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Variances and Covariances of the Nuclear Matrix Elements for the $0\nu 2\beta$ decay

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Based on work done in collaboration with

G.L. Fogli, E. Lisi **arXiv: 0905.1832**

A. Faessler, V. Rodin, F. Simkovic **arXiv: 0810.5733**

Contents

- $0\nu 2\beta$ decay and QRPA free parameters
- Nuclear matrix element uncertainties and their correlations
- Results and implications
- Probing particle and nuclear physics models
- Conclusions

Neutrinoless double beta decay ($0\nu 2\beta$)

A **second-order** ($\Delta Z = 2$) weak process:

$$(Z, A) \quad \rightarrow \quad (Z+1, A) \quad \rightarrow \quad (Z+2, A) + 2e^-$$

with lifetime:

$$[T_i^{0\nu}]^{-1} = G_i^{0\nu} \left| M_i' \right|^2 m_{\beta\beta}^2$$

- in a given candidate nucleus i = (Z,A)
- assuming that the process is due to light Majorana neutrinos.

$G^{0\nu}$ → known phase space factor

$\left| M' \right|^2$ → **nuclear matrix element (N.M.E.)**

$m_{\beta\beta}^2$ → "effective Majorana neutrino mass"

$M'(g_{pp}, g_A)$ must be evaluated in a nuclear model

NME uncertainties and correlations

NME ***variances*** are important to compare $0\nu 2\beta$ limits (or signals) in different nuclei. NME ***covariances*** are important as well: e.g.

$$T_j^{0\nu} = T_i^{0\nu} \frac{G_i^{0\nu} \left| M_i' \right|^2}{G_j^{0\nu} \left| M_j' \right|^2}$$

half-life expected in a nucleus j, if T_i is measured in a nucleus i.

"most favorable case" (experimentally)

→ shortest T_j, i.e. smallest $\left| M_i' \right|$ and largest $\left| M_j' \right|$.

However, NME positive correlations prevent such favorable case (NME go all "up" or "down")

Our focus is to show that, indeed:

- there are high correlations between NME uncertainties in any couple of nuclei
- these correlations play an important role in the phenomenology of $0\nu 2\beta$ decay

Notation:

$$\tau_i = \log_{10}(T_i / y)$$
$$-\gamma_i = \log_{10}[G_i^{0\nu} / (\mathrm{y}^{-1}\mathrm{eV}^{-2})]$$
$$\eta_i = \log_{10} |M_i'|$$
$$\mu = \log_{10}(m_{\beta\beta} / \mathrm{eV})$$

$\longrightarrow$ **Linearization**:

$$\boxed{[T_i^{0\nu}]^{-1} = G_i^{0\nu} |M_i'|^2 m_{\beta\beta}^2 \longrightarrow \tau_i = \gamma_i - 2\eta_i - 2\mu}$$

This is more appropriate to deal with relatively large NME errors.

E.g., consider a "factor of two" uncertainty:

$$|M_i'| = \left(\frac{1}{2} \ldots 2\right) |M_i^{0\prime}| \longrightarrow |M_i'| = |M_i^{0\prime}| \times \left(1^{+1.0}_{-0.5}\right) \quad \text{at } 1\sigma$$

The above error parametrization is **not convenient**, because:

- asymmetric errors are difficult to manage with usual statistical tools;
- the unphysical region $|M_i'| < 0$ is hit at $> 2\sigma$

Both drawbacks are avoided by taking logarithms:

$$\log_{10} |M_i'| = \log_{10} |M_i^{0\prime}| \pm 0.30$$

$$\eta_i = \eta_i^0 \pm 0.30$$

Input Ingredients

We consider a set of eight $0\nu2\beta$ candidate nuclei:

^{76}Ge ^{82}Se ^{96}Zr ^{100}Mo ^{116}Cd ^{128}Te ^{130}Te ^{136}Xe

Within the quasiparticle random phase approximation (**QRPA**), we consider*:

- **2 values** for the **axial coupling**: g_A = 1.25 (bare) and g_A = 1.00 (quenched);
- **2 approaches** to **short range correlations**** (s.r.c.): the so-called Jastrow-type s.r.c., and the unitary correlation operator method (UCOM);
- **3 size** for model **basis**: small, intermediate and large;
- **2 many-body models**: QRPA and its renormalized version (RQRPA).

2 × 2 × 3 × 2 = 24 variants

supplemented by errors induced by g_{pp} errors through the experimentally observed $2\nu2\beta$ processes.

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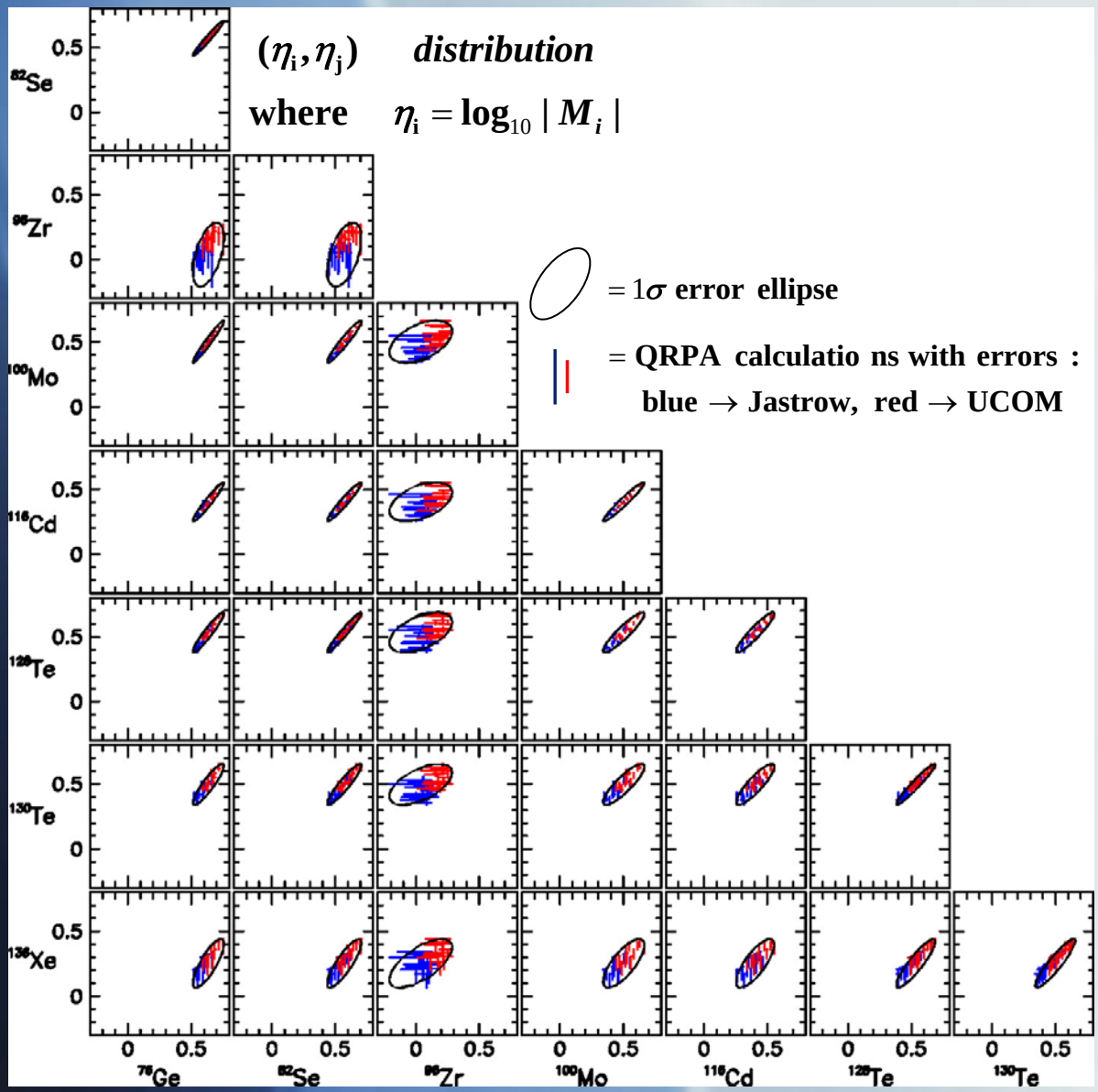
* As taken from Faessler et al., arXiv: 0706.4304, arXiv: 0710.2055.

** s.r.c. are peculiar of the $0\nu2\beta$ decay and account for the repulsive nucleon-nucleon interaction at small distances.

RESULTS:



-Strong positive correlations among theoretical estimates

-QRPA calculations scattered along the ellipse major axis, as a result of s.r.c. model (Jastrow, UCOM)

-Dispersion along the ellipse minor axis due to g_{pp} variations

- ^{96}Zr big errors: QRPA collapse

Note: errors conservatively inflated to embrace min/max values of NME

From: arXiv: 0810.5733

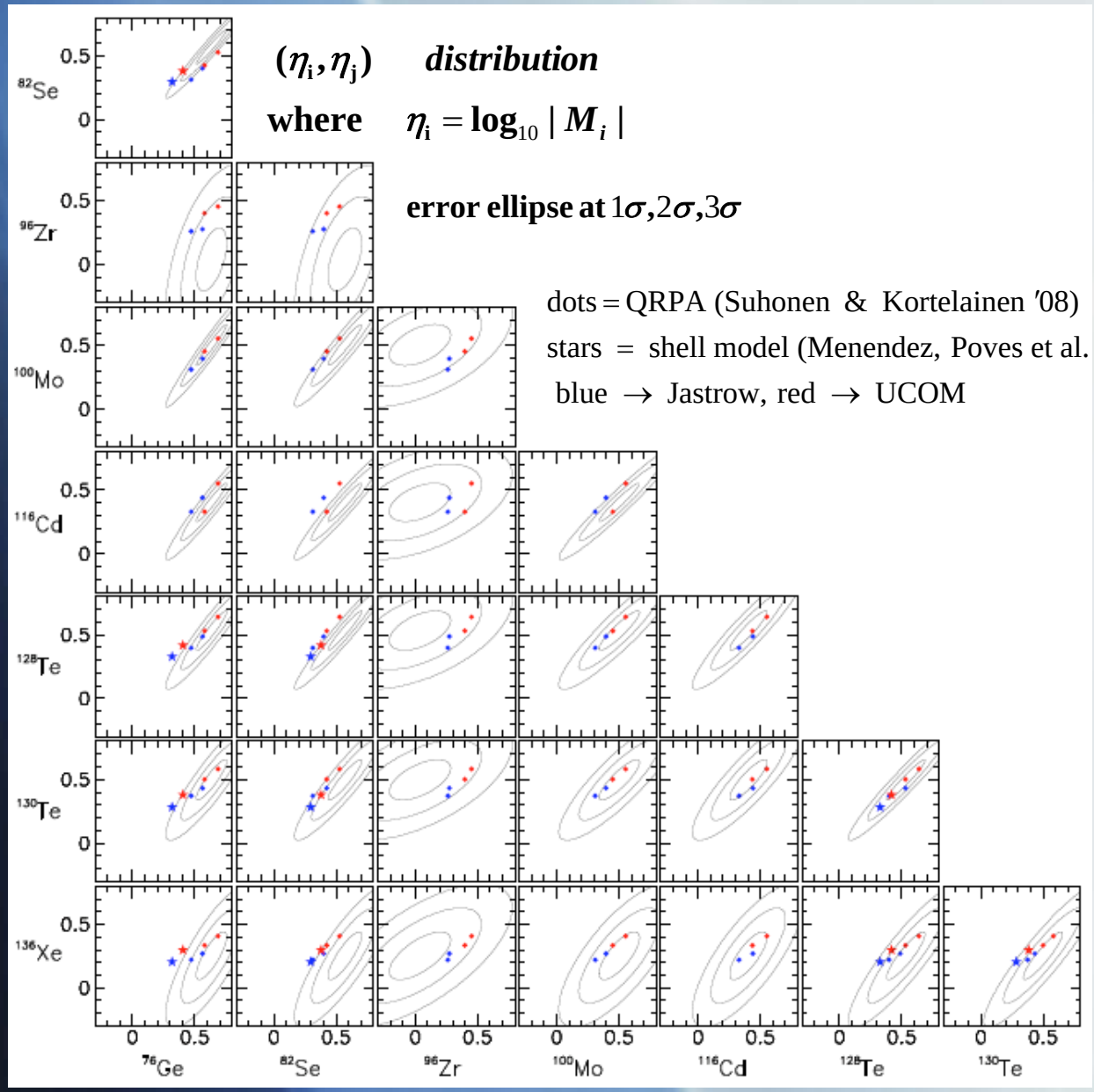
TABLE I: For each nucleus i, we report the phase space factor γ_i, the central value of the nuclear matrix error η_i, and the error σ_i, together with the (symmetric) error correlation matrix ρ_{ij}, according to the QRPA estimates in this work. See the text for definitions.

				correlation matrix ρ_{ij}							
i	γ_i	η_i^0	σ_i	^{76}Ge	^{82}Se	^{96}Zr	^{100}Mo	^{116}Cd	^{128}Te	^{130}Te	^{136}Xe
^{76}Ge	25.517	0.635	0.122	1							
^{82}Se	24.870	0.571	0.135	0.978	1						
^{96}Zr	24.550	0.038	0.247	0.518	0.506	1					
^{100}Mo	24.660	0.503	0.162	0.973	0.957	0.491	1				
^{116}Cd	24.622	0.404	0.150	0.961	0.961	0.474	0.965	1			
^{128}Te	26.073	0.534	0.154	0.947	0.968	0.515	0.916	0.930	1		
^{130}Te	24.674	0.498	0.158	0.899	0.927	0.575	0.862	0.870	0.964	1	
^{136}Xe	24.644	0.254	0.187	0.805	0.846	0.663	0.747	0.773	0.898	0.916	1

where:

$$\mathrm{cov}(\eta_i, \eta_j) = \rho_{ij}\sigma_i\sigma_j$$ **covariance matrix**

Comparison with other (2008) state-of-the-art NME estimates



-Independent η_i values fall within our estimated 3σ range

-The points appear roughly aligned to the major axis of each ellipse

-Independent confirmation of positive correlations between NME.

Application 1: not involving correlations

We translate:

90% limits on half-lives → 90% limits on the Majorana neutrino mass

TABLE II: Best current limits on half-lives at 90% C.L. ($T_i > T_i^{90}$ and $\tau_i > \tau_i^{90}$) for different nuclei i, from [26].

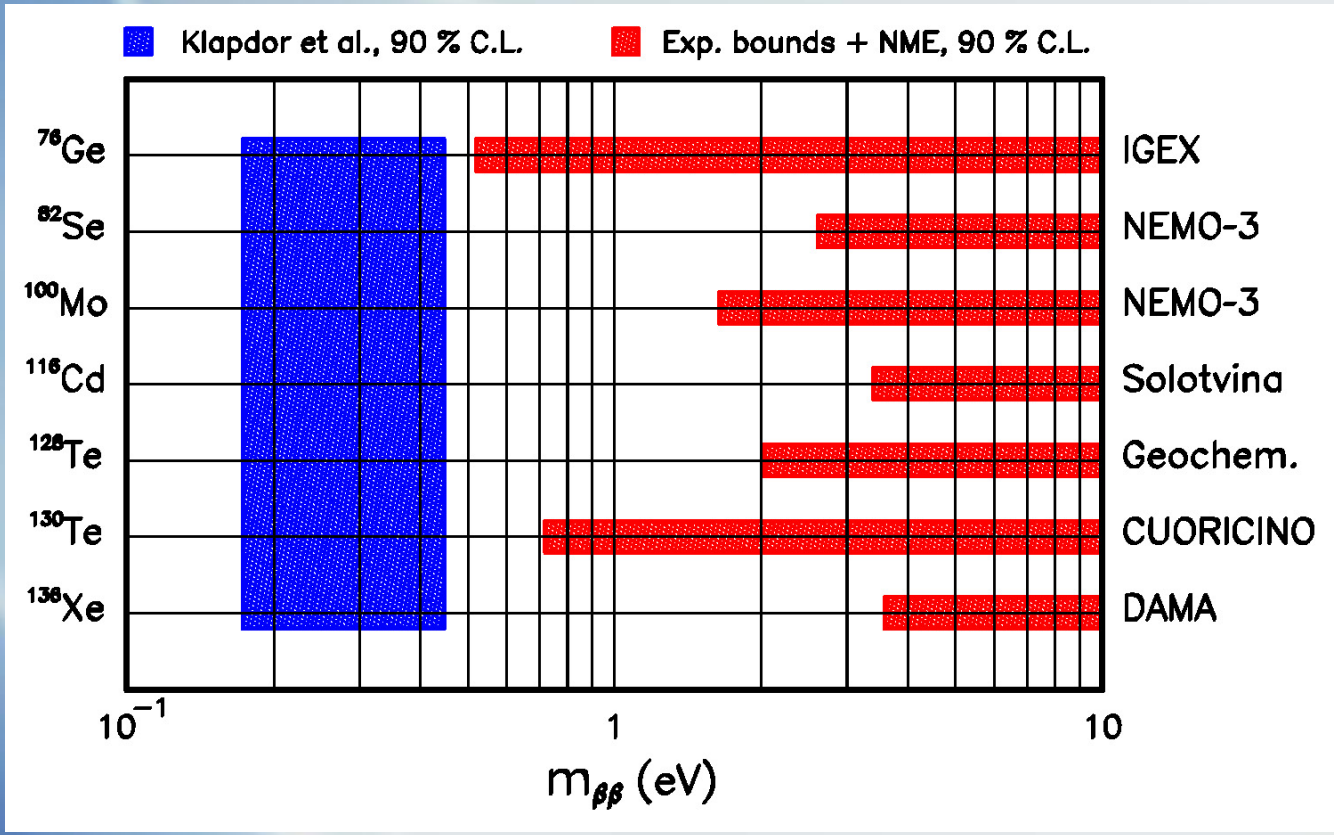
i	T_i^{90}/y	τ_i^{90}	Experiment	Ref.
^{76}Ge	1.6×10^{25}	25.204	IGEX	[27]
^{82}Se	2.1×10^{23}	23.322	NEMO-3	[28]
^{96}Zr	8.6×10^{21}	21.934	NEMO-3	[28]
^{100}Mo	5.8×10^{23}	23.763	NEMO-3	[28]
^{116}Cd	1.7×10^{23}	23.230	Solotvina	[29]
^{128}Te	7.7×10^{24}	24.886	Geochem.	[30]
^{130}Te	3.0×10^{24}	24.477	CUORICINO	[31]
^{136}Xe	4.5×10^{23}	23.653	DAMA	[32]

$$\mu < \frac{1}{2}(\gamma_i - \tau_i^{90}) - \eta_i$$

$$< \frac{1}{2}(\gamma_i - \tau_i^{90}) - \eta_i^0 + 1.64\sigma_i = \mu^{90}$$

where $\eta_i = \log_{10} |M_i|$

$\mu = \log_{10}(m_{\beta\beta}/\mathrm{eV})$

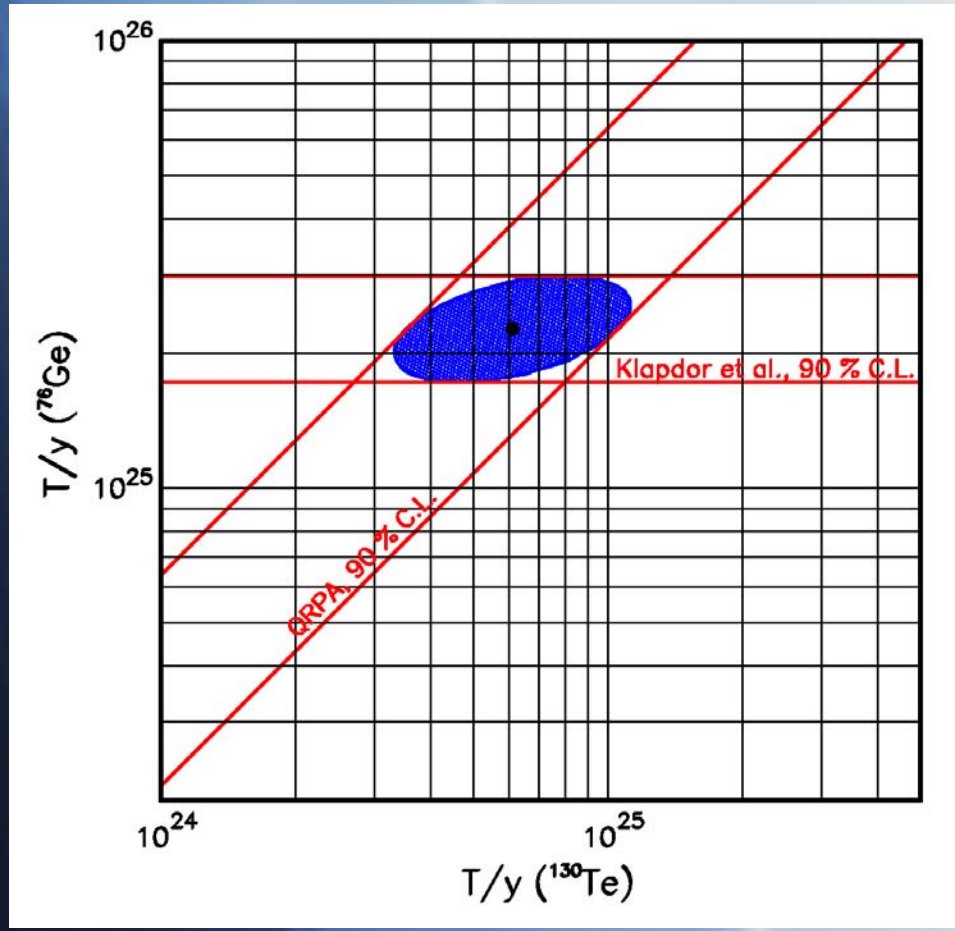


Range of $m_{\beta\beta}$ allowed at 90% C.L. by Klapdor $0\nu 2\beta$ claim:

$$T_i / y = 2.23^{+0.44}_{-0.31} \times 10^{25}$$

compared with the 90% limits placed by other experiments. The comparison involves the NME and their errors, as estimated in our work.

Application 2: involving correlations
comparison of half-life in a couple of nuclei . . .



Theoretical and experimental constraints at 90% C.L.

Horizontal band: Klapdor et al. claim.

Slanted band: our QRPA constraint.

The combination provides the shaded ellipse.

Ellipse projection gives the

^{130}Te prediction expected from ^{76}Ge claim

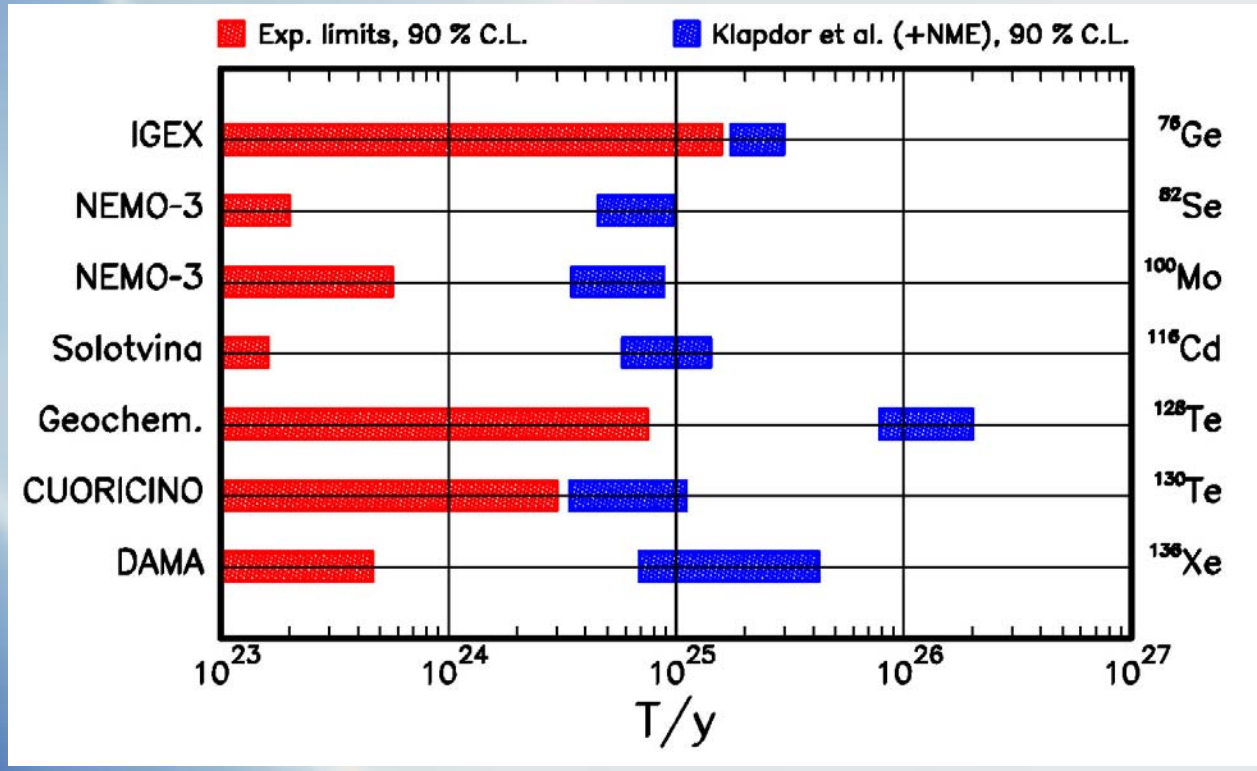
Numerically:

If: $\tau_i(^{76}\mathrm{Ge}) = 25.355 \pm 0.118$ (90% C.L.)

then $\tau_j(^{130}\mathrm{Te}) = 24.786 \pm 0.161$ (90% C.L.)

with correlation: $r_{ij} = 0.447$

. . . repeat for more nuclei



Range of T_i at 90% C.L. by Klapdor $0\nu 2\beta$ claim, compared with the 90% limits placed by other experiments. The comparison involves the NME, their errors and correlations, as estimated in our work.

NOTE: None of the existing limits reaches the range preferred by Klapdor's claim.

Application 3: combination of several nuclei and degeneracy effects

Assumptions: N different nuclei with measured half-life $\tau_i = \tau_i^0 \pm s_i$ $(i = 1, \ldots, N)$

light Majorana neutrino exchange

We obtain N linear equations

See arXiv: 0810.5733 for details

LEAST SQUARE METHOD

μ value estimation $\mu = \log_{10}(m_{\beta\beta}/\mathrm{eV})$

$$\mu_i = \mu_i^0 \pm \delta$$

Exercise: We consider: ^{76}Ge ^{82}Se ^{130}Te ^{136}Xe and

we fix $\mu_0 = -0.7$ ($m_{\beta\beta} = 0.2$ MeV) and 20% experimental uncertainty for predicted half-lives

The relevant output parameter is then the uncertainty δ.

1, 2, 3, 4 nuclei combinations

TABLE IV: Combination of any among the four hypothetical half-life data T_i in Eq. (35) with experimental uncertainty $\delta T_i/T_i = 20\%$. Results are given in terms of the total 1σ error δ on the parameter $\mu = \log_{10}(m_{\beta\beta}/\mathrm{eV})$, including theoretical uncertainties without and with correlations. Bullets indicate the data included in the evaluation (from 1 to 4 data).

# of data	^{76}Ge	^{82}Se	^{130}Te	^{136}Xe	δ (w/o corr.)	δ (with corr.)
1	•				0.128	0.128
1		•			0.141	0.141
1			•		0.163	0.163
1				•	0.191	0.191
2	•	•			0.095	0.128
2	•		•		0.100	0.128
2	•			•	0.106	0.127
2		•	•		0.107	0.141
2		•		•	0.114	0.141
2			•	•	0.124	0.163
3	•	•	•		0.082	0.127
3	•	•		•	0.085	0.127
3	•		•	•	0.089	0.127
3		•	•	•	0.093	0.140
4	•	•	•	•	0.075	0.127

W/o correlations: error δ decreases, by increasing the data sample

With correlations: error δ cannot be much better than the smallest theoretical uncertainty. A **degeneracy effect** appear, induced by correlations (i.e., error on Majorana mass is degenerate with smallest NME error).

Application 4: probing nonstandard particle physics models

Refs: literature recently reviewed by Deppisch and Pas, hep-ph/0612165
Gehman and Elliott, hep-ph/0701099

Half-life for a given candidate nucleus:

$$[T_i^{0\nu}]^{-1} = G_i^l \left| M_i^l \right|^2 \lambda_l^2$$

different decay mechanism

NME
Nuclear physics

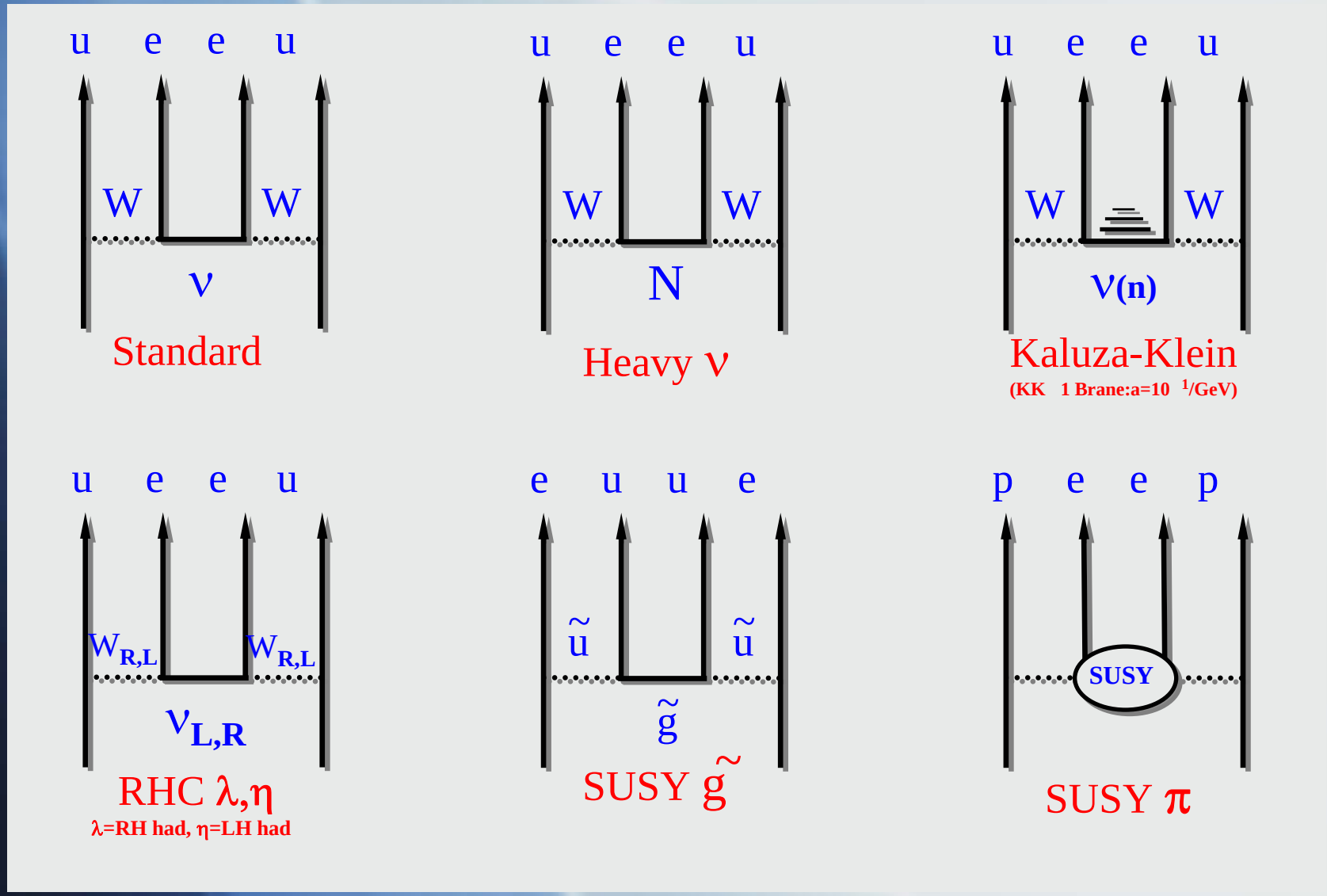
lepton number violation param
Particle physics

"Standard scenario": light Majorana neutrino exchange and QRPA

Other mechanism $0\nu 2\beta$ decay:

- **Heavy ν**: exchange of Majorana neutrinos with heavy masses (> 1 GeV)
- **SUSY π:** supersymmetric models with R-parity violation and pion exchange
- **SUSY g:** as above, but with gluino exchange
- **RHC η:** left-right symm models with RH leptonic currents coupled to LH hadronic currents
- **RHC λ**: as above but coupled to RH hadronic currents
- **KK+1:** Kaluza-Klein neutrino exchange with brane-shift param. a = 10^{+1} GeV^{-1}
- **KK-1:** as above, but with a = 10^{-1} GeV^{-1}

Each (non)standard case is assumed to be dominant (no interference)



Null Hypothesis: we focus on 4 candidate nuclei ^{76}Ge ^{82}Se ^{130}Te ^{136}Xe

and we fix: $T(^{76}Ge) = 10^{26}$ y

Within the standard scenario, we obtain a set of predictions for log half-lives and their 1σ errors:

$$\tau_i = \tau_i^0 \pm s_i^0 \qquad \text{(with correlations } \rho_{ij}\text{)}$$

Then, we assume that several experiments will observe positive $0\nu 2\beta$ signals, with half-life accuracy $dT_i/T_i = 20\%$

Statistical approach: **χ^2 test** to compare the **null hypothesis** with **mock data**

Centered at predictions for several nonstandard mechanisms, rescaled to match a benchmark value of $T(^{76}Ge) = 10^{26}$ y

See: arXiv: 0905.1832 for details

Results:

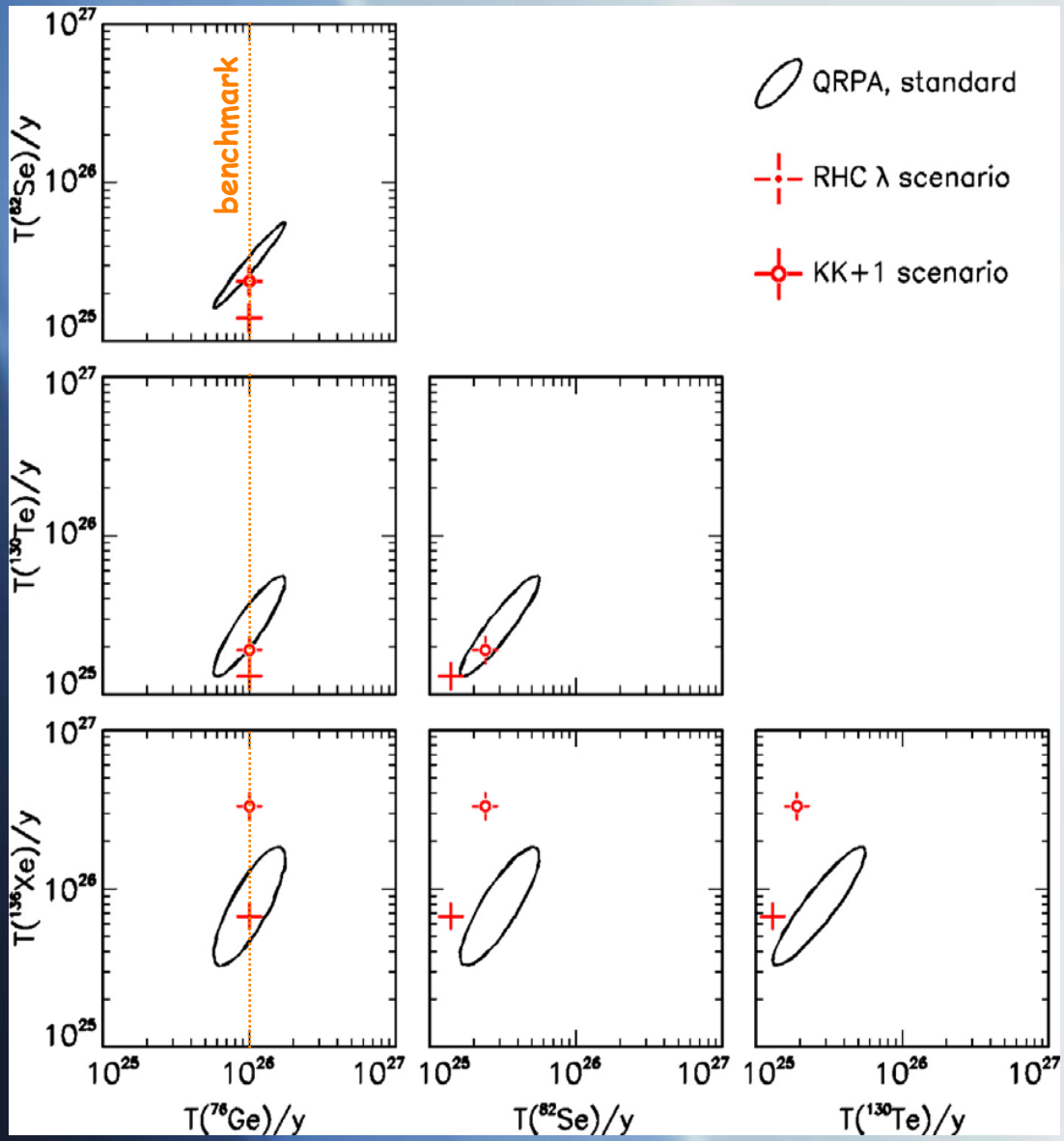
TABLE IV: Test of the null hypothesis (QRPA, standard), assuming $0\nu\beta\beta$ half-life data as predicted by different particle physics models, with 20% experimental uncertainties. Second column: rejection probability P_r (%) with four nuclei. Third column: rejection probability range for three nuclei (^{76}Ge plus any other two). The null hypothesis can be rejected at > 95% C.L. in two models, marked by *. Fourth and fifth columns: as in the previous two columns, but without correlations.

Particle physics model			4 nuclei, %	3 nuclei, %	4 nuclei, % (no ρ_{ij})	3 nuclei, % (no ρ_{ij})
Heavy ν			53.3	49.6 – 71.3	29.8	28.0 – 49.4
SUSY π		MIN	5.2	5.6 – 14.7	0.2	0.7 – 2.0
SUSY $\tilde{g}$			27.1	18.8 – 47.7	9.8	9.4 – 24.4
RHC η			10.5	17.9 – 24.1	4.5	7.7 – 14.9
RHC λ *	Rejection		97.6	89.5 – 97.9	50.1	38.3 – 69.1
KK+1 *	Rejection	MAX	99.9	35.4 – 99.9	61.8	15.7 – 77.0
KK−1			37.0	14.9 – 57.2	4.8	2.6 – 15.7

From: arXiv: 0905.1832v1

Conclusion:

- with 4 nuclei, at least 2 non standard mechanism (RHC λ and KK+1) can be distinguished from the standard one at > 95% C.L.
- RHC λ and KK+1 models can reach P > 95% C.L. even with some 3 nuclei combinations.
  - correlations of theoretical errors are crucial for the statistical power of the test: w/o correlations, significant discrimination is never obtained.



NULL HYPOTHESIS

-if $\rho_{ij} = \delta_{ij}$ all the ellipses would cover a much larger fraction of the plane

- if $\rho_{ij} = 1$ all the ellipses would shrink to slanted segments: theoretical errors would exactly cancel and the analysis would be dominated by experimental errors

- detailed estimates of the theoretical covariance matrix is crucial.

Application 5: probing nuclear physics models

Half-life for a given candidate nucleus:

$$[T_i^{0\nu}]^{-1} = G_i^l \left| M_i^l \right|^2 \lambda_l^2$$

different decay mechanism

NME

Nuclear physics

lepton number violation param

Particle physics

"Standard scenario": light Majorana neutrino exchange and QRPA

Alternative NME calculations::

- **QRPA** calc by Suhonen and Kortelainen (S&K) for fixed g_A= 1.00 and Jastrow s.r.c.
- **QRPA** (S&K) for g_A= 1.00 and UCOM s.r.c.
- **QRPA** (S&K) for g_A= 1.25 and Jastrow s.r.c.
- **QRPA** (S&K) for g_A= 1.25 and UCOM s.r.c.
- **Shell Model calc for fixed** g_A= 1.25 and Jastrow s.r.c.
- **Shell Model with** g_A= 1.25 and UCOM s.r.c.

Null Hypothesis: the same as before (standard Majorana mechanism and QRPA).

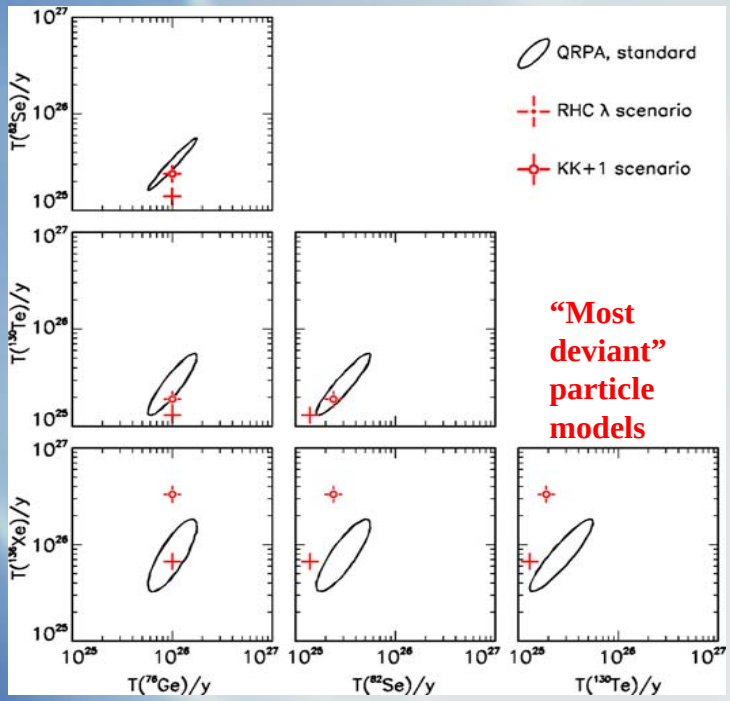
Mock data: centered on the state-of-the-art calculations by nuclear physics models (for light Majorana neutrino exchange), normalized to: $T(^{76}Ge) = 10^{26}$ y .

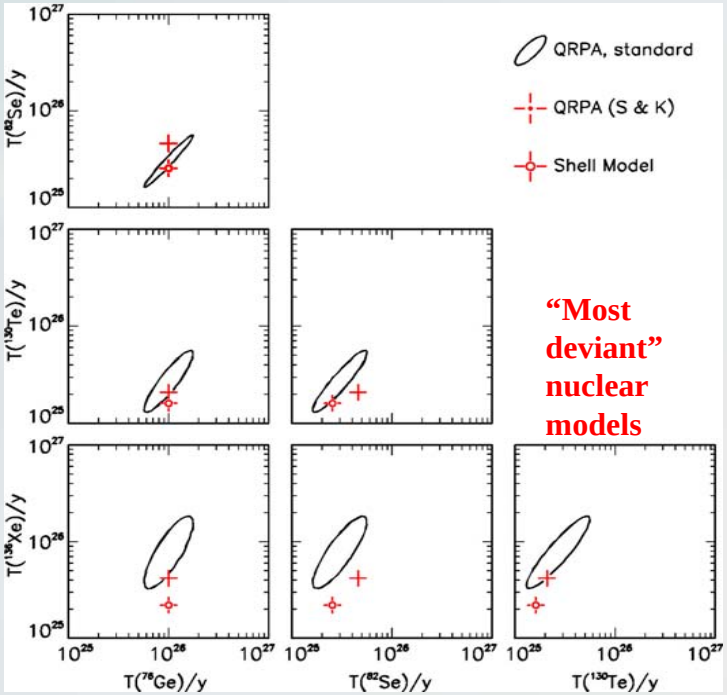
Results:

TABLE VI: Test of the null hypothesis (QRPA, standard), assuming $0\nu\beta\beta$ half-life data as predicted by different nuclear physics models, with 20% experimental uncertainties. Second column: rejection probability P_r (%) with four nuclei. Third column: rejection probability range for three nuclei (^{76}Ge plus any other two). The null hypothesis is not rejected, within 95% C.L. Fourth and fifth columns: as in the previous two columns, but without correlations of theoretical errors.

Nuclear physics model	4 nuclei, %	3 nuclei, %	4 nuclei, % ($\rho_{ij} = 0$)	3 nuclei, % ($\rho_{ij} = 0$)
QRPA (S&K), $g_A = 1.00$, Jastrow	86.8	45.0 – 93.7	19.4	22.5 – 37.0
QRPA (S&K), $g_A = 1.00$, UCOM	87.0	43.0 – 93.1	20.5	23.6 – 36.5
QRPA (S&K), $g_A = 1.25$, Jastrow	79.9	30.8 – 90.1	12.9	10.0 – 29.8
QRPA (S&K), $g_A = 1.25$, UCOM	80.9	28.1 – 89.9	14.2	14.5 – 30.3
Shell Model, $g_A = 1.25$, Jastrow	78.9	46.4 – 89.4	50.3	21.0 – 68.4
Shell Model, $g_A = 1.25$, UCOM	81.1	52.5 – 90.7	53.6	23.5 – 71.3

- none of the six nuclear models can reject the null hypothesis at > 95% C.L.
- the six nuclear models are phenomenologically indistinguishable from our reference QRPA model.





- the two most deviant nuclear physics models are closer to standard predictions than the two most deviant particle physics models;
- in any given panel, the largest deviations between theory and data are generally opposite in the two fig., implying that nuclear physics variations do not mimic particle physics variations.

Conclusion: using prospective data in 4 promising nuclei, the discrimination in some particle mechanism is not spoiled by estimated uncertainties in state-of-the-art nuclear theory calculations.

Conclusion

- NME for $0\nu2\beta$ decay are affected by relatively large theoretical uncertainties.
- We have shown that, within a given set of nuclei, the correlations among NME errors are as important as their size.
- We have made a first attempt to quantify the covariance matrix of the NME. Its effects have been clarified through a series of examples. Some consequences:
  - Correlations are important in comparing signals or limits from different nuclei.
  - They may severely limit the accuracy in the reconstruction of $m_{\beta\beta}$ from any number of $0\nu2\beta$ observations in different nuclei, due to a degeneracy effect.
  - State-of-the-art calculations for light Majorana neutrino exchange would not be distinguishable within errors (at 95% CL), even using four different nuclei
  - However, light Majorana neutrino exchange could be distinguished from at least two alternative, nonstandard mechanisms (RHCλ and KK+1 scenarios) at > 95% CL, using 4 (or even 3) prospective data from different nuclei.
  - Correlations appear to be crucial ingredients in this context.

THANK YOU FOR YOUR ATTENTION

Backup Slides

The g_A and g_{pp} parameter

Experimentally, the observed GT strength ($\propto g_A^2$) in nuclei is weaker than in vacuum (**"quenching"**): $g_A < 1.25$.

The $0\nu 2\beta$ matrix element components $M_X^{0\nu}$ (X = F, GT, T) may also depend on model-specific parameters. In **QRPA (quasiparticle random phase approximation**) the most important is the so-called particle-particle strength parameter g_{pp} :

$$M^{0\nu}(g_{pp}, g_A) = M_{GT}^{0\nu}(g_{pp}) + M_T^{0\nu}(g_{pp}) - \frac{M_F^{0\nu}(g_{pp})}{g_A^2}$$

g_A and g_{pp} **expected to be of O(1)**, but precise values not fixed by first principles. May be different in different nuclei. Must be **constrained by experimental data**.