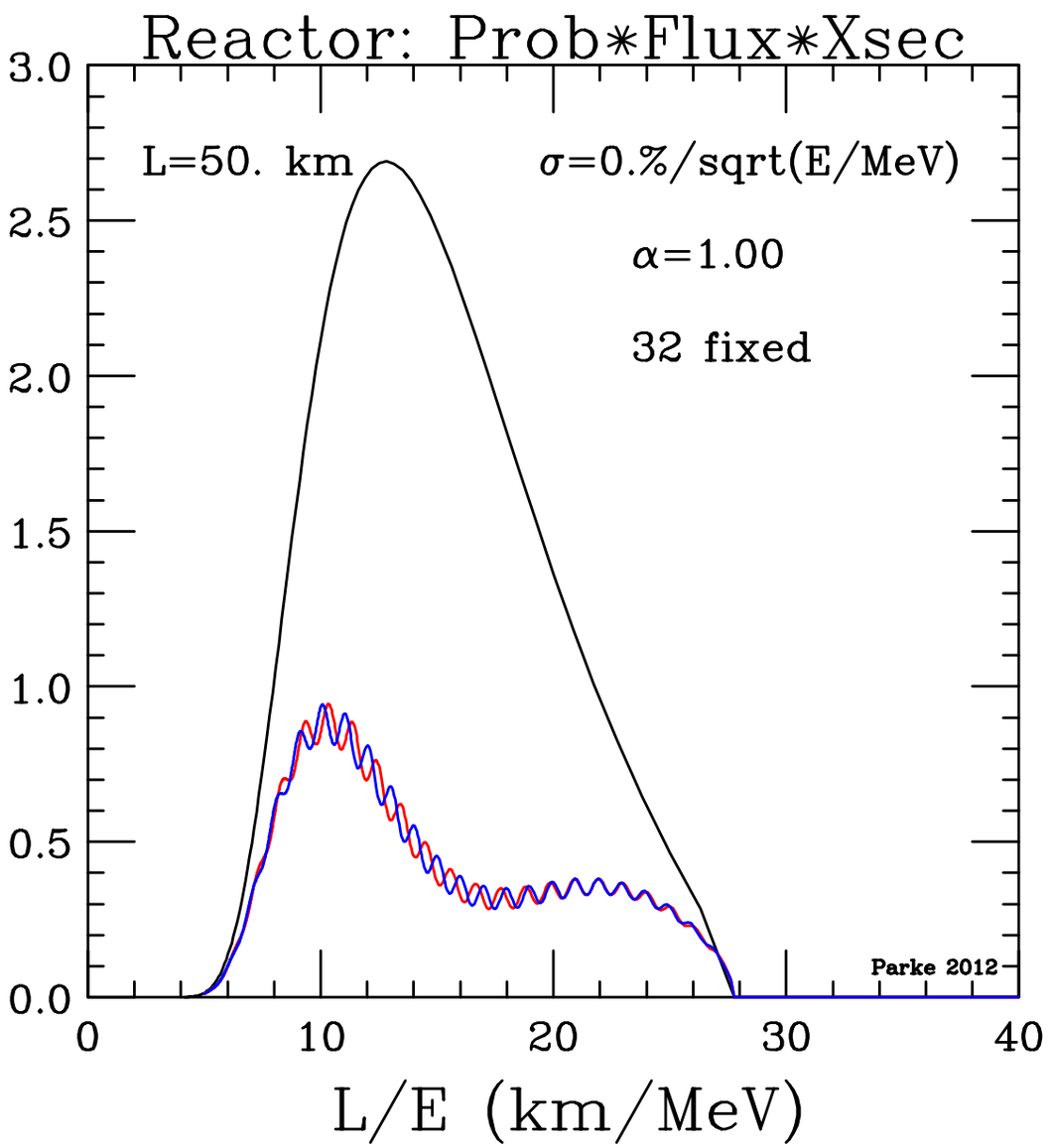


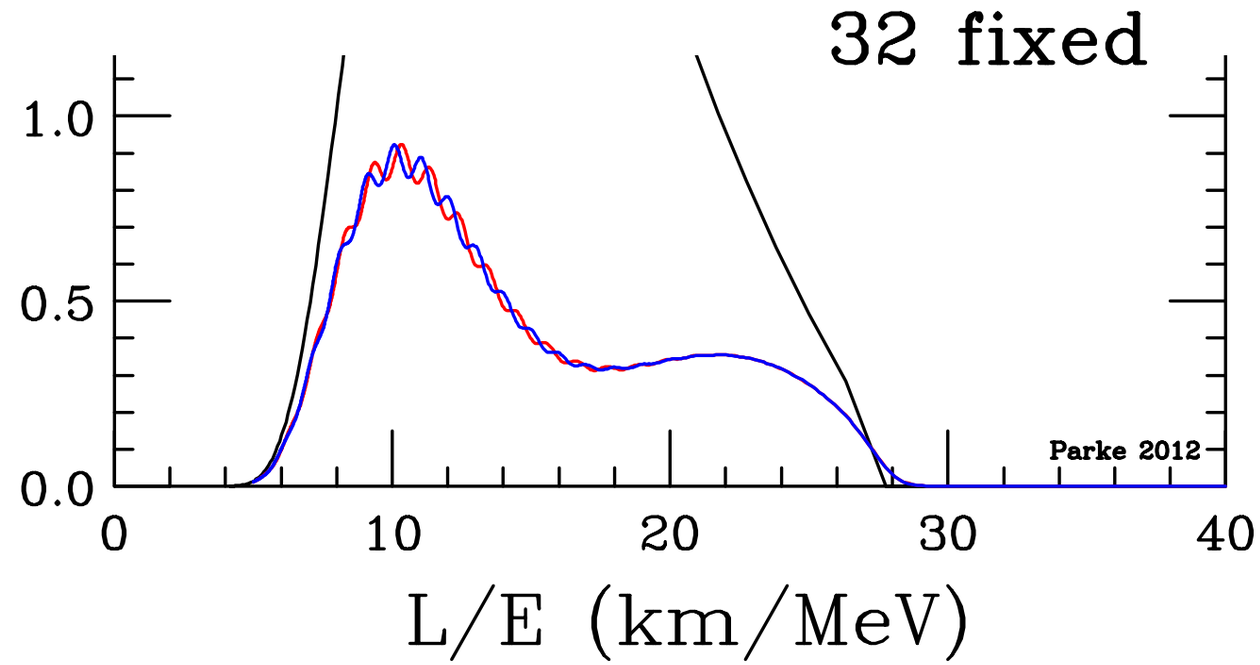
Reactors & the Mass Ordering:

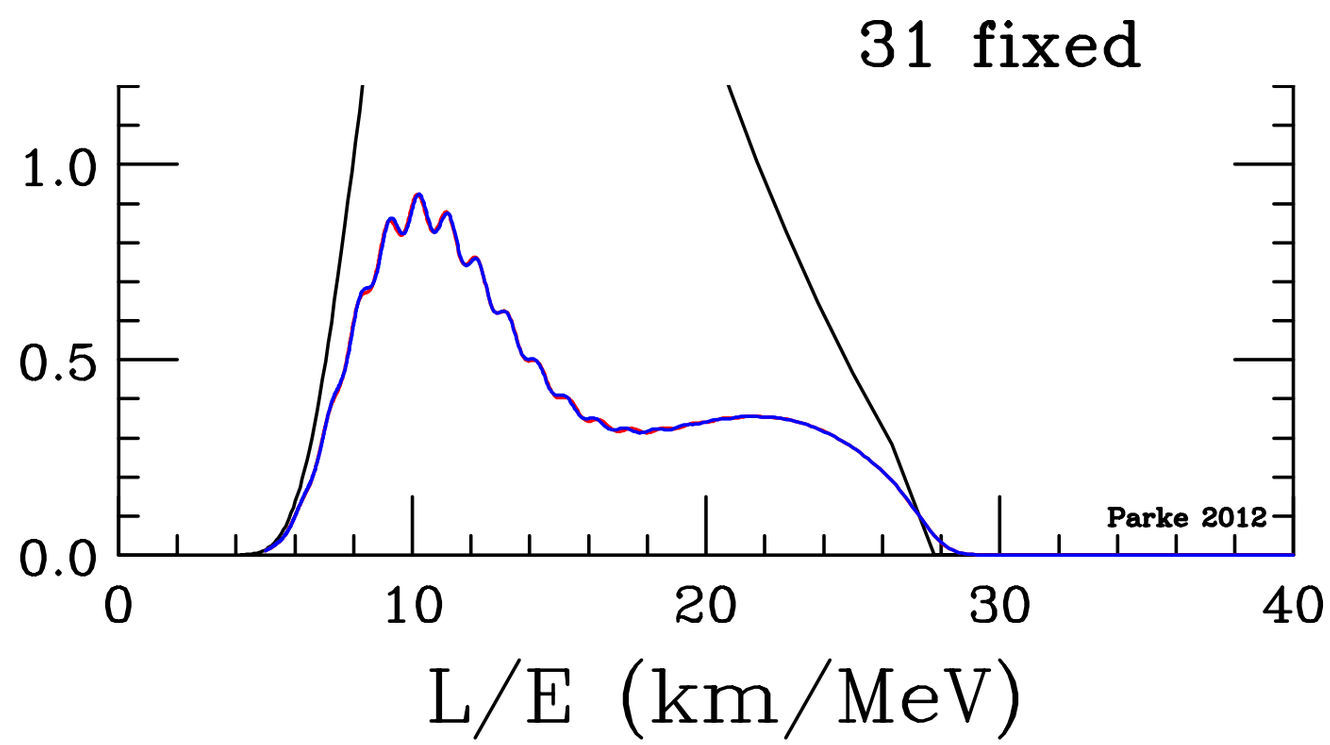


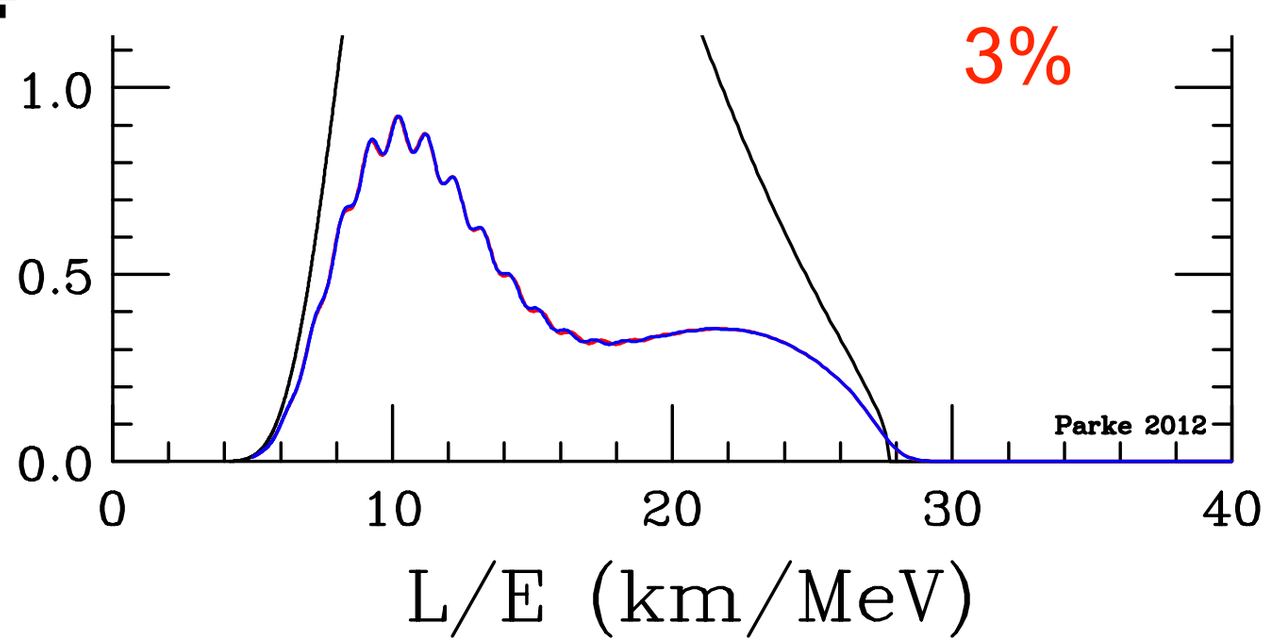


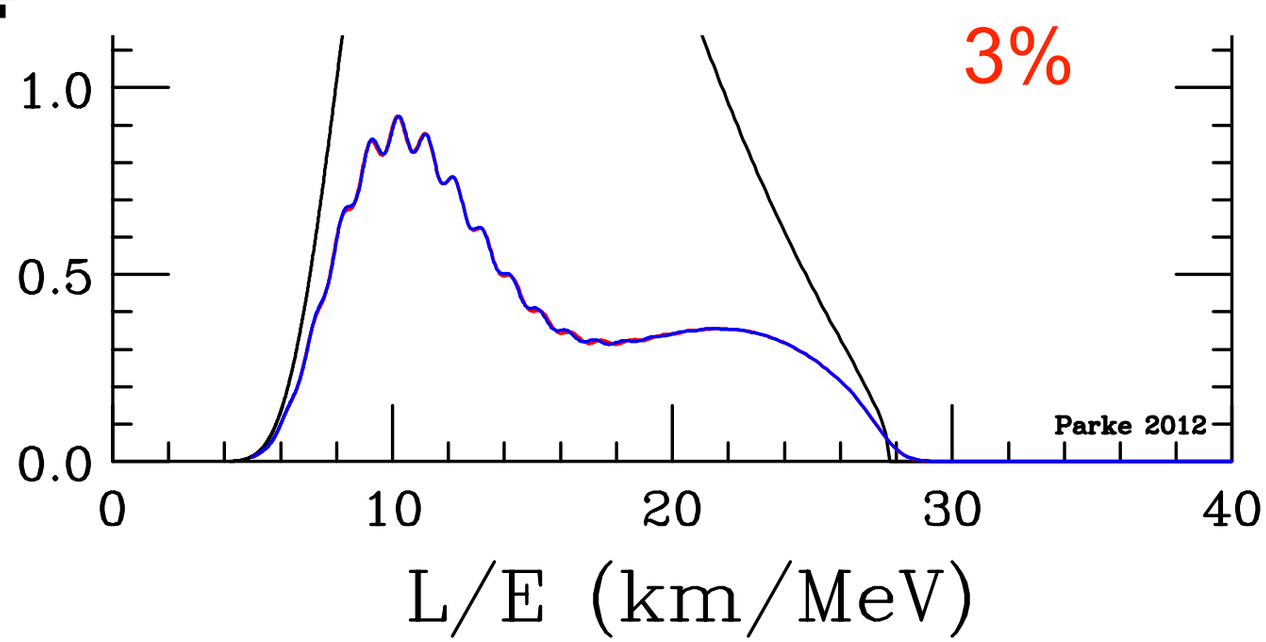
Reactors & the Mass Ordering:

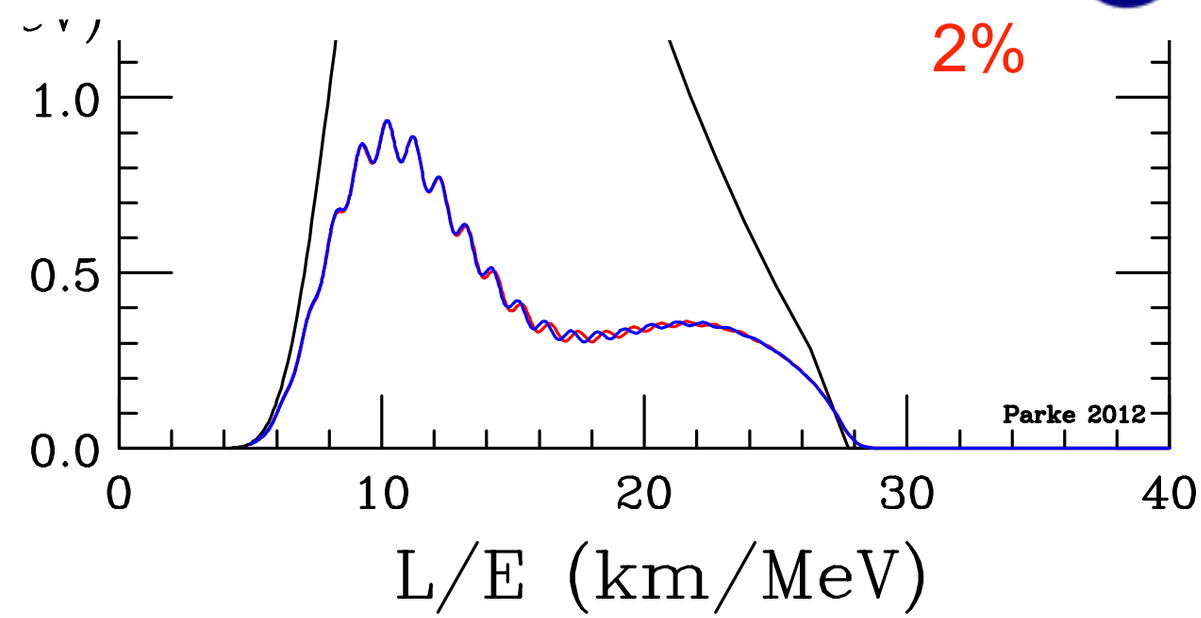


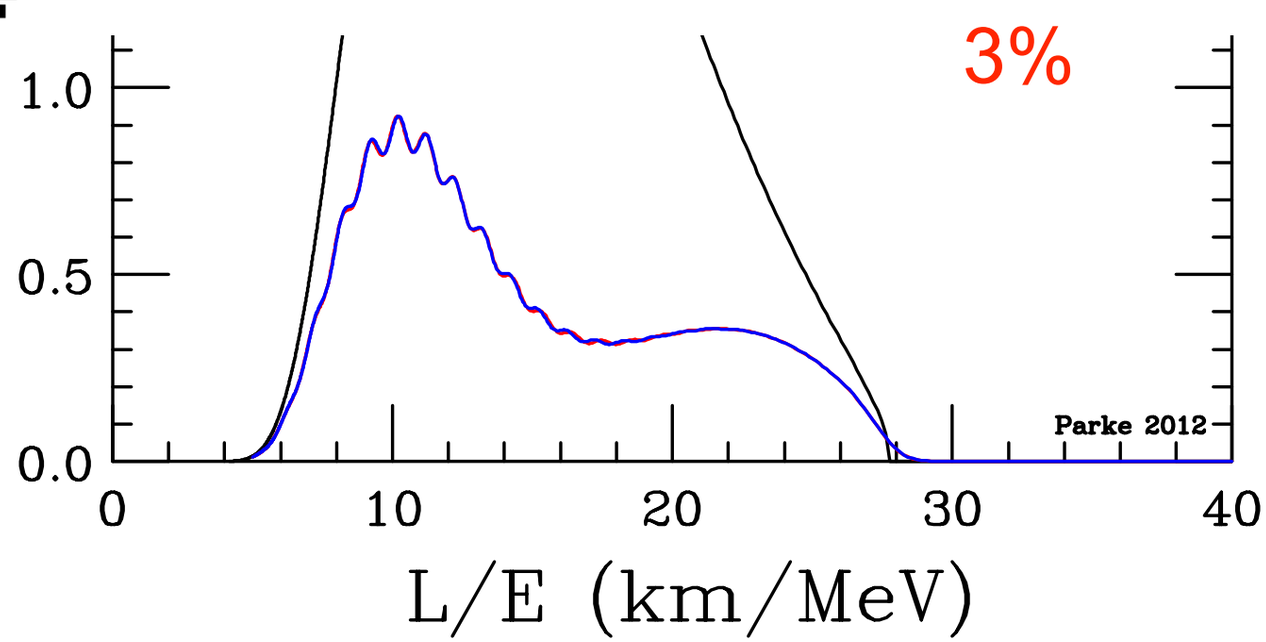


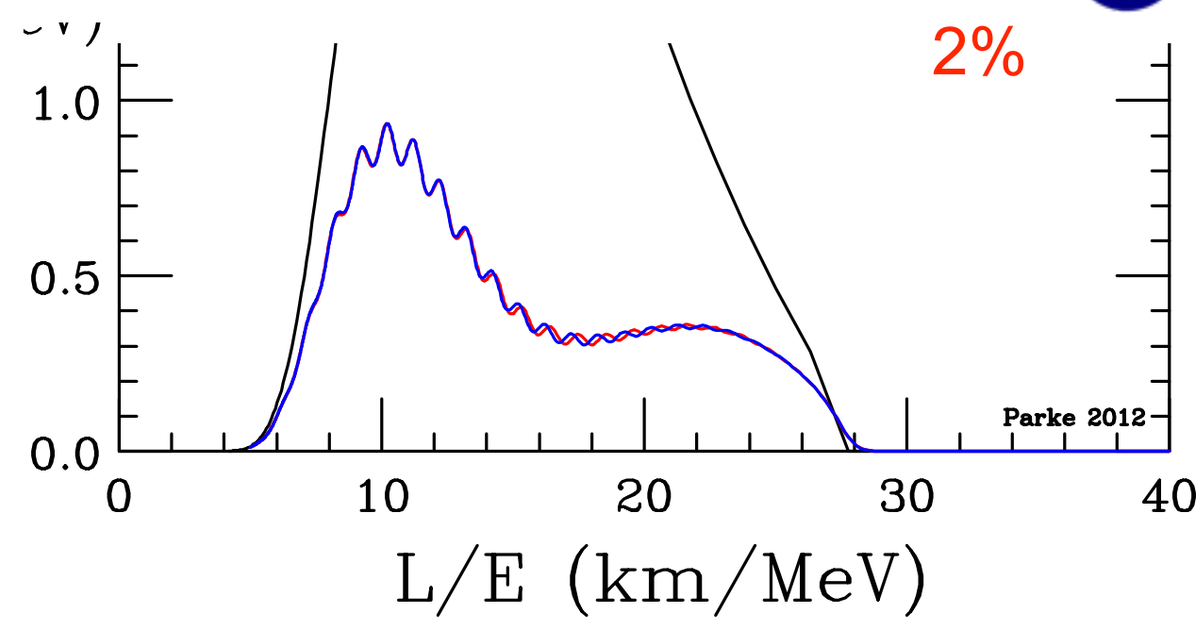



3%
1.0
0.5
0.0
0
10
20
30
40
Parke 2012
L/E (km/MeV)

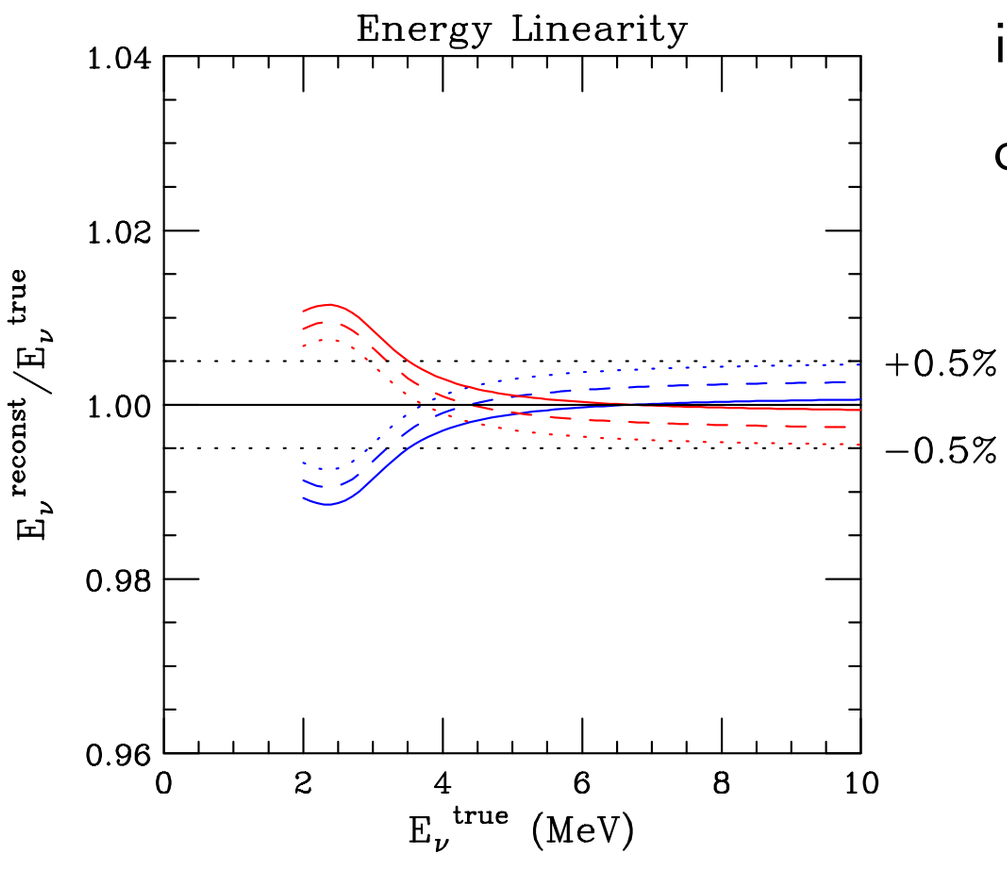

3%
1.0
0.5
0.0
0
10
20
30
40
Parke 2012
L/E (km/MeV)


2%
1.0
0.5
0.0
0
10
20
30
40
Parke 2012
L/E (km/MeV)





Neutrino Energy



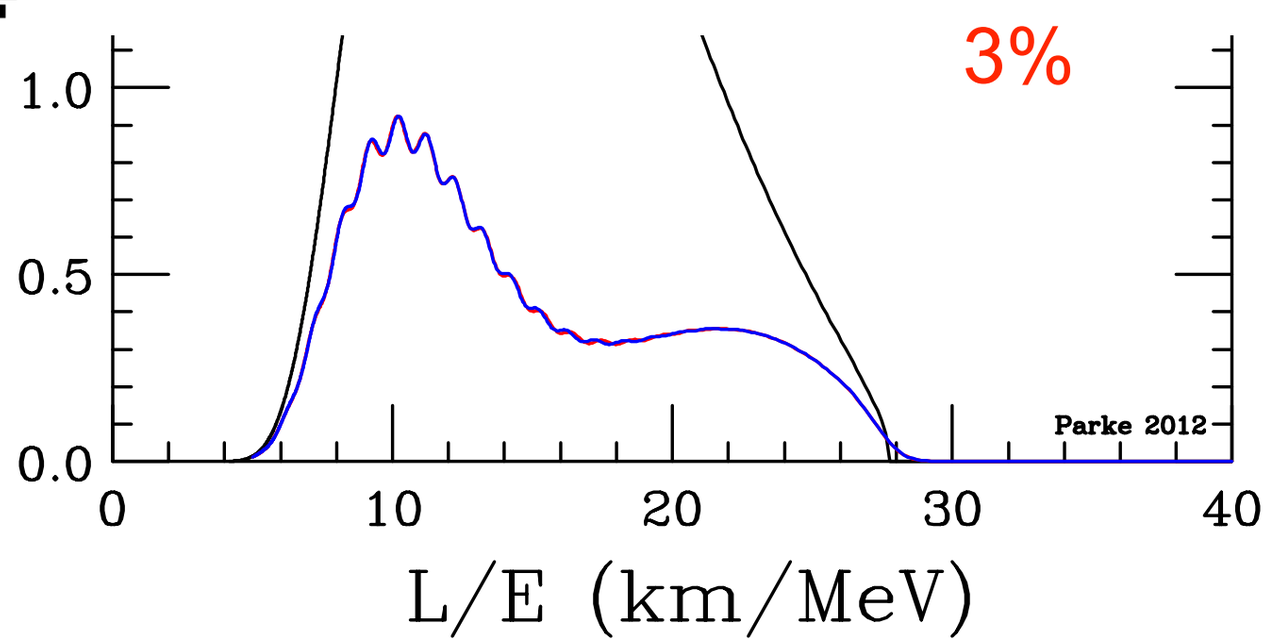
Mass Ordering Indetermined:

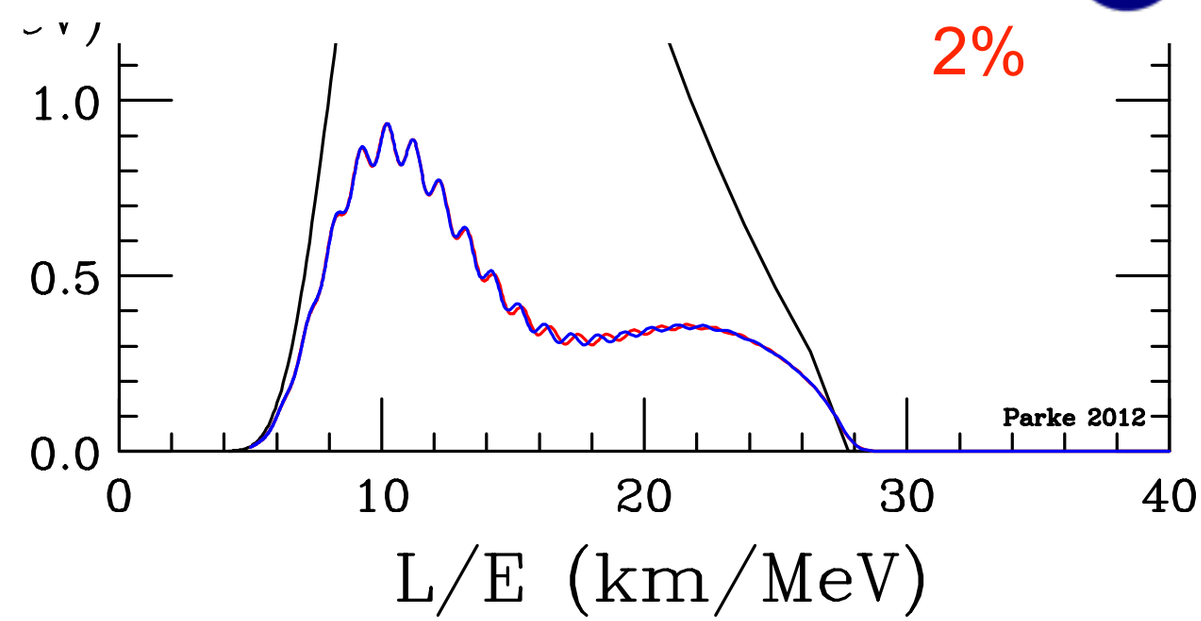
if $2\Delta_{ee} + \phi = 2\Delta'_{ee}$

or $2\Delta_{ee} - \phi = 2\Delta'_{ee}$

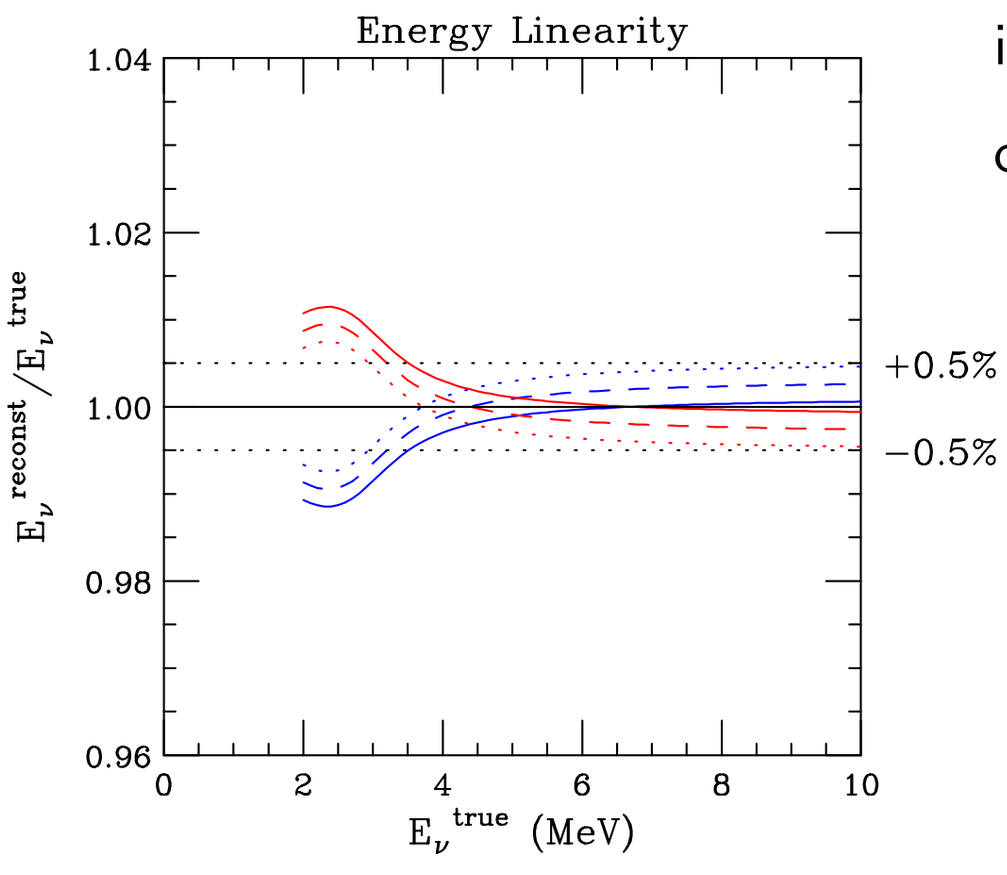
Parke @ NOW 2008

$$\delta\ (\Delta m^2_{ee}) \sim 0.5\%$$





Neutrino Energy



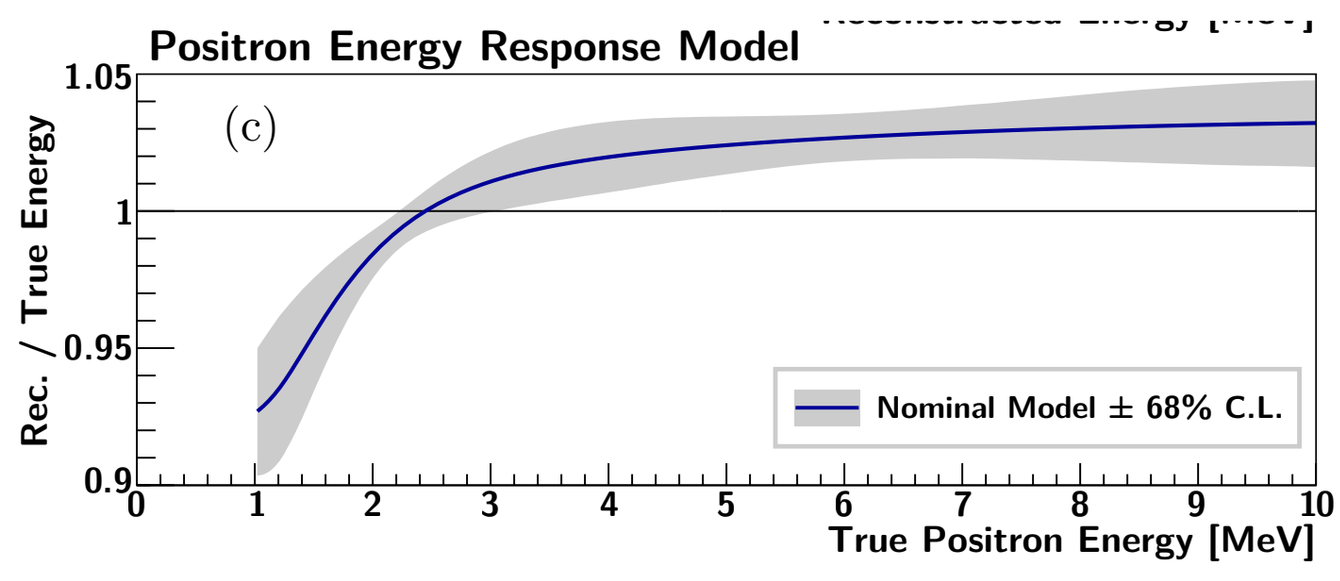
Mass Ordering Indetermined:

Parke @ NOW 2008

if $2\Delta_{ee} + \phi = 2\Delta'_{ee}$

or $2\Delta_{ee} - \phi = 2\Delta'_{ee}$

$$\delta\ (\Delta m^2_{ee}) \sim 0.5\%$$

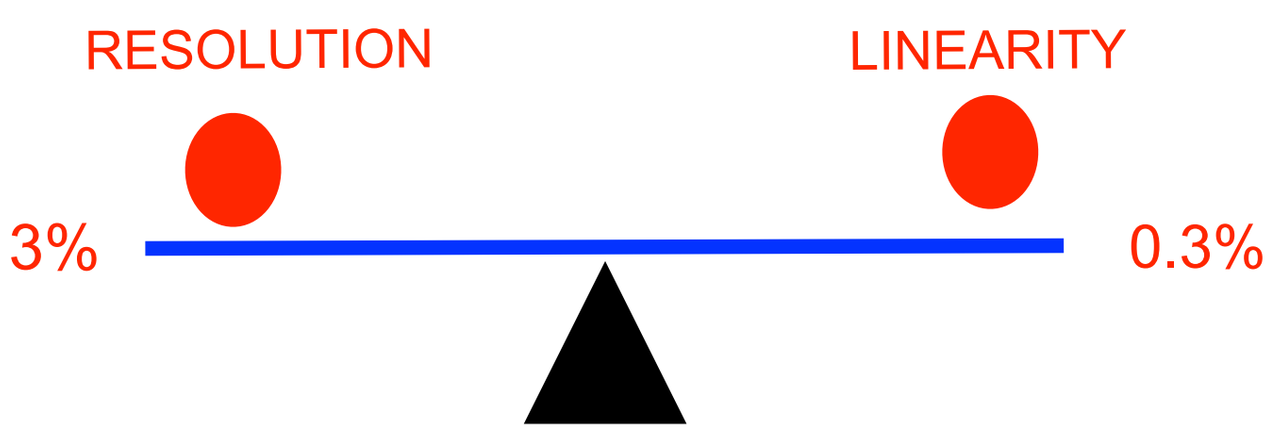


- Energy Resolution to 3% or lower at 1 MeV
- Linearity to sub 1% precision for the reconstructed neutrino energy

	KamLAND	JUNO	RENO-50	
LS mass	~1 kt	20 kt	18 kt	20 x
Energy Resolution	6%/	~3%/	~3%/	
Light yield	250 p.e./MeV	1200 p.e./MeV	>1000 p.e./MeV	4 x
Linearity	1.9%	< 0.5%	< 0.5%	> 4 x

- Energy Resolution to 3% or lower at 1 MeV
- Linearity to sub 1% precision for the reconstructed neutrino energy

	KamLAND	JUNO	RENO-50	
LS mass	~1 kt	20 kt	18 kt	20 x
Energy Resolution	6%/	~3%/	~3%/	
Light yield	250 p.e./MeV	1200 p.e./MeV	>1000 p.e./MeV	4 x
Linearity	1.9%	< 0.5%	< 0.5%	> 4 x



$$P_{ee} = 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21}$$

$$-\frac{1}{2}\sin^2 2\theta_{13}\left[1 - \sqrt{(1-\sin^2 2\theta_{12}\sin^2\Delta_{21})}.\cos(\ 2|\Delta_{ee}|\ \pm\phi\ )\right]$$

$$\text{remember} \quad \phi = \{\arctan(\cos 2\theta_{12}\tan\Delta_{21}) - \Delta_{21}\cos 2\theta_{12}\}$$

$$P_{ee} = 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21}$$

$$-\frac{1}{2}\sin^2 2\theta_{13}\left[1 - \sqrt{(1-\sin^2 2\theta_{12}\sin^2\Delta_{21})}.\cos(\ 2|\Delta_{ee}|\ \pm\phi\ )\right]$$

Daya Bay
Double Chooz
RENO

≤ 1 but $> (1-10^{-3})$

$< 10^{-5}$

may as well use: $-\sin^2 2\theta_{13}\sin^2\Delta_{ee}$

$$\text{remember} \quad \phi = \{\arctan(\cos 2\theta_{12}\tan\Delta_{21}) - \Delta_{21}\cos 2\theta_{12}\}$$

$$P_{ee} = 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21}$$

$$-\frac{1}{2}\sin^2 2\theta_{13}\left[1 - \sqrt{(1-\sin^2 2\theta_{12}\sin^2\Delta_{21})}.\cos(\ 2|\Delta_{ee}|\ \pm\phi\ )\right]$$

Daya Bay
Double Chooz
RENO

≤ 1 but $> (1 - 10^{-3})$

$< 10^{-5}$

may as well use: $-\sin^2 2\theta_{13}\sin^2\Delta_{ee}$

JUNO
RENO50

$\sim \cos 2\theta_{12} \approx 0.4$

$< 1\%$ shift in phase
challenging to observe ! ! !

$$\text{remember} \quad \phi = \{\arctan(\cos 2\theta_{12}\tan\Delta_{21}) - \Delta_{21}\cos 2\theta_{12}\}$$

$$\Delta m^2_{ee} \equiv c^2_{12}\ \Delta m^2_{31} + s^2_{12}\ \Delta m^2_{32}$$

$$\Delta m^2_{ee} \equiv c^2_{12}\ \Delta m^2_{31} + s^2_{12}\ \Delta m^2_{32}$$

- Is a simple combination of fundamental parameters and is independent of L/E for all values of L/E.

$$\Delta m^2_{ee} \equiv c^2_{12}\ \Delta m^2_{31} + s^2_{12}\ \Delta m^2_{32}$$

- Is a simple combination of fundamental parameters and is independent of L/E for all values of L/E.

- Has a direct, simple, physical interpretation:
  Δm^2_{ee} is "the ν_e weighted average of Δm^2_{31} and Δm^2_{32},"
  since the ratio of the ν_e fraction in $\nu_1 : \nu_2$ is $c^2_{12} : s^2_{12}$.

$$\Delta m^2_{ee} \equiv c^2_{12}\ \Delta m^2_{31} + s^2_{12}\ \Delta m^2_{32}$$

- Is a simple combination of fundamental parameters and is independent of L/E for all values of L/E.

- Has a direct, simple, physical interpretation:
  Δm^2_{ee} is "the ν_e weighted average of Δm^2_{31} and Δm^2_{32}," since the ratio of the ν_e fraction in $\nu_1 : \nu_2$ is $c^2_{12} : s^2_{12}$.

- Can be used in short baseline reactor experiments, $L/E < 1$ km/MeV, using the approximate oscillation probability,

  $$P(\bar{\nu}_e \to \bar{\nu}_e) \ \approx\ 1 - 4c^4_{13}s^2_{12}c^2_{12}\sin^2\Delta_{21} - 4s^2_{13}c^2_{13}\sin^2\Delta_{ee},$$

  which is accurate to better than one part in 10^4.

$$\Delta m^2_{ee} \equiv c^2_{12}\ \Delta m^2_{31} + s^2_{12}\ \Delta m^2_{32}$$

- Is a simple combination of fundamental parameters and is independent of L/E for all values of L/E.

- Has a direct, simple, physical interpretation: Δm^2_{ee} is "the ν_e weighted average of Δm^2_{31} and Δm^2_{32}," since the ratio of the ν_e fraction in $\nu_1 : \nu_2$ is $c^2_{12} : s^2_{12}$.

- Can be used in short baseline reactor experiments, $L/E < 1$ km/MeV, using the approximate oscillation probability,

  $$P(\bar{\nu}_e \to \bar{\nu}_e) \ \approx\ 1 - 4c^4_{13}s^2_{12}c^2_{12}\sin^2\Delta_{21} - 4s^2_{13}c^2_{13}\sin^2\Delta_{ee},$$

  which is accurate to better than one part in 10^4.

- Can be used in medium baseline reactor experiments, $L \approx 50$ km, and determines the Δm^2 of the fundamental oscillation. The advancement or retardation of the phase of this fundamental oscillation determines the neutrino mass ordering.

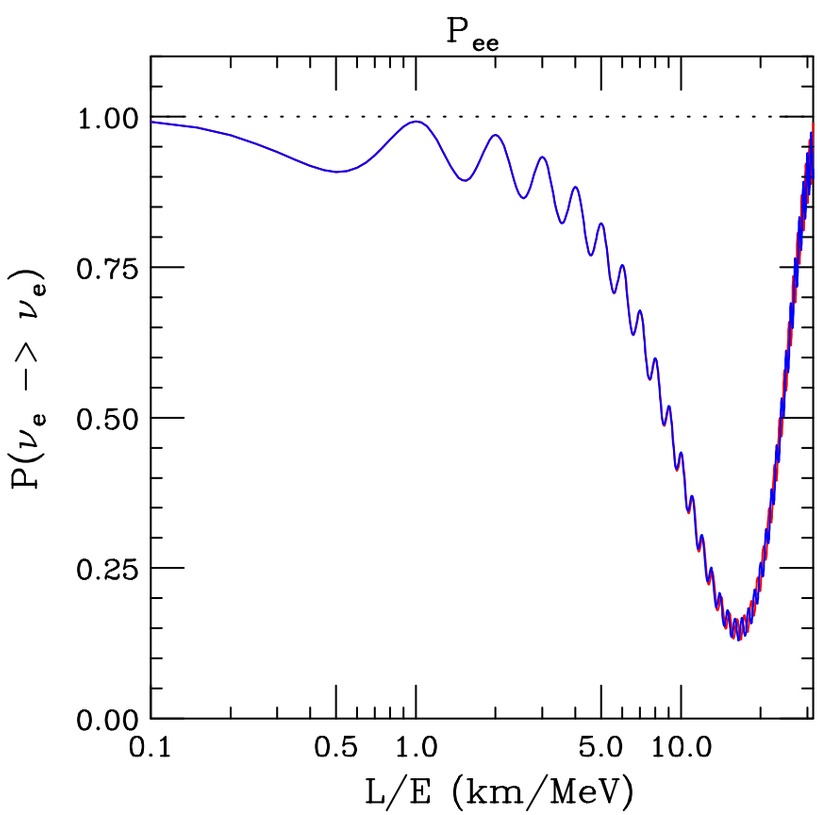
What is δm^2_{ee} ?

$$\delta m^2_{ee} \equiv c^2_{12}\delta m^2_{31} + s^2_{12}\delta m^2_{32}$$

is simpler and has obvious physical meaning:

the ν_e weighted average δm^2_{31} and δm^2_{32}

H. Nunokawa, S. J. Parke and R. Zukanovich Funchal, Phys. Rev. D **72**, 013009 (2005), hep-ph/0503283



Solar parameters: $\theta_{12},\ \Delta m^2_{21} > 0$:

Atmospheric parameters: $\theta_{13},\ \Delta m^2_{ee}$:

(sign of Δm^2_{ee} determines the atmospheric mass ordering)

Additional

arXiv:1310.6732

ν_j and ν_i. Since $\Delta m^2_{21} \ll |\Delta m^2_{31}| \approx |\Delta m^2_{32}|$ [1], the short-distance (~km) reactor $\overline{\nu}_e$ oscillation is due primarily to the Δ_{3i} terms and naturally leads to the definition of the effective mass-squared difference $\sin^2 \Delta_{ee} \equiv \cos^2 \theta_{12} \sin^2 \Delta_{31} + \sin^2 \theta_{12} \sin^2 \Delta_{32}$ [11].

1505.03456v1

[8] $\sin^2 \Delta_{ee} \equiv \cos^2 \theta_{12} \sin^2 \Delta_{31} + \sin^2 \theta_{12} \sin^2 \Delta_{32}$, where $\Delta_{ji} \equiv 1.267 \Delta m^2_{ji}(\mathrm{eV}^2)[L(\mathrm{m})/E(\mathrm{MeV})]$, and Δm^2_{ji} is the difference between the mass-squares of the mass eigenstates ν_j and ν_i.

What is $\Delta m^2_{\mu\mu}$?

"the ν_μ weighted average of Δm^2_{31} and Δm^2_{32}"

$$\Delta m^2_{\mu\mu} \equiv \frac{|U_{\mu 1}|^2}{1-|U_{\mu 3}|^2}\,\Delta m^2_{31} + \frac{|U_{\mu 2}|^2}{1-|U_{\mu 3}|^2}\,\Delta m^2_{32}$$

for vacuum ν_μ-disappearance !

And similarly for ν_τ.