Thermal Axion Production in the Primordial Quark-Gluon Plasma

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Abstract

If the strong CP problem is solved via the Peccei-Quinn mechanism, axions will pervade the Universe as an extremely weakly interacting light particle species. We calculate the rate for thermal production of axions via scattering of quarks and gluons in the primordial plasma. To obtain a finite result in a gauge-invariant way that is consistent to leading order in the strong gauge coupling, we use systematic field theoretical methods such as hard thermal loop resummation and the Braaten–Yuan prescription. The thermally produced yield and the density parameter are computed for axions with a mass below 10 meV. In this regime, with a Peccei–Quinn scale above $6\times10^{8}\,\mathrm{GeV}$, the associated axion population can still be relativistic today and can coexist with the axion cold dark matter condensate.

Introduction

- The unmotivated smallness of CP violation in QCD is the strong CP problem.
- The axion provides the most promising solution.
- The basic features of the axion are its mass (see, e.g. [1])

$$m_a = 0.6\,\mathrm{meV}\left(\frac{10^{10}\,\mathrm{GeV}}{f_{\mathrm{PQ}}}\right)$$

and its interactions with gluons

$$\mathcal{L}_a = \frac{g_s}{32\pi^2}\frac{1}{f_{\mathrm{PQ}}} a G^a_{\mu\nu}\widetilde{G}^{a\,\mu\nu},$$

where g_s is the strong coupling constant and $G^a_{\mu\nu}$ the gluon field strength tensor with its dual $\widetilde{G}^a_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma}G^{a\,\rho\sigma}/2$. All other interactions are model dependent. Current laboratory, astrophysical, and cosmological limits require [2]

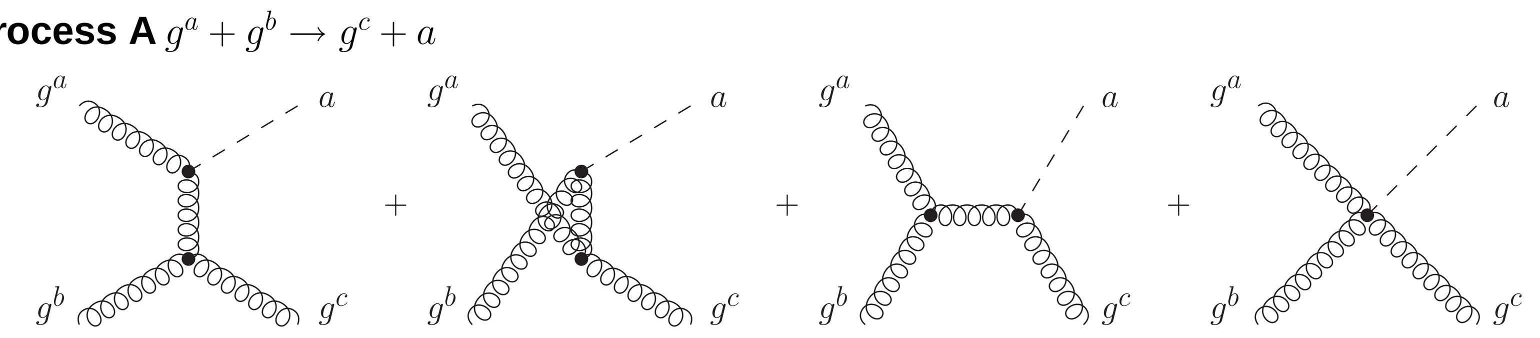
$$f_{\mathrm{PQ}} \gtrsim 6\times10^{8}\,\mathrm{GeV}.$$

- Axions can be produced via scattering processes in the early Universe and would still be around today for this f_{PQ} range.
- We calculate the thermal production rate of axions in the early Universe. Here we use Hard Thermal Loop resummation techniques [3] and the Braaten–Yuan prescription [4] to account for possible IR divergences in a gauge-invariant way. The result is consistent to leading order in the coupling constant.

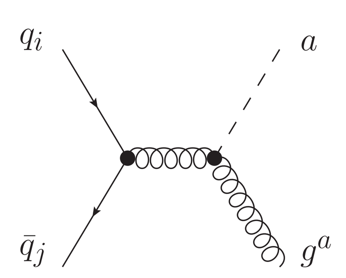
Thermal Axion Production

Axions are produced thermally in the quark-gluon plasma via these three $2\to2$ processes:

Process A $g^a + g^b \to g^c + a$



Process B $q_i + \bar{q}_j \to g^a + a$



Process C $q_i + g^a \to q_j + a$ (crossing of B)

The resulting squared matrix elements depend on the mandelstam variables s and t:

Label i	Process i	$\lvert M_i\rvert^2/\left(\frac{g_s^6}{128\pi^4 f_{\mathrm{PQ}}^2}\right)$
A	$g^a + g^b \to g^c + a$	$-4\frac{(s^2+st+t^2)^2}{st(s+t)}\lvert f^{abc}\rvert^2$
B	$q_i + \bar{q}_j \to g^a + a$	$\left(\frac{2t^2}{s} + 2t + s\right)\lvert T^a_{ji}\rvert^2$
C	$q_i + g^a \to q_j + a$	$\left(-\frac{2s^2}{t} - 2s - t\right)\lvert T^a_{ji}\rvert^2$

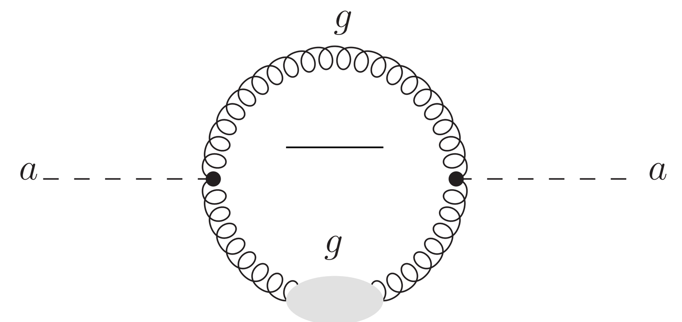
These matrix elements are potentially infrared divergent for $t\to0$. Here screening effects of the plasma become relevant. One could model the screening by introducing a finite gluon mass, but this spoils gauge invariance. In contrast, we employ the HTL resummation technique and the Braaten-Yuan prescription which allow for a gauge-invariant treatment.
First we introduce a momentum scale k_{cut} such that $g_s T \ll k_{\mathrm{cut}} \ll T$ in the weak coupling limit $g_s \ll 1$. This allows us to split the axion production rate W_a into one part for processes with soft momentum transfer ($k\propto g_s T$) and one with hard momentum transfer ($k\propto T$):

$$E\frac{dW_a}{d^3p} = E\frac{dW_a}{d^3p}\bigg|_{\mathrm{soft}} + E\frac{dW_a}{d^3p}\bigg|_{\mathrm{hard}},$$

In the soft momentum regime, we use the imaginary part of the axion self energy

$$E\frac{dW_a}{d^3p}\bigg|_{\mathrm{soft}} = -\frac{f_{\mathrm{B}}(E)}{(2\pi)^3}\,\mathrm{Im}\,\Pi_a(E+i\epsilon,\vec{p})|_{k<k_{\mathrm{cut}}}$$

where we use a HTL resummed propagator for the soft gluon:



The hard part is obtained using the above matrix elements, so with bare gluon propagators, weighted with the respective phase-space densities:

$$E\frac{dW_a}{d^3p}\bigg|_{\mathrm{hard}} = \frac{1}{2(2\pi)^3}\int\frac{d\Omega_p}{4\pi}\int\left[\prod_{j=1}^{3}\frac{d^3p_j}{(2\pi)^3 2E_j}\right] \times(2\pi)^4\delta^4(P_1+P_2-P_3-P)\Theta(k-k_{\mathrm{cut}}) \times\sum f_1(E_1)f_2(E_2)[1\pm f_3(E_3)]\lvert M_{1+2\to3+a}\rvert^2.$$

With any initial axion population diluted away by inflation and T well below the axion decoupling temperature T_{D} so that axions are not in thermal equilibrium, the axion phase space density f_a is negligible in comparison to the equilibrium distributions f_i of the other particles. Thereby, axion disappearance reactions ($\propto f_a$) are neglected as well as the respective Bose enhancement ($1+f_a\approx1$).

Axion Density Parameter

For T sufficiently below T_{D}, the evolution of the thermally produced axion number density n_a with cosmic time t is governed by the Boltzmann equation

$$\frac{dn_a}{dt} + 3Hn_a = \int d^3p\frac{dW_a}{d^3p} = W_a.$$

Here H is the Hubble expansion rate, and the collision term is the integrated thermal production rate

$$W_a = \frac{\zeta(3)g_s^6T^6}{64\pi^7 f_{\mathrm{PQ}}^2}\left[\ln\left(\frac{T^2}{m_g^2}\right) + 0.406\right].$$

Assuming conservation of entropy per comoving volume element, this can be rewritten in terms of the axion yield $Y_a^{\mathrm{TP}} = n_a/s$. With initial temperature taken to be the reheating temperature T_{R} at which $Y_a^{\mathrm{TP}}(T_{\mathrm{R}}) = 0$, the relic axion yield today is given by

$$Y_a^{\mathrm{TP}} \approx Y_a^{\mathrm{TP}}(T_{\mathrm{mat=rad}}) = \int_{T_{\mathrm{mat=rad}}}^{T_{\mathrm{R}}} dT\frac{W_a(T)}{Ts(T)H(T)} = 18.6\times g_s^6\ln\left(\frac{1.501}{g_s}\right)\left(\frac{10^{10}\,\mathrm{GeV}}{f_{\mathrm{PQ}}}\right)^2\left(\frac{T_{\mathrm{R}}}{10^{10}\,\mathrm{GeV}}\right).$$

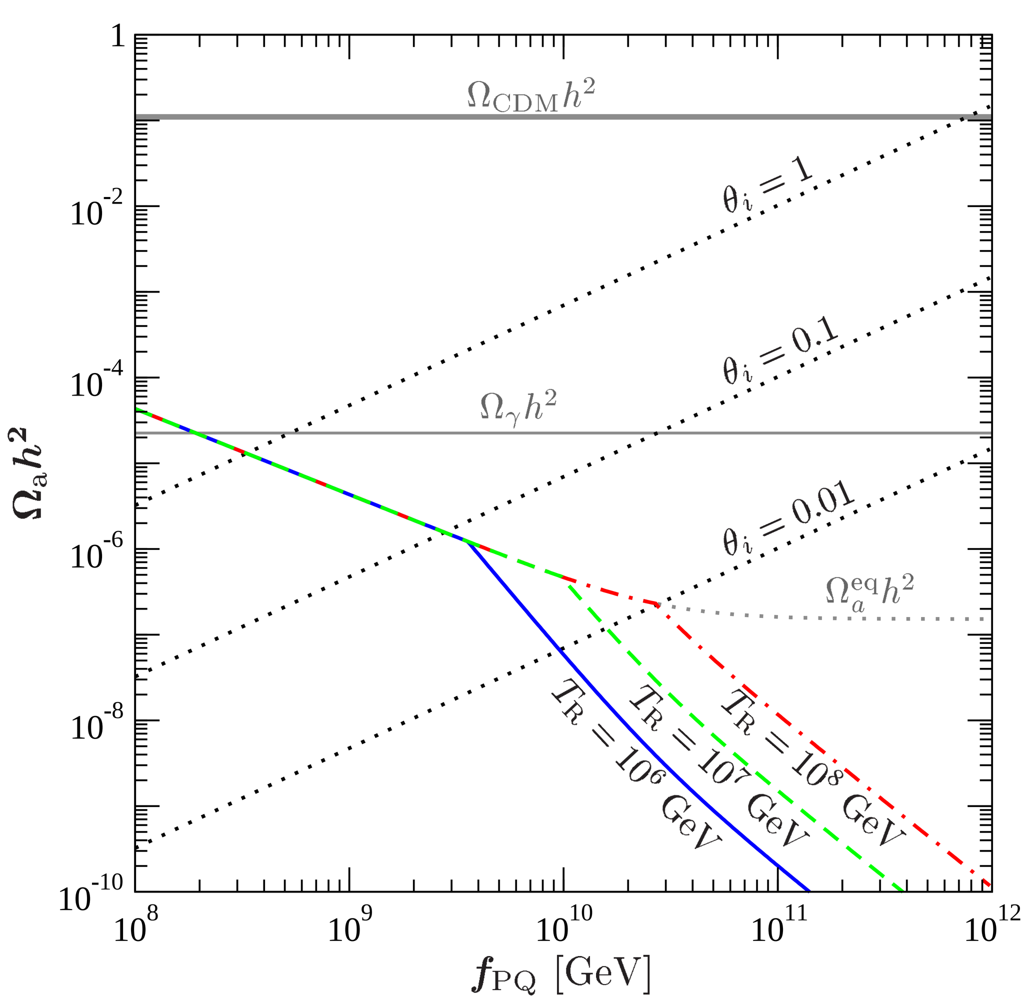
This result describes the axion production for cosmological scenarios where the axion has never been in thermal equilibrium, i.e. for $T_{\mathrm{R}} \ll T_{\mathrm{D}}$.
If there was an early period where axions were in thermal equilibrium and then decoupled from the plasma, their yield today is given by

$$Y_a^{\mathrm{eq}} = \frac{n_a^{\mathrm{eq}}}{s} \approx 2.6\times10^{-3}.$$

The decoupling temperature is then approximately the value of T_{R} where $Y_a^{\mathrm{TP}} \approx Y_a^{\mathrm{eq}}$.
Since also thermally produced axions have basically a thermal spectrum, the density parameter from thermal processes can be described approximately by

$$\Omega_a^{\mathrm{TP/eq}}h^2 \simeq \sqrt{\langle p_{a,0}\rangle^2 + m_a^2}\,\frac{Y_a^{\mathrm{TP/eq}}s(T_0)h^2}{\rho_c}$$

with present average momentum $\langle p_{a,0}\rangle = 2.701\,T_{a,0}$ and temperature $T_{a,0} = 0.332\,T_0 \simeq 0.08\,\mathrm{meV}$. A comparison of $T_{a,0}$ with the axion mass shows that this axion population is still relativistic today for $f_{\mathrm{PQ}} \gtrsim 10^{11}\,\mathrm{GeV}$.



In this figure we show the energy density parameter of thermally produced axions as a function of f_{PQ} for different values of the reheating temperature T_{R}. We also show the density parameter of axions that where in thermal equilibrium in the early Universe. For comparison, the energy density of axions from the misalignment mechanism is shown. For larger values of f_{PQ}, this axion population can account for the cold dark matter in the Universe.

References

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