Testing the sterile neutrino hypothesis at the solar sector

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Preamble
News flash on standard mixing: Evidence of $\theta_{13}>0$

Introduction
Hints of light sterile $\nu$'s

Theoretical framework
The MSW mechanism in a 3+1 scheme

Phenomenological implications
Testing sterile $\nu$'s with solar sector data

Conclusions
News flash on standard mixing: Evidence of $\theta_{13} > 0$
All the three $\theta_{ij}$ are (most likely) non-zero

Evidence of $\theta_{13} > 0$ is no less than $3\sigma$

To be taken into account when considering any perturbation of the standard scheme

Fogli, Lisi, Marrone, A.P., Rotunno
INTRODUCTION

Hints of light sterile ν’s
Hint #1: The Gallium calibration anomaly

Deficit observed in calibration performed with radioactive sources ~ 2.5 sigma effect

But it could be due to overestimate of $\nu_e + {}^{71}\text{Ga} \rightarrow {}^{71}\text{Ge} + e^-$ cross section (its recalculation highly desirable)
Hint #2: The reactor antineutrino anomaly

Mention et al., PRD 83 073006 (2011)

With new reactor fluxes deficit of all the short-baseline reactor measurement ~ 2 sigma effect

But new calculations, like older ones, are still anchored to (one single) $\beta$-spectrum experiment (ILL)

Mueller et al., PRC 83 054615 (2011)
Huber, PRC 84 024617 (2011)
Fit to gallium expts + SBL reactor data

\[ \sin^2 2\theta_{new} \simeq 0.1 \quad \Delta m^2_{new} \gtrsim 1 \text{ eV}^2 \]

Mention et al., PRD 83 073006 (2011)
**Hint #3: Less tension within VSBL results**

Kopp, Maltoni, Schwetz, arXiv1103.4570

Giunti and Laveder, arXiv1107.1452

With new reactor fluxes fit improves both in 3+1 and 3+2 schemes

But tension between $\nu_\mu \rightarrow \nu_e$ (positive) appearance and disappearance [$\nu_e \rightarrow \nu_e$ (positive) & $\nu_\mu \rightarrow \nu_\mu$ (negative)] searches is still considerable.

$$\sin^2 2\theta_{e\mu} \approx \frac{1}{4} \sin^2 2\theta_{ee} \sin^2 2\theta_{\mu\mu} \approx 4|U_{e4}|^2|U_{\mu4}|^2$$
Hint #4: Cosmology favors extra radiation

CMB + LSS tend to prefer extra relativistic content
~ 2 sigma effect
[Hamann et al., PRL 105, 181301 (2010)]

But one must bear in mind that:

- It may be a statistical artifact driven by prior effects
  (Gonzalez-Morales et al., arXiv:1106.5052)

- eV masses acceptable only abandoning standard $\Lambda$CDM

- $N_{\text{eff}} > 4$ at BBN strongly disfavored (Mangano & Serpico PLB 701, 296, 2011)

- $N_{\text{eff}}$ is not specific of $\nu_s$ and can also depend on the epoch
  (new light particles, decay of dark matter particles, quintessence, ...)

[Hamann & Pastor later]
THEORETICAL FRAMEWORK

The MSW mechanism in a 3+1 scheme
Perturbing the 3-flavor scheme

The 4\textsuperscript{th} \( \nu \) state induces a small perturbation of the 3-flavor framework

\[ |U_{s4}| \sim 1 \]

\[ \Delta m_{\text{new}}^2 > 1 \text{eV}^2 \]

* Solar sector alone cannot distinguish the 3+1 scheme from a scheme where also \( U_{s3} \) is big (but this disfavored by the atmospheric sector)

* Hierarchy: reciprocal ordering of \((\nu_3, \nu_4)\) & respect to \((\nu_1, \nu_2)\) unknown
VSBL $\nu_e$ disappearance in a 3+1 scheme

In a 2$\nu$ framework:

$$P_{ee} \sim 1 - \sin^2 2\theta_{new} \sin^2 \frac{\Delta m_{new}^2 L}{4E}$$

$$\sin^2 2\theta_{new} \sim 0.17 \pm 0.1 \ (95\%)$$

In a 3+1 scheme:

$$P_{ee} = 1 - 4 \sum_{j<k} U_{ej}^2 U_{ek}^2 \sin^2 \frac{\Delta m_{jk}^2 L}{4E}$$

$$\Delta m_{sol}^2 \ll \Delta m_{atm}^2 \ll \Delta m_{new}^2$$

$$\sin^2 \theta_{new} \sim U_{e4}^2 = \sin^2 \theta_{14}$$

3+1 scheme has several consequences: solar, atm, react., accel.

We will focus on the implications for Solar (S) & KamLAND (K)
LBL $\nu_e$ disappearance in a 3+1 scheme

\[ P_{ee} = 1 - 4 \sum_{j>k} U_{e j}^2 U_{e k}^2 \sin^2 \frac{\Delta m_{jk}^2 L}{4E} \]

\[ \Delta m_{sol}^2 \ll \Delta m_{atm}^2 \ll \Delta m_{new}^2 \]

$\Delta m_{atm}^2$-driven osc. averaged
$\Delta m_{new}^2$

\[ P_{ee} = (1 - U_{e 3}^2 - U_{e 4}^2)^2 P_{ee}^{2\nu} + U_{e 3}^4 + U_{e 4}^4 \]

\[ U_{e 3}^2 = c_{14}^2 s_{13}^2 \quad U_{e 4}^2 = s_{14}^2 \]

Exact degeneracy between $U_{e 3}$ and $U_{e 4}$
Solar $\nu$ conversion in a 3+1 scheme

\[
i \frac{d}{dx} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \nu_s \end{pmatrix} = H \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \\ \nu_s \end{pmatrix} \quad H = U K U^T + V(x)
\]

\[
K = \frac{1}{2E} \text{diag}(k_1, k_2, k_3, k_4) \quad k_i = \frac{m_i^2}{2E} \quad \text{wavenumbers in vacuum}
\]

Useful to write the mixing matrix as*:

\[
U = R_{23} \ S \ R_{13} \ R_{12} \quad S = R_{24} \ R_{34} \ R_{14}
\]

\[
\theta_{14} = \theta_{24} = \theta_{34} = 0 \quad \rightarrow \quad S = I \quad \rightarrow \quad 3\text{-flavor case}
\]

\[
V = \text{diag}(V_{CC}, 0, 0, -V_{NC}) \quad \text{MSW potential}
\]

\[
V_{CC} = \sqrt{2} \ G_F \ N_e \quad V_{NC} = \frac{1}{2} \sqrt{2} \ G_F \ N_n
\]

* We assume $U$ to be real but in general it can be complex due to CP phases
Change of basis: \( \nu' = (R_{23} S R_{13})^T \nu = A^T \nu = R_{12} U^T \)

In the new basis: \( H' = A^T H A = R_{12} K R_{12}^T + R_{13}^T S^T V S R_{13} \)

At zero\(^{th}\) order in: \( \frac{V}{k_{atm}} \) and \( \frac{V}{k_{new}} \)

\[
H' \approx \begin{pmatrix}
H'_{2\nu} & \hline \\
\hline & k_3
\end{pmatrix}
\begin{pmatrix}
k_4
\end{pmatrix}
\]

The 3\(^{rd}\) & 4\(^{th}\) state evolve independently from the 1\(^{st}\) & 2\(^{nd}\)

The dynamics reduces to that of a 2x2 system
Diagonalization of the Hamiltonian

The 2x2 Hamiltonian is diagonalized by a 1-2 rotation

$$\tilde{R}_{12}^T H_{2\nu}' \tilde{R}_{12} = \text{diag}(\tilde{k}_1, \tilde{k}_2)$$

which defines the solar mixing angle in matter

$$\tilde{\theta}_{12}(x)$$

wavenumbers in matter

$$\tilde{k}_i$$

The starting Hamiltonian is then diagonalized by

$$\tilde{U} = A \tilde{R}_{12}$$

$$\tilde{U}^T H \tilde{U} = \text{diag}(\tilde{k}_1, \tilde{k}_2, k_3, k_4)$$

For $\nu_3$ and $\nu_4$ (averaged) vacuum-like propagation
The 2x2 Hamiltonian: \[ H_{2\nu} = H_{2\nu}^{\text{kin}} + H_{2\nu}^{\text{dyn}} \]

\[ H_{2\nu}^{\text{kin}} = \begin{pmatrix} c_{12} & s_{12} \\ -s_{12} & c_{12} \end{pmatrix} \begin{pmatrix} -k_{\text{sol}}/2 & 0 \\ 0 & k_{\text{sol}}/2 \end{pmatrix} \begin{pmatrix} c_{12} & -s_{12} \\ s_{12} & c_{12} \end{pmatrix} \]

\[ k_{\text{sol}} = \frac{m_2^2 - m_1^2}{2E} \]

\[ H_{2\nu}^{\text{dyn}} = V_{CC}(x) \begin{pmatrix} \gamma^2 + r(x) \alpha^2 & r(x) \alpha \beta \\ r(x) \alpha \beta & r(x) \beta^2 \end{pmatrix} \]

\[ r(x) = \frac{V_{NC}(x)}{V_{CC}(x)} \]

\[ \begin{align*}
\alpha^2 + \beta^2 &= U_{s1}^2 + U_{s2}^2 \\
\gamma^2 &= 1 - (U_{e3}^2 + U_{e4}^2)
\end{align*} \]

All the dynamical effects induced by the 4\textsuperscript{th} (and 3\textsuperscript{rd}) state are 2\textsuperscript{nd} order in the \( s_{ij} \): small deviations from the standard MSW.

But important new kinematical effects are present ...

For adiabatic propagation (valid for small deviations around the LMA)

\[ P_{ee} = \sum_{i=1}^{4} U_{ei}^2 \tilde{U}_{ei}^2 = U_{e1}^2 \tilde{U}_{e1}^2 + U_{e2}^2 \tilde{U}_{e2}^2 + U_{e3}^4 + U_{e4}^4 \]

\[ P_{es} = \sum_{i=1}^{4} U_{si}^2 \tilde{U}_{ei}^2 = U_{s1}^2 \tilde{U}_{e1}^2 + U_{s2}^2 \tilde{U}_{e2}^2 + U_{s3}^2 U_{e3}^2 + U_{s4}^2 U_{e4}^2 \]

Expressions for \( U_{ei} \)'s
(always valid)

\[
\begin{align*}
U_{e1}^2 &= c_{14}^2 c_{13}^2 c_{12}^2 \\
U_{e2}^2 &= c_{14}^2 c_{13}^2 s_{12}^2 \\
U_{e3}^2 &= c_{14}^2 s_{13}^2 \\
U_{e4}^2 &= s_{14}^2
\end{align*}
\]

\[ \sim 1 - s_{14}^2 - s_{13}^2 \]

Expressions for \( U_{si} \)'s
valid for \( \theta_{24} = \theta_{34} = 0 \)

\[
\begin{align*}
U_{s1}^2 &= s_{14}^2 c_{13}^2 c_{12}^2 \\
U_{s2}^2 &= s_{14}^2 c_{13}^2 s_{12}^2 \\
U_{s3}^2 &= s_{14}^2 s_{13}^2 \\
U_{s4}^2 &= c_{14}^2 c_{13}^2 \
\end{align*}
\]

\[ \sim s_{14}^2 \]

\[ \sim s_{13}^2 \]

\[ \sim 0 \]

\[ \sim 1 - s_{14}^2 \]

The elements of \( \tilde{U} \) are obtained replacing \( \theta_{12} \) with \( \tilde{\theta}_{12} \) calculated in the production point (near the sun center)
PHENOMENOLOGICAL IMPLICATIONS

Testing sterile $\nu$'s at the solar sector
Two simple limit cases

\( \theta_{13} \neq 0 \quad \theta_{14} = 0 \) \quad (3\nu)

\[
\begin{align*}
\begin{cases}
    P_{ee} &= c_{13}^4 P_{ee}^{2\nu} \bigg|_{V \rightarrow Vc_{13}^2} + s_{13}^4 \\
    P_{es} &= 0
\end{cases}
\end{align*}
\]

\( \theta_{13} = 0 \quad \theta_{14} \neq 0 \) \quad (4\nu)

\[
\begin{align*}
\begin{cases}
    P_{ee} &= c_{14}^4 P_{ee}^{2\nu} \bigg|_{V \rightarrow Vc_{14}^2} + s_{14}^4 \\
    P_{es} &\sim s_{14}^2 P_{ee}^{2\nu} \bigg|_{V \rightarrow Vc_{14}^2} + s_{14}^2
\end{cases}
\end{align*}
\]
$(\theta_{13}, \theta_{12})$ vs $(\theta_{14}, \theta_{12})$ constraints (new reactor fluxes)

Two similar indications at $1.8\sigma$ ($1.3\sigma$ with old fluxes)

We expect a degeneracy among $\theta_{13}$ and $\theta_{14}$

CC $\sim \Phi_B P_{ee}$

$\text{NC} \sim \Phi_B (1-P_{es})$

$\text{ES} \sim \Phi_B (P_{ee} + 0.15 P_{ea})$

Solar $\nu$ sensitive to $P_{es}$

CC/NC (SNO) & ES (SK)

Different correlations

(\theta_{13}, \theta_{14}) constraints (new reactor fluxes)

Complete degeneracy
\theta_{13}-\theta_{14} indistinguishable

Solar sector essentially sensitive to \sim U_{e3}^2 + U_{e4}^2

Hint for \nu_e mixing with states others than (\nu_1, \nu_2)

Different probes are necessary to determine if \nu_e mixes with \nu_3 or \nu_4

How $\theta_{13} > 0$ eats up $\theta_{14} > 0$

A. P., work in preparation

Upper limit $\Rightarrow \sin^2 \theta_{14} < 0.05$ (90% C.L.)

Bound is not incompatible with gallium & reactor anomalies...
In that case non-zero $\theta_{14}$ is rescued and we would have a double indication: $\theta_{13} > 0$ & $\theta_{14} > 0$ at $\sim 3\sigma$ level.
Summary

- $3\nu$ standard paradigm acquires a new piece: $\theta_{13} > 0$ ($>3\sigma$); to be taken into account when studying its perturbations.

- A few anomalies suggest active $\nu$'s mix with new sterile states and require an enlarged framework (3+1 or 3+n).

- However, each indication is problematic per se and/or conflicting with other ones. Further scrutiny is needed.

- Evidence of $\theta_{13} > 0$ reduces solar hint of sterile $\nu$'s, which now becomes a robust upper limit: $\sin^2 \theta_{14} < 0.05$ (90% C.L.)

- If we tolerate tension among app.& disapp. we have an intriguing indication of non-zero $\theta_{14}$ at the $3\sigma$ level

- A new VSBL experiment indispensable to settle the issue.
BACK UP
Can $\theta_{14} > 0$ explain the LBL $\nu_e$ appearance?

One may ask if the excess of events in MINOS and T2K can be imputable to sterile neutrino oscillations and not to $\theta_{13}$.

The answer is negative: sterile mixing is too small.

\begin{align*}
P^{3\nu}(\nu_\mu \rightarrow \nu_e) & \propto |U_{e3}|^2|U_{\mu3}|^2 \propto \sin^2 \theta_{13} \sin^2 \theta_{23} & \nu_3 \text{-driven} \\
P^{4\nu}(\nu_\mu \rightarrow \nu_e) & \propto |U_{e4}|^2|U_{\mu4}|^2 \propto \sin^2 \theta_{14} \sin^2 \theta_{24} & \nu_4 \text{-driven}
\end{align*}

$|U_{e3}|^2|U_{\mu3}|^2 \simeq 10^{-2}$ is able to explain the excess

$|U_{e4}|^2|U_{\mu4}|^2 < 10^{-3}$ is at least one order of magnitude smaller than required

\textbf{from 3+1 fit (disapp. + app).} (e.g., Giunti & Laveder arXiv:1107.1452)