Analytical expressions for the kinetic decoupling of WIMPs

Luca Visinelli
University of Bologna
Physics and Astronomy Department

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Kinetic decoupling of WIMPs: analytic expressions

Luca Visinelli[*] and Paolo Gondolo[†]

Department of Physics and Astronomy, University of Utah,

115 South 1400 East #201, Salt Lake City, Utah 84112-0830, USA

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We present a general expression for the values of the average kinetic energy and of the temperature of kinetic decoupling of a WIMP, valid for any cosmological model. We show an example of the usage of our solution when the Hubble rate has a power-law dependence on temperature, and we show results for the specific cases of kination cosmology and low-temperature reheating cosmology.

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WIMP freeze-out

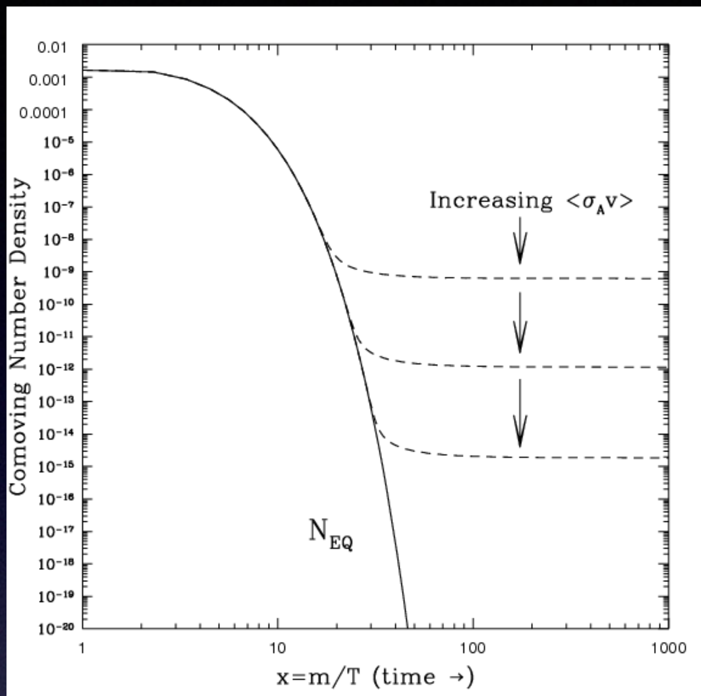


Figure: Kolb & Turner

Regulated by Boltzmann equation

$$\frac{d\,n_\chi}{dt} + 3H\,n_\chi = -\langle \sigma\,v \rangle\,\left(n_\chi^2 - n_{\chi\,\mathrm{eq}}^2\right)$$

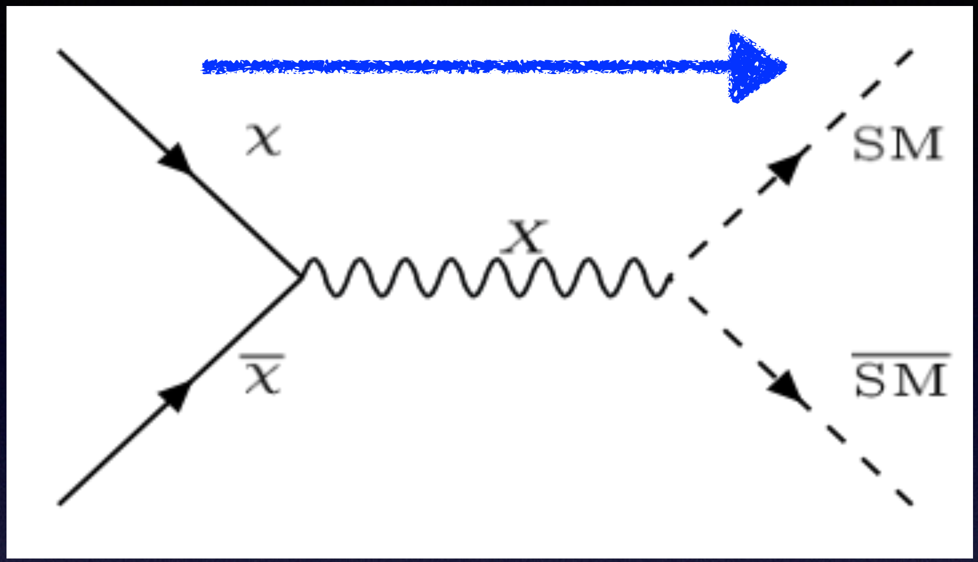
$\langle \sigma\,v \rangle$ thermal average: $\chi\,\chi \to \mathrm{SM}\,\mathrm{SM}$

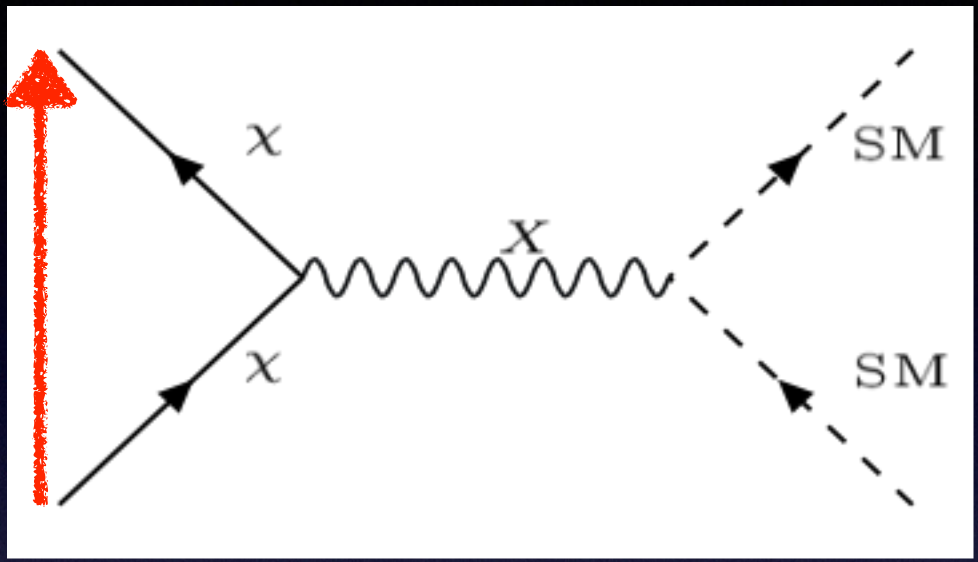
"Late-time" behavior $n_\chi \approx n_{\chi\,\mathrm{eq}}$, so $a^3\,n_\chi \approx \mathrm{constant}$

Present DM density $\Omega_\chi\,h^2 \approx \dfrac{3 \times 10^{-27}\,\mathrm{cm}^3/\mathrm{s}}{\langle \sigma\,v \rangle}$

The "WIMP miracle": weak interactions $\Omega_\chi\,h^2 = O\left(10^{-1}\right)$

Freeze-out vs. kinetic decoupling





WIMP annihilation
into SM particles

- Annihilation rate $\Gamma_{\rm ann}$
- Changes n_χ
- Process ends at

$$\Gamma_{\rm ann} \approx H(T_{\rm freeze})$$

with $T_{\rm freeze} \approx M_\chi/20$

Scattering of
WIMP and SM particles

- Momentum exchange rate $\Gamma_{\rm exc}$
- Does not change n_χ
- Process ends at

$$\Gamma_{\rm exc} \approx H(T_{\rm kd})$$

with $T_{\rm kd} \ll T_{\rm freeze}$

Temperature of kinetic decoupling

- T_{kd} is a key parameter in cosmological models.
- determines the cutoff in the density spectrum
- links the size of the smallest halos with WIMP nature

The mass of the smallest halo M_{cut} is the largest between:

- from WIMP free-streaming $M_{\mathrm{fs}} = \frac{4\pi}{3}\,\rho_\chi\,\left(\frac{\pi}{k_{\mathrm{fs}}}\right)^3$
- from acoustic oscillations $M_{\mathrm{ao}} = \frac{4\pi}{3}\,\rho_\chi\,\left(\frac{1}{H(T_{\mathrm{kd}})}\right)^3$

More on kinetic decoupling

The scattering of WIMPs of mass M_χ off a plasma at temperature $T \ll M_\chi$ is a Brownian motion (random walk)

$$\sqrt{N_e}\,\Delta p = p \approx \sqrt{M_\chi T}$$

- p : WIMP momentum
- Δp: WIMP momentum spread
- N_e : number of collisions required to change the momentum by p .

Fokker-Planck equation

The WIMP occupation number $f_\chi = f_\chi(p)$ follows a Fokker-Planck (FP) equation

$$\frac{\partial f_\chi}{\partial t} - H(T)\,\mathbf{p}\cdot\frac{\partial f_\chi}{\partial \mathbf{p}} = \gamma(T)\,\frac{\partial}{\partial \mathbf{p}}\cdot\left[\mathbf{p}\, f_\chi\,(1+f_\chi) + M_\chi\, T\,\frac{\partial f_\chi}{\partial \mathbf{p}}\right]$$

Hubble Rate

Relaxation Rate

Statistics

We solve FP for any $H(T)$ and $\gamma(T)$, assuming $f_\chi \ll 1$

For $f_\chi \ll 1$, we rewrite FP as the Master Equation

$$\boxed{a\,\frac{dT_\chi}{da} + 2\left[1+\Upsilon(T)\right]T_\chi = 2\Upsilon(T)\,T}$$

where we defined the kinetic temperature

$$T_\chi = \frac{2}{3}\int \frac{\mathbf{p}_\chi^2}{2M_\chi}\, f_\chi(\mathbf{p}_\chi)\, d^3\mathbf{p}_\chi$$

WIMP scattering off the plasma is regulated by the ratio

$$\Upsilon(T) = \frac{\gamma(T)}{H(T)}$$

The general solution to the Master Equation is the sum of a homogeneous and an inhomogeneous part

$$T_\chi(a) = \boxed{T_i \left(\frac{a_i}{a}\right)^2 e^{-G(a,a_i)} +}$$

$$\boxed{+\frac{2}{a^2}\int_{a_i}^{a} e^{-G(a,a')}\,\Upsilon(a')\,T(a')\,a'\,da'}$$

where we introduced $T_i = T_\chi(a_i)$ and

$$G(a,a') = 2\int_{a'}^{a}\Upsilon(a'')\,\frac{da''}{a''} \quad \text{or} \quad G(t,t') = 2\int_{t'}^{t}\gamma(t'')\,dt''$$

Limits at early- and late-time

- $\gamma(T) \gg H(T)$ or $\Upsilon(T) \gg 1$:

WIMPs are tightly coupled to the plasma

$$a\,T_\chi = \mathrm{constant} \quad \text{and} \quad T_\chi \approx T$$

- $\gamma(T) \ll H(T)$ or $\Upsilon(T) \ll 1$:

WIMPs decouple from the plasma

$$a^2\,T_\chi = \mathrm{constant} \quad \text{and} \quad T_\chi \propto \frac{T^2}{T_i}$$

Applications: power-law cosmology (I)

We choose the parametrization for the cosmology:

$$H(T) = H_i \left(\frac{T}{T_i}\right)^{\nu}$$

$$a^{\alpha}\, T = \mathrm{constant}$$

We also set $\gamma(T) = \gamma_i \left(\frac{T}{T_i}\right)^{4+n}$ and $\Upsilon_i = \frac{\gamma_i}{H_i}$

Applications: power-law cosmology (2)

The solution to the Master Equation is

$$T_\chi = T\, s^\lambda\, e^s\, \left[\Gamma(1-\lambda, s) + \lambda\, \Gamma(-\lambda, s_i)\right]$$

with $s = \dfrac{2\Upsilon}{\alpha\,(4+n-\nu)} = \dfrac{2}{\alpha\,(4+n-\nu)}\,\Upsilon_i\,\left(\dfrac{T}{T_i}\right)^{4+n-\nu}$

If WIMPs are initially tightly coupled to the plasma,

$$T_\chi = T\, s^\lambda\, e^s\, \Gamma\,(1-\lambda, s) \qquad (s_i \to +\infty)$$

For $\alpha = 1, \nu = 2$ (RD), and for $n = 2$ (p-wave)

$$T_\chi = T\, s^{1/4}\, e^s\, \Gamma\left(\frac{3}{4}, s\right)$$ (Berschinger, PRD 74 2006)

Temperature of kinetic decoupling (I)

We define $\gamma(T_{\rm kd}) = H(T_{\rm kd})$

In power-law models $T_{\rm kd} = T_i \, \Upsilon_i^{-\frac{1}{4+n-\nu}}$

Standard RD: $T_{\rm kd,std} = T_i \left(\frac{H(T_i)}{\gamma_i}\right)^{\frac{1}{2+n}}$

Temperature of kinetic decoupling (II)

The late-time behavior of T_χ during RD is

$$T_\chi = \frac{T^2}{T_i}\left(\frac{2\Upsilon_i}{2+n}\right)^{\frac{1}{2+n}} \Gamma\left(\frac{1+n}{2+n}\right)$$

Compare with (Bringmann and Hofmann, JCAP 0407, 016 2007)

$$T_\chi = \frac{T^2}{T_{\rm kd,std}}\left(\frac{2}{2+n}\right)^{\frac{1}{2+n}} \Gamma\left(\frac{1+n}{2+n}\right)$$

gives $T_{\rm kd} = \left(\frac{T_{\rm kd,std}^{n+2}}{T_i^{\nu-2}}\right)^{\frac{1}{4+n-\nu}}$

$$T_{\rm kd} = T_{\rm kd,std} \text{ for } \nu = 2 \text{ (RD)}$$

$= \frac{T_{\rm kd,std}^2}{T_{\rm RH}}$ for $\nu = 4, n = 2$ Low − Temperature Reheating model

Gelmini and Gondolo, JCAP 0810, 002 2008

Summary and conclusions

- T_{kd} is a key parameter in cosmological models, since it sets the size of the smallest protohalos.
- T_{kd} is related to the kinetic temperature T_χ through the Fokker-Planck equation;
- The Fokker-Planck equation is solved in terms of the kinetic temperature T_χ;
- The solution is in terms of T , and can be implemented in numerical models like DarkSUSY;
- In power-law models (RD, LTR, Kination), generic expression for T_χ and T_{kd} are obtained.