The first second of leptons

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with
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Description for evolution of SM particles in the early universe between electroweak transition (\( T_{ew} \approx 200\text{GeV}, \ t_H \approx 10 \text{ ps}, \ r_H \approx 10 \text{ mm} \)) and neutrino oscillation (\( T_{osc} \approx 10 \text{ MeV}, \ t_H \approx 1 \text{ s}, \ r_H \approx 10^5 \text{ km} \)).

Influence of large neutrino asymmetries on the QCD transition and on the relic abundance of WIMPs?
Why study neutrino asymmetries?  
- $Z^0$ decay $\rightarrow$ three neutrino flavour $N_\nu = 3$, but experiments observe $N_{\nu}^{\text{eff}} > 3$.
- Neutrinos relativistic, huge part of the radiation energy density.
- $\simeq$ 650 billions Neutrinos per second and cm$^2$, still difficult to observe.
- Neutrino background, $C_{\nu B}$ at $T_\nu \simeq (4/11)^{1/3} T_\gamma$?
- Neutrinos or Anti-neutrinos? A large asymmetry $n_{\nu_f} - n_{\bar{\nu}_f}$ is compatible with observations...
- ...and would induce an additional radiation energy via their large chemical potentials.

Consequences for evolution of early universe?

Expanding universe $\rightarrow$ SM particles inside $\rightarrow$ describe evolution.
How to describe

- Take all Standard Model Particles incl all their physical values.
- Find the correct thermodynamic description and
- evolve them in the early universe for $T_{ew} > T > T_{\nu_{osc}}$.

Cosmic (SM) particle fluid in chemical equilibrium:

- Hubble time $t_H = 1/H$ is the scale of interest.
- All interaction rates larger for $T \geq$ few MeV.
- Thermal and chemical equilibrium excellent approximation.
Net particle density for a species $i$ with distribution function $f(i)$:

$$n_i = \frac{g_i}{2\pi^2} \int_{m_i}^{\infty} E \sqrt{E^2 - m_i^2} \left( \frac{1}{\exp \frac{E-\mu_i}{T} \pm 1} - \frac{1}{\exp \frac{E+\mu_i}{T} \pm 1} \right) dE$$

$(T \gg m_i, \mu_i)$

$$= \begin{cases} \frac{1}{3} g T^2 \mu_i + O(\mu_i^2) & \text{for bosons} \\ \frac{1}{6} g T^2 \mu_i + O(\mu_i^2) & \text{for fermions} \end{cases}$$

$(T \ll m_i)$

$$= 2g \left( \frac{m_i T}{2\pi} \right)^{3/2} \sinh \left( \frac{\mu_i}{T} \right) \exp \left( -\frac{m_i}{T} \right).$$

Energy density:

$$\epsilon_{i,\bar{i}} = \frac{g_i}{2\pi^2} \int_{m_i}^{\infty} E^2 \sqrt{E^2 - m_i^2} \left( \frac{1}{\exp \frac{E-\mu_i}{T} \pm 1} + \frac{1}{\exp \frac{E+\mu_i}{T} \pm 1} \right) dE$$

$(T \gg m_i, \mu_i)$

$$= \begin{cases} \sum_i g_i \left[ \frac{\pi^2}{15} T^4 + \frac{1}{2} T^4 \left( \frac{\mu_i}{T} \right)^2 + O \left( \frac{\mu_i}{T}^4 \right) \right] & \text{for bosons} \\ \sum_i g_i \left[ \frac{7\pi^2}{120} T^4 + \frac{1}{4} T^4 \left( \frac{\mu_i}{T} \right)^2 + \frac{1}{8\pi^2} T^4 \left( \frac{\mu_i}{T} \right)^4 \right] & \text{for fermions} \end{cases}$$

$(T \ll m_i)$

$$= m_i n_i$$
All particle interactions follow:

- charge conservation and neutrality $q = 0$.  
  \[ \text{Siegel & Frye, 2007} \]

- baryon number conservation, $b = (8.85 \pm 0.24) \times 10^{-11}$.  
  \[ \text{Komatsu et al. WMAP Coll., 2009} \]

- lepton flavour number conservation, $l_f = ???$

Lifting global conservation laws to local ones:

\[ l_f = \frac{n_f + n_{\nu_f}}{s} \quad \text{with } f = e, \mu, \tau, \]
\[ b = \sum_i b_i n_i \quad \text{with } b_i = \text{baryon number of species } i, \]
\[ 0 = \sum_i q_i n_i \quad \text{with } q_i = \text{charge of species } i, \]

with $s = s(T)$: entropy.

\[ \Rightarrow \text{Solvable system of equations for net particle densities } n_i(T, \mu_i) \text{ in the QP and the HG.} \]
Lepton flavour number???

For $T < T_{osc} \simeq 10\text{MeV}$

experimental: $|l_f| \leq 0.02$, after $T_{osc}$. Simha & Steigmann; Popa & Vasile, 2008

theoretical: $b \ll |l_f|$ possible. Several leptogenesis models in which $l$ may be larger than $b$.

Flanz et al., 1996; L.Covi & E.Roulet, 1996; Casas et al., 1997; T.Hambye, 2002

Assume $l_e = l_\mu = l_\tau$.

For $T > T_{osc}$

A scenario with single flavour much larger is possible.

For example $\sum_f l_f = b$ but $l_e \simeq b$ and $l_\mu = -l_\tau = O(1)$.

Numerous theories can explain a large lepton flavour asymmetry with the observer small baryon asymmetry.

B.A.Campbell et al., 1998; J.March-Russell et al., 1999; J.McDonald, 2000

A.D.Dolgov et al., 2002; Y.Y.Y.Wong, 2002; G.Mangany et al., 2011

What are experimental evidences?
The cosmic QCD transition – quarks confine to hadrons:

- Starting condition for BBN.
- Possible generation of relics like QCD dark matter (quark nuggets, . . .), modification of gravitational waves, . . .
- Crossover or 1st order? Knowing the order would rule out many scenarios.

So far most reliable description via lattice simulation with three quarks at almost physical masses and vanishing chemical potential.

Inclusion of leptons and cosmic QCD phase transition?
Chemical potentials

Each conserved quantum number associated with a chemical potential. The particle contribution to the free energy:

\[ \mu_Q n_Q + \mu_B n_B + \sum_f \mu_L f n_L f \]

\[ \begin{align*}
T > T_{QCD} & \Rightarrow \sum_q \mu_q n_q + \sum_l \mu_l n_l + \sum_g \mu_g n_g \\
T < T_{QCD} & \Rightarrow \sum_b \mu_b n_b + \sum_m \mu_m n_m + \sum_l \mu_l n_l,
\end{align*} \]

Analytical approach:
\[ \rightarrow T \gg T_{QCD} \]
\[ \rightarrow \text{u,d,c,s-quarks and e,}\mu,\tau, m_i = 0, \]
\[ \rightarrow 3 l_f = \sum_f l_f \]
\[ \Rightarrow \mu_B(T) = \left( \frac{39}{4} b - l \right) \frac{s(T)}{4 T^2}. \]

\[ T \gg m, \mu \]
\[ \mu_B = \mu_B(b, l) \]
\[ b \ll l \Rightarrow O(\mu_B) = O(l) \]

Small \( T \): \( \mu_B(T < m_\pi/3) \approx m - T \ln \left[ \frac{c(T)}{2 bs(T)} \right] \) independent of \( l \).
Baryochemical potential

Trajectories of the baryochemical potential $\mu_B$

The sign dependence of the trajectory.

Evolution of the baryochemical potential for negative lepton asymmetries.
WIMPs

WIMP particle most promising DM candidate:

\[ 1 \text{ TeV} > m_\chi > 10 \text{ GeV} \]
leads to \( 40 \text{ GeV} \)

\[ T_{f_0} \approx \frac{m_\chi}{20} < 0.4 \text{ GeV}. \]

What is the relic abundance today?

- Assume \( \chi \bar{\chi} \to \cdots \) with \( \sigma \propto G_F^2 \).
- For the number density:

\[
\dot{n} + 3Hn = - \langle \sigma |v| \rangle (n^2 - n_{\text{eq}}^2).
\]

- \( \left( \frac{n_\chi}{s} \right)_0 = \left( \frac{n_\chi}{s} \right)_{f_0} \)
- \( \Omega_\chi h^2 = \frac{m_\chi n_\chi}{\rho_c} \)

\[
\Rightarrow \Omega_\chi h^2 = \frac{1}{0.264m_{Pl}} \left( \frac{s}{\rho_c h^{-1}} \right)_0 \left( \frac{1}{\langle \sigma |v| \rangle g_*^{1/2}} \right)_{f_0}
\]

A.Green, S.Hofmann, D.J.Schwarz, 2005

\[
g_* (T, \{\mu_i\}) \equiv \frac{30}{\pi^2 T^4} \epsilon(T, \{\mu_i\})
\]

\[ m=0 \quad \frac{15}{T^4 \pi^4} \sum_i g_i \int_0^\infty \frac{E_3}{\exp\left[\frac{E-\mu}{T}\right]+1} \text{d}E
\]

\[ = \sum_F \frac{7}{4} g_F + \frac{15}{2} g_F \left( \frac{\mu_F}{\pi T} \right)^2 + \frac{15}{4} g_F \left( \frac{\mu_F}{\pi T} \right)^4 + \sum_B g_B
\]
Relic abundance $\Omega_{\chi}$ and $l_f$
Conclusions

- Large lepton asymmetries do have an impact on the evolution of the early universe.
- \( b \ll |l| \leq 0.01 \) can significantly influence dynamics of the QCD phase transition and maybe even the order of the transition in the \( \mu_B - T \)-plane.
- The cosmic QCD phase transition lives at least in 5 dimensions \((l_f, b, q = 0)\) and even more for an inhomogeneous universe.
- Large lepton asymmetries might allow larger WIMP annihilation cross section and exclude smaller.
- For several (a)symmetric flavour asymmetries \( l_f \geq \mathcal{O}(0.01) \) we found a few percent effect on the relic WIMP abundance.