Searches for gravitational wave signals form rotating neutron stars

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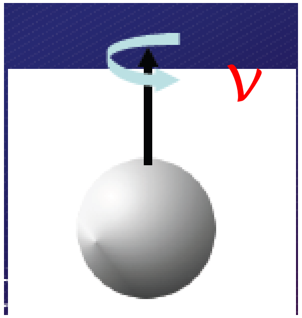
Plan of the talk

- Mechanisms of gravitational wave radiation from rotating neutron stars
- Data analysis methods
- Recent results of searches in the LIGO and VIRGO data

Continuous gravitational waves from neutron stars



- Non-axisymmetric distortions:

$$h_0 = \frac{16\pi^2 G}{c^4} \frac{I_{zz}\nu^2}{d} \epsilon$$

I_{zz} - moment of inertia
$\varepsilon = (I_{xx} - I_{yy})/I_{zz}$ - ellipticity
d - distance

Breaking strain

a) supported by elastic stresses

$$\epsilon_{\rm max} \approx 5 \times 10^{-7} \left(\frac{\sigma}{10^{-2}}\right)$$

normal matter

$$\epsilon_{\rm max} \approx 4 \times 10^{-4} \,(\sigma/10^{-2})$$

Strange-quark matter

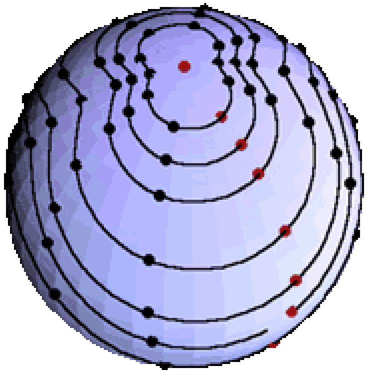
a) due to magnetic field ($\varepsilon \leq 10^{-6}$)
b) due to accretion from a companion ($\varepsilon \leq 10^{-6}$)

GW frequencies: $f_{GW} = 2\nu$ and $f_{GW} = \nu$

- Non-axisymmetric instabilities

r-mode instability

GW frequency: $f_{GW} = 4/3\ \nu$

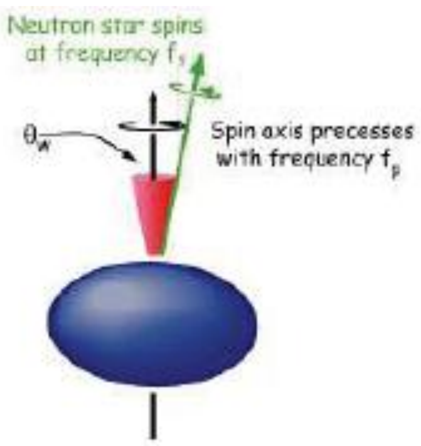


Courtesy: B. J. Owen

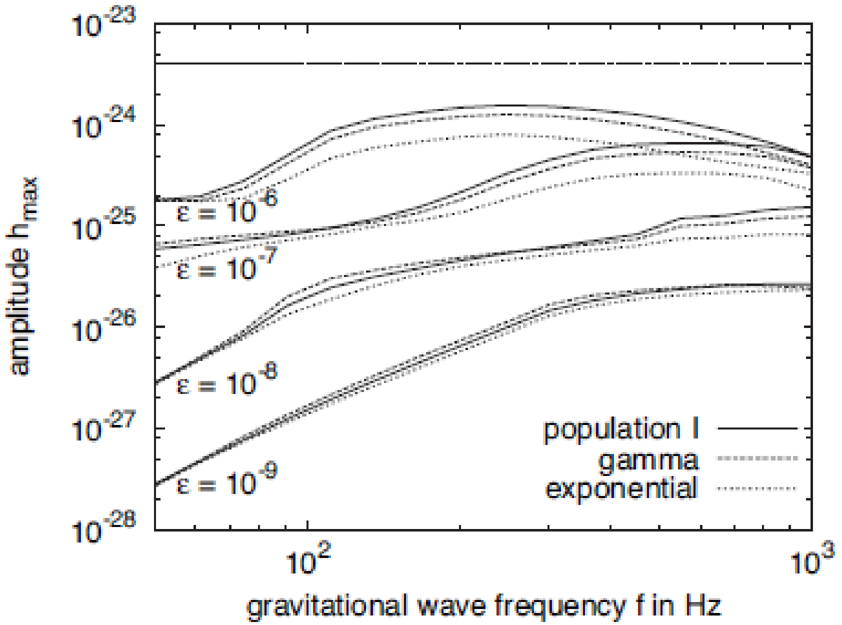
- Free precession

GW frequencies:

$f_{GW} = 2\,(\nu\ +/-\ \nu_{prec})$ and
$f_{GW} = \nu\ +/- \nu_{prec}$



- Loudest expected signal from unknown isolated neutron stars (Allen & Knispel 2008):



- The spin down limit for known pulsars

$$h_0 \leq h_{\rm sd} = \frac{1}{d}\sqrt{\frac{5G\,I_{zz}}{2c^3}\,\frac{|\dot{\nu}|}{\nu}}$$

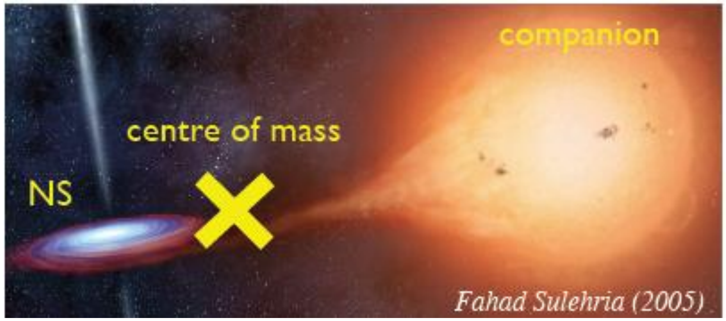
$$= 8\times 10^{-24}\sqrt{\left(\frac{I}{10^{38}\,kg\,m^2}\right)\left(\frac{|\dot{\nu}|}{10^{-10}\,Hz/s}\right)\left(\frac{100Hz}{\nu}\right)}\left(\frac{d}{100pc}\right)^{-1}$$

If a star's age τ is known but its spin is unknown, one can define an indirect spindown limit by assuming pulsar was losing its energy through gravitational radiation:

$$\tau = \frac{\nu}{4|\dot{\nu}|}$$

$$h_{isd} = 2\times10^{-23}\sqrt{\left(\frac{I}{10^{38}\,kg\,m^2}\right)\left(\frac{1000yr}{\tau}\right)}\left(\frac{d}{100pc}\right)^{-1}$$

Maximum expected signal from accreting neutron stars, e.g. Scorpius X-1 (another indirect limit)



$$h_0 \approx 5\times10^{-27}\left(\frac{300\,\mathrm{Hz}}{\nu}\right)^{1/2}\left(\frac{F_\mathrm{x}}{10^{-8}\,\mathrm{erg\,cm^{-2}\,s^{-1}}}\right)^{1/2}$$

Spin down limits on ellipticity

Spin down limit

$$\varepsilon_{sd} = 2\times10^{-5}\sqrt{\left(\frac{10^{38}\,kg\,m^2}{I}\right)\left(\frac{100Hz}{\nu}\right)^5\left(\frac{|\dot{\nu}|}{10^{-10}\,Hz/s}\right)}$$

Indirect spin down limit

$$\varepsilon_{isd} = 5\times10^{-5}\sqrt{\left(\frac{10^{38}\,kg\,m^2}{I}\right)\left(\frac{100Hz}{\nu}\right)^4\left(\frac{1000yr}{\tau}\right)}$$

Response of the detector to a continuous gravitational wave

$$s(t) = \sum_{k=1}^{4} A_k \, h_k(t)$$

$$h_1(t) = a(t;\delta,\alpha)\cos\phi(t;\mathrm{f},\delta,\alpha),$$

$$h_2(t) = b(t;\delta,\alpha)\cos\phi(t;\mathrm{f},\delta,\alpha),$$

$$h_3(t) = a(t;\delta,\alpha)\sin\phi(t;\mathrm{f},\delta,\alpha),$$

$$h_4(t) = b(t;\delta,\alpha)\sin\phi(t;\mathrm{f},\delta,\alpha),$$

frequency and s spin down parameters

$$\mathrm{f} = (f_o, f_1, \ldots, f_s)$$

Detector ephemeris

$$\phi(t) \simeq 2\pi \sum_{k=0}^{s} f_k \frac{t^{k+1}}{(k+1)!} + \frac{2\pi}{c}\mathbf{n}_0 \cdot \mathbf{r}_\mathrm{d}(t) \sum_{k=0}^{s} f_k \frac{t^k}{k!};$$

Source position

$A_k = A_k(h_o,\phi_o,\psi,\iota) \sim h_o$, **an exceedingly weak signal**:

$$h_o = 4\times 10^{-25}\left(\frac{\varepsilon}{10^{-6}}\right)\left(\frac{I}{10^{38}\,kg\,m^2}\right)\left(\frac{\nu}{100Hz}\right)^2\left(\frac{100pc}{d}\right)$$

Data analysis of continuous gravitational waves

Likelihood function: $\Lambda = \frac{p_\theta(x)}{p(x)}$

Maxiumum likelihood method : $\frac{\partial \Lambda}{\partial \theta} = 0$ → $\mathcal{F}$-statistic

Depends only on a subset of parameters called the intrinsic parameters

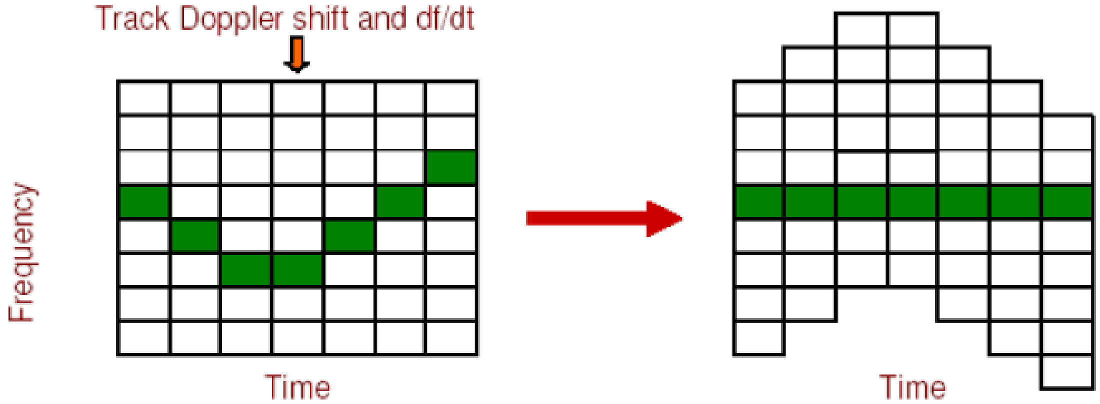
$$\boldsymbol{\xi} = (\mathbf{f}, \delta, \alpha)$$

$$\mathcal{F}[x;\boldsymbol{\xi}] \cong \frac{2}{S_0 T_o} \frac{B|F_a|^2 + A|F_b|^2 - 2C\Re(F_a F_b^*)}{D}$$

$$F_a := \int_0^{T_o} x(t)a(t)\exp[-\mathrm{i}\phi(t)]\,\mathrm{d}t$$

$$F_b := \int_0^{T_o} x(t)b(t)\exp[-\mathrm{i}\phi(t)]\,\mathrm{d}t$$

Semi-coherent methods



The dark pixels represent a signal in the data. Its frequency changes with time due to Doppler shift and intrinsic evolution of the source. By appropriately sliding the frequency bins of successive coherent stacks the power of the signal can be enhanced.

1. Stack slide method - add powers
2. Power flux method - weighted power (non-stationarities, antenna pattern)
1. Hough transform - binary number counts

Computing cost: $C_{coh} \sim \mathbf{T^{6+}_{obs} f^2}$ vs. $C_{semicoh} \sim \mathbf{T^{2}_{obs} T^{4}_{coh} f^2}$

Hierarchical methods

Neither the matched-filtering nor the semi-coherent methods described optimize the sensitivity of a wide-parameter search given finite computing power. The sensitivity can be improved by combining several stages of coherent and semi-coherent steps.

Usually there is **a first step** where a **coherent search** is used with observation time of the order of ***days*** followed by a **second step** of **semi-coherent analysis** for observation time of the order of ***months.***

> Optimization subject to intensive research: Brady and Creighton (2000), Frasca et al. (2005), Cutler et al. (2005), Allen and Pletsch (2009), Pletsch (2010,2011)
>
> Very promising and already implemented in the real search is optimization by Allen and Pletsch that exploits large scale parameter-space correlation in the coherent detection statistic.

Bayesian analysis: posterior probability

$$p(\theta \mid x) = \frac{\pi(\theta)\, p_\theta(x)}{p(x)}$$

Bayes theorem

Posterior probability of GW amplitude h_o:

Nuisance parameters:
phase ϕ, polarization ψ, inlination ι,
Variance of the noise σ

$$p(h_o \mid x) = \frac{\int \pi(\theta)\, p_\theta(x)\, d\phi\, d\psi\, d\cos\iota\, d\sigma^2}{p(x)}$$

Marginalization

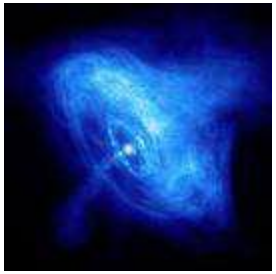
Unknown noise

Application of Bayesian methodology to targeted searches **(Dupuis and Woan, Phys. Rev. D72, 102002, 2005)**

Search for continuous gravitational wave signals

- Targeted searches- known pulsars (frequency, sky position, sometimes polarization known)
- All-sky (wide-parameter) searches-unknown sky position and frequency evolution
- Directed searches - known position, unknown frequency

ν ~30.2Hz



Targeted searches

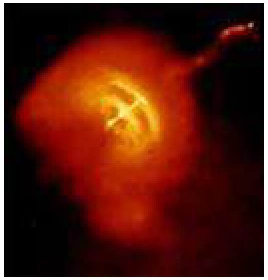
Spin-down limit beaten for two pulsars so far:

Crab (ApJ, 713, 671, 2010), LIGO S5 data, Bayesian analysis

Epoch	$h_0^{95\%}$		Ellipticity		$h_0^{95\%} / h_0^{sd}$	
	Uniform	Restricted[a]	Uniform	Restricted[a]	Uniform	Restricted[a]
			Crab pulsar			
Model (1)[b]	2.6×10^{-25}	2.0×10^{-25}	1.4×10^{-4}	1.1×10^{-4}	0.18	0.14
Model (2)[c]	2.4×10^{-25}	1.9×10^{-25}	1.3×10^{-4}	9.9×10^{-5}	0.17	0.13
1.	4.9×10^{-25}	3.9×10^{-25}	2.6×10^{-4}	2.1×10^{-4}	0.34	0.27
2.	2.4×10^{-25}	1.9×10^{-25}	1.3×10^{-4}	1.0×10^{-4}	0.15	0.13

Vela (ApJ, 737, 93, 2011), Virgo VSR2 data, Bayesian analysis, matched filtering

ν~11.2Hz



Analysis method	95% upper limit for h_0
Heterodyne, restricted priors	$(2.1 \pm 0.1) \times 10^{-24}$
Heterodyne, unrestricted priors	$(2.4 \pm 0.1) \times 10^{-24}$
$\mathcal{G}$-statistic	$(2.2 \pm 0.1) \times 10^{-24}$
$\mathcal{F}$-statistic	$(2.4 \pm 0.1) \times 10^{-24}$
MF on signal Fourier components, 2 d.o.f.	$(1.9 \pm 0.1) \times 10^{-24}$
MF on signal Fourier components, 4 d.o.f.	$(2.2 \pm 0.1) \times 10^{-24}$

~ 40% of spin down limit

ε (ellipticity) ~ 10^{-3}

Bayesian analysis for the Vela pulsar

Figure from ApJ, 737, 93, 2011.

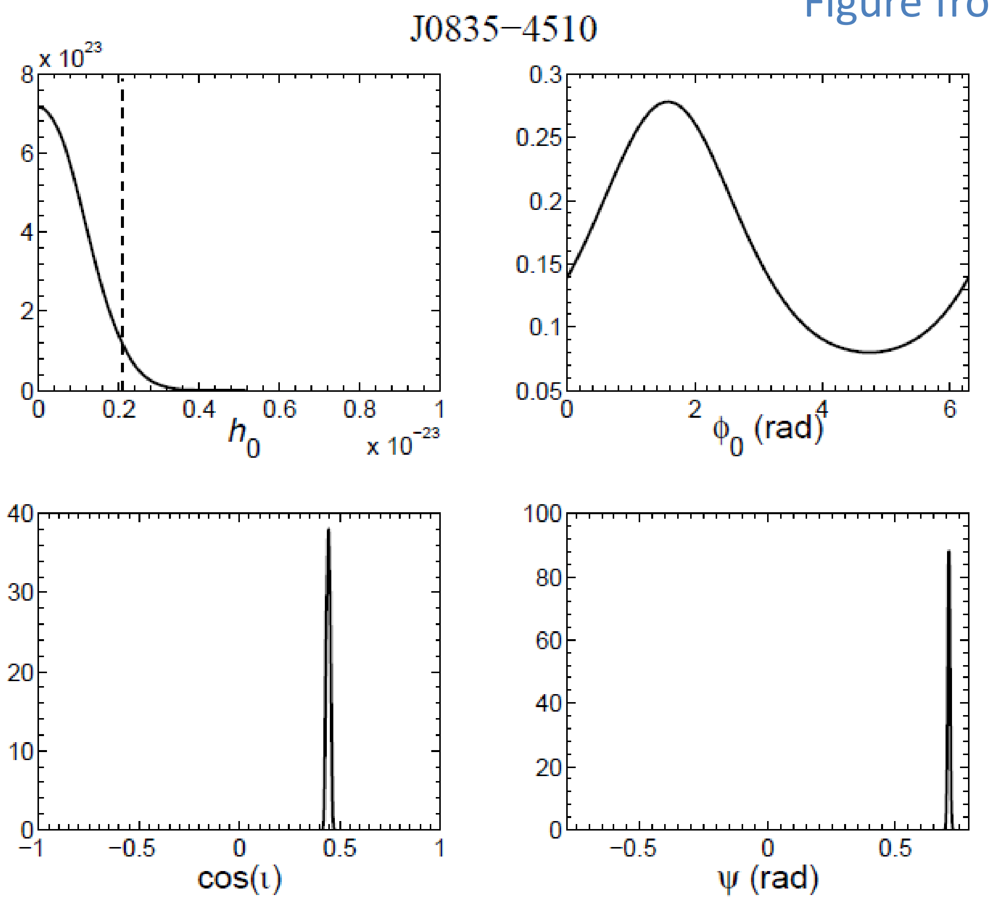


Fig. 4.— The posterior PDFs for the pulsar parameters h_0, Φ_0, $\cos\iota$ and ψ for PSR J0835−4510, produced using restricted priors on $\cos\iota$ and ψ with the complex heterodyne method. The vertical dashed line shows the 95% upper limit on h_0.

Einstein@Home

Distributed computing project using the BOINC technology

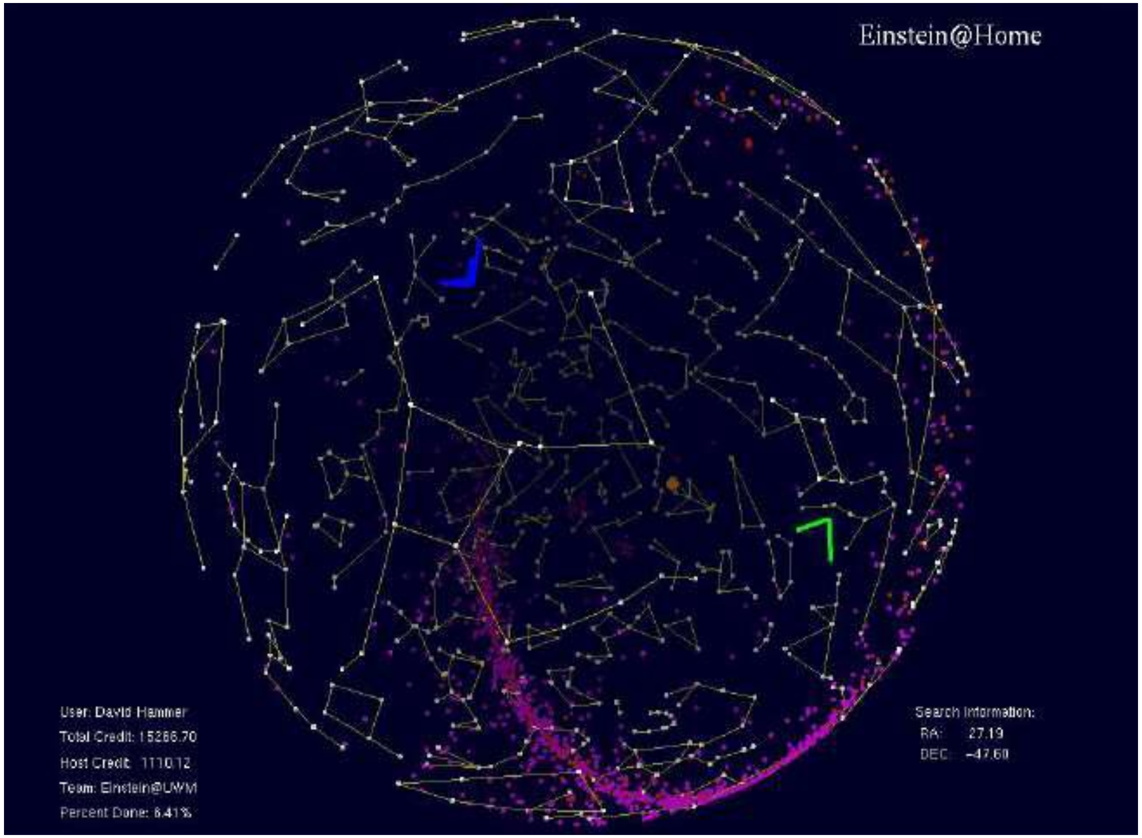
http://einstein.phys.uwm.edu

Development led by **Bruce Allen** at AEI and UWM

Launched in 2005

~10^5 volunteers

~100s Tflops computing power



All-sky searches: Einstein@Home

2 month of S5 data analyzed (Phys. Rev. D80, 042003, 2009)

Analysis method consisted of two steps: coherent $\mathcal{F}$-statistic search of 30 hour data segments followed by analysis of coincidences among the segments.

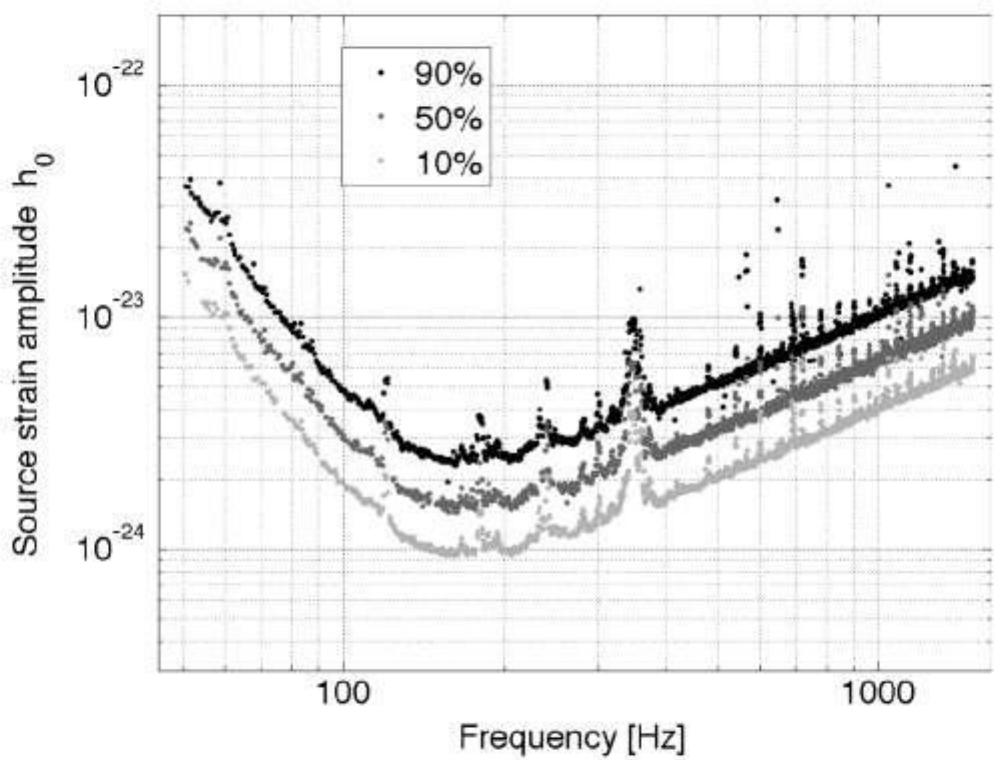


FIG. 4. Estimated sensitivity of the Einstein@Home search for isolated periodic GW sources in the early S5 LIGO data. The set of three curves shows the source strain amplitudes h_0 at which 10% (bottom), 50% (middle) and 90% (top) of simulated sources would be confidently detected (i.e., would produce at least 20 coincidences out of 28 possible) in this Einstein@Home search.

Einstein@Home cont.

Full S5 data all-sky search using a hierarchical method with $\mathcal{F}$-statistics coherent step followed by a Hough search is completed (paper under review).

Another full S5 data search using optimized hierarchical method of Pletsch and Allen currently running.

All-sky searches: semi coherent method

8 month of S5 data analyzed with power flux method (Phys. Rev. Lett 102, 111102, 2009)

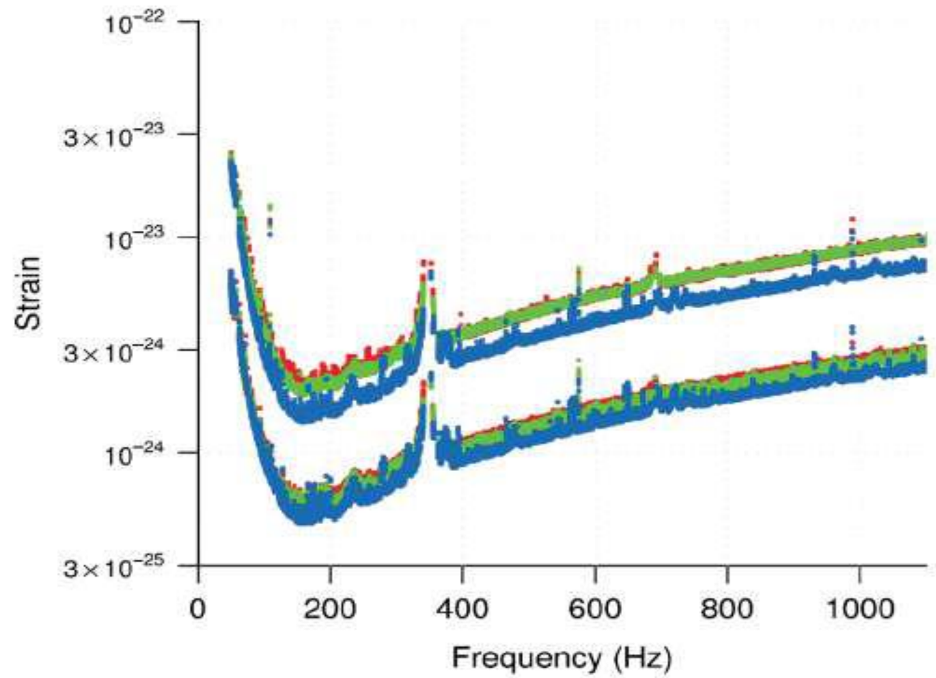
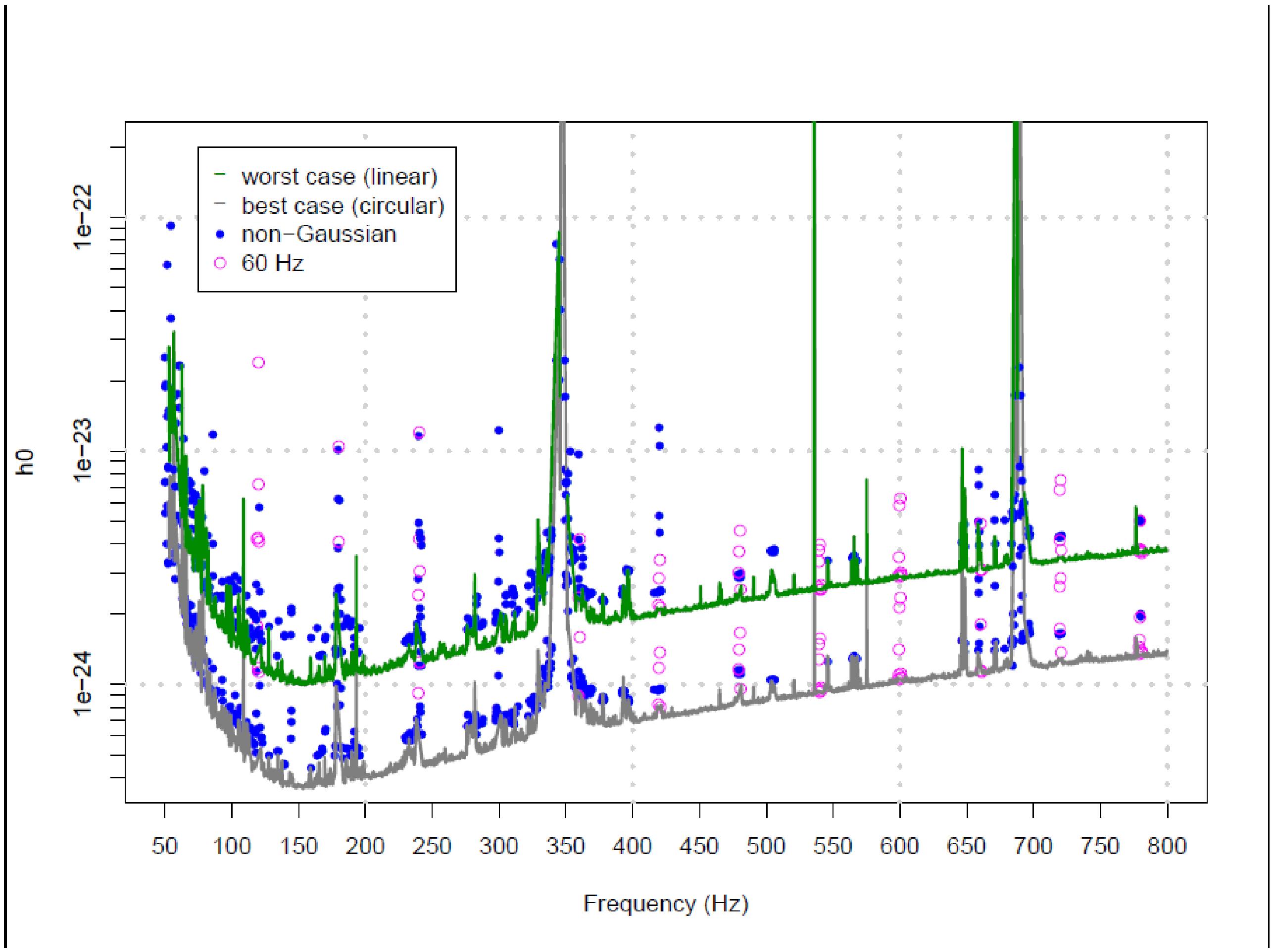


FIG. 1 (color). Minimum (H1 or L1) upper limits (95% C.L.) on pulsar gravitational-wave amplitude h_0 for the equatorial (red), intermediate (green), and polar (blue) declination bands for best-case (lower curves) and worst-case (upper curves) pulsar orientations. Shown are all the minimum limits for each of the 11 spin-down values from -5×10^{-9} Hz s^{-1} to zero in steps of 5×10^{-10} Hz s^{-1}.

Full S5 completed, results under review. Perliminary results:



Directed searches

12 days of LIGO S5 data coherently searched for GW form Cas A supernova (ApJ 722, 1504, 2010). Known position but unknown frequency. 100-300 Hz frequency band searched and a wide range of first and second spin down

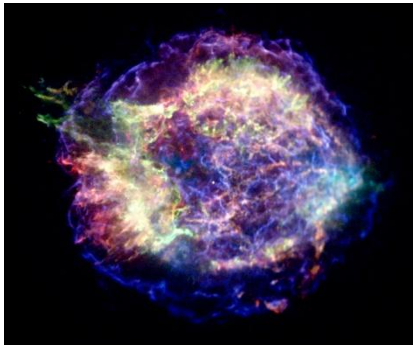


Image: Chandra/NASA

Upper limit beats the indirect limit on GW amplitude

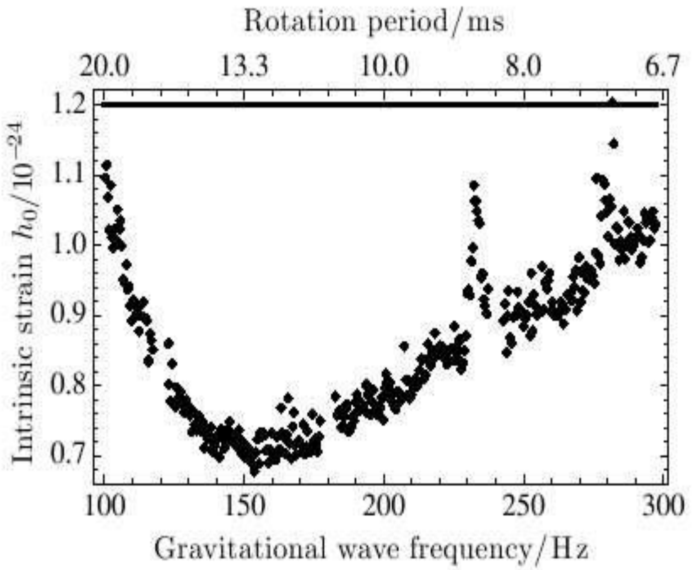


Figure 3. Upper limits at 95% confidence (dots) on the intrinsic strain h_0 of gravitational waves from Cas A and the indirect limit (line). The gravitational-wave frequency is assumed to be twice the rotation frequency. Systematic uncertainties are not included; see Section 3 for discussion.

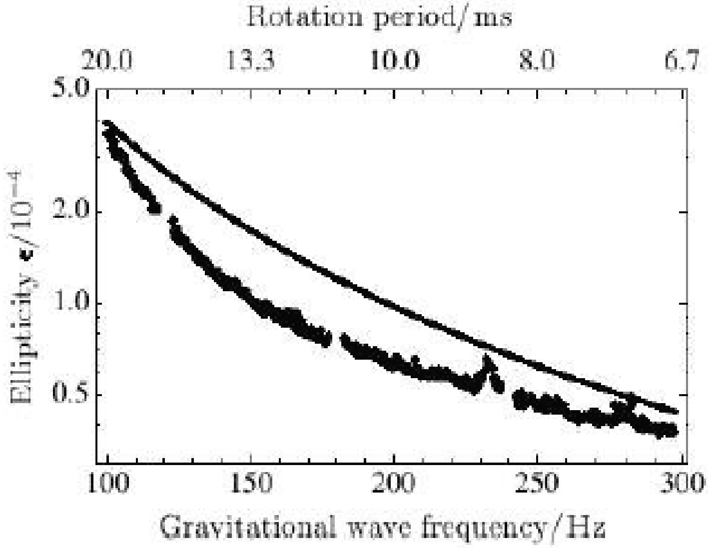


Figure 4. Upper limits at 95% confidence (dots) on the equatorial ellipticity ϵ of Cas A and the indirect limit (line). The gravitational-wave frequency is assumed to be twice the rotation frequency. Systematic uncertainties are not included; see Section 3 for discussion.

Directed searches cont.

Cas A search has also established the first upper limit on r-mode amplitude

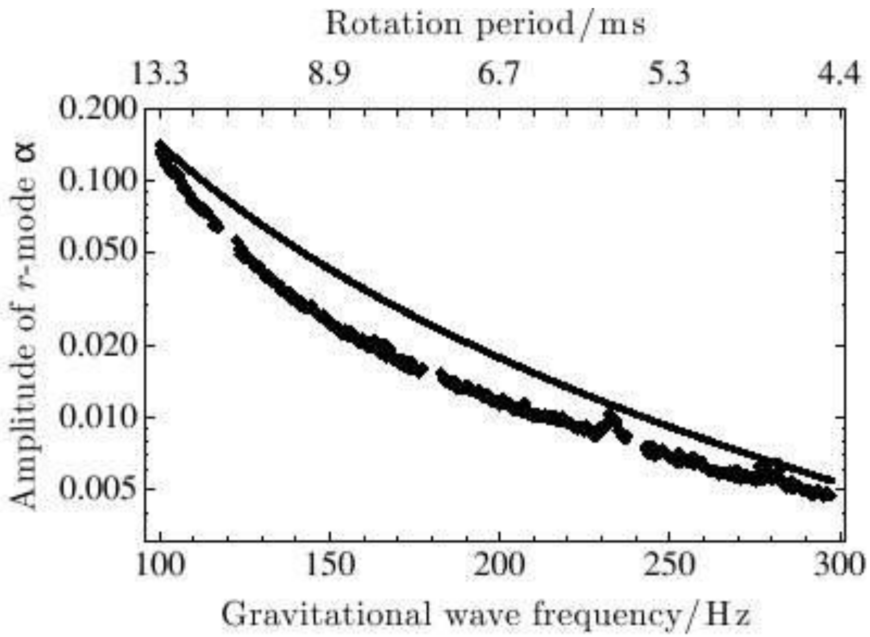


Figure 5. Upper limits at 95% confidence (dots) on the amplitude α of r-mode oscillations of Cas A and the indirect limit (line). The gravitational-wave frequency is assumed to be 4/3 times the rotation frequency. Systematic uncertainties are not included; see Section 3 for discussion.

Summary

No gravitational waves from spinning neutron stars have been detected in the LIGO /GEO600 and VIRGO data yet.

Astrophysically nontrivial and interesting upper limits on the level of the gravitational wave emission have been obtained.

We continue to search in the full S5 data and in the more recent LIGO and Virgo data with algorithms that are steadily improving.