Variances and Covariances of the Nuclear Matrix Elements for the $0\nu 2\beta$ decay

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Based on work done in collaboration with

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Contents

- $0\nu2\beta$ decay and QRPA free parameters
- Nuclear matrix element uncertainties and their correlations
- Results and implications
- Probing particle and nuclear physics models
- Conclusions
Neutrinoless double beta decay ($0\nu2\beta$)

A second-order ($\Delta Z = 2$) weak process:

$$(Z, A) \to (Z+1, A) \to (Z+2, A) + 2e^-$$

with lifetime:

$$[T_i^{0\nu}]^{-1} = G_i^{0\nu} |M'_i|^2 m^2_{\beta\beta}$$

- in a given candidate nucleus $i = (Z,A)$
- assuming that the process is due to light Majorana neutrinos.

known phase space factor $G^{0\nu}$ nuclear matrix element (N.M.E.) $|M'|^2$ “effective Majorana neutrino mass” $m^2_{\beta\beta}$

$M'(g_{pp}, g_A)$ must be evaluated in a nuclear model
NME uncertainties and correlations

NME variances are important to compare $0\nu2\beta$ limits (or signals) in different nuclei. NME covariances are important as well: e.g.

$$T_{j}^{0\nu} = T_{i}^{0\nu} \frac{G_{i}^{0\nu} |M_{i}'|^2}{G_{j}^{0\nu} |M_{j}'|^2}$$

half-life expected in a nucleus j, if $T_{i}$ is measured in a nucleus i.

“most favorable case” (experimentally)
→ shortest $T_{j}$, i.e. smallest $|M_{i}'|$ and largest $|M_{j}'|$

However, NME positive correlations prevent such favorable case (NME go all “up” or “down”)

Our focus is to show that, indeed:
- there are high correlations between NME uncertainties in any couple of nuclei
- these correlations play an important role in the phenomenology of $0\nu2\beta$ decay
Notation:

\[ \tau_i = \log_{10}(T_i/y) \]
\[ -\gamma_i = \log_{10}[G_{i}^{0\nu}/(y^{-1}\text{eV}^{-2})] \]
\[ \eta_i = \log_{10}|M'_i| \]
\[ \mu = \log_{10}(m_{\beta\beta}/\text{eV}) \]

**Linearization:**

\[ [T_{i}^{0\nu}]^{-1} = G_{i}^{0\nu}\left|M'_i\right|^2 m_{\beta\beta}^{2} \rightarrow \tau_i = \gamma_i - 2\eta_i - 2\mu \]

This is more appropriate to deal with relatively large NME errors.

E.g., consider a “factor of two” uncertainty:

\[ \left|M'_i\right| = \left(\frac{1}{2} \ldots 2\right) \rightarrow \left|M'_i\right| = \left|M_i^{0\nu}'\right| \times \left(1^{+1.0}_{-0.5}\right) \text{ at } 1\sigma \]

The above error parametrization is **not convenient**, because:
- asymmetric errors are difficult to manage with usual statistical tools;
- the unphysical region \( |M'_i| < 0 \) is hit at \( > 2\sigma \)

Both drawbacks are avoided by taking logarithms:

\[ \log_{10}|M'_i| = \log_{10}|M_i^{0\nu}'| \pm 0.30 \]

\[ \eta_i = \eta_i^{0} \pm 0.30 \]
**Input Ingredients**

We consider a set of eight $0\nu2\beta$ candidate nuclei:

$$^{76}\text{Ge} \quad ^{82}\text{Se} \quad ^{96}\text{Zr} \quad ^{100}\text{Mo} \quad ^{116}\text{Cd} \quad ^{128}\text{Te} \quad ^{130}\text{Te} \quad ^{136}\text{Xe}$$

Within the quasiparticle random phase approximation (QRPA), we consider*:

- **2 values** for the *axial coupling*: $g_A = 1.25$ (bare) and $g_A = 1.00$ (quenched);
- **2 approaches** to *short range correlations*\(^{**}\) (s.r.c.): the so-called Jastrow-type s.r.c., and the unitary correlation operator method (UCOM);
- **3 size** for model *basis*: small, intermediate and large;
- **2 many-body models**: QRPA and its renormalized version (RQRPA).

$$2 \times 2 \times 3 \times 2 = 24 \text{ variants}$$

supplemented by errors induced by $g_{pp}$ errors through the experimentally observed $2\nu2\beta$ processes.

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** s.r.c. are peculiar of the $0\nu2\beta$ decay and account for the repulsive nucleon-nucleon interaction at small distances.
RESULTS:

\[(\eta_i, \eta_j) \quad \text{distribution}\]

where \(\eta_i = \log_{10} |M_i|\)

- Strong positive correlations among theoretical estimates
- QRPA calculations scattered along the ellipse major axis, as a result of s.r.c. model (Jastrow, UCOM)
- Dispersion along the ellipse minor axis due to \(g_{pp}\) variations
- \(^{96}\text{Zr}\) big errors: QRPA collapse

Note: errors conservatively inflated to embrace min/max values of NME

From: arXiv: 0810.5733
where:

\[
\text{cov}(\eta_i, \eta_j) = \rho_{ij} \sigma_i \sigma_j \quad \text{covariance matrix}
\]
Comparison with other (2008) state-of-the-art NME estimates

\[(\eta_i, \eta_j) \quad \text{distribution}\]

where \( \eta_i = \log_{10} |M_i| \)

error ellipse at 1\(\sigma\), 2\(\sigma\), 3\(\sigma\)

dots = QRPA (Suhonen & Kortelainen '08)
stars = shell model (Menendez, Poves et al. '08)
blue \(\rightarrow\) Jastrow, red \(\rightarrow\) UCOM

- Independent \(\eta_i\) values fall within our estimated 3\(\sigma\) range
- The points appear roughly aligned to the major axis of each ellipse
- Independent confirmation of positive correlations between NME.
Application 1: not involving correlations

We translate:

90% limits on half-lives → 90% limits on the Majorana neutrino mass

\[
\begin{align*}
\mu &< \frac{1}{2} (\gamma_i - \tau_i^{90}) - \eta_i \\
\eta_i &< \frac{1}{2} (\gamma_i - \tau_i^{90}) - \eta_i^0 + 1.64 \sigma_i = \mu^{90}
\end{align*}
\]

where \[\eta_i = \log_{10} |M_i|\]

\[
\mu = \log_{10} (m_{\beta\beta} / \text{eV})
\]
Range of $m_{\beta\beta}$ allowed at 90% C.L. by Klapdor $0\nu2\beta$ claim:

$$T_i / y = 2.23^{+0.44}_{-0.31} \times 10^{25}$$

compared with the 90% limits placed by other experiments. The comparison involves the NME and their errors, as estimated in our work.
Application 2: involving correlations

comparison of half-life in a couple of nuclei . . .

Theoretical and experimental constraints at 90% C.L.
Horizontal band: Klapdor et al. claim.
Slanted band: our QRPA constraint.
The combination provides the shaded ellipse.
Ellipse projection gives the

\[ \text{\(^{130}\text{Te prediction expected from }^{76}\text{Ge claim}} \]

Numerically:

If: \( \tau_i(^{76}\text{Ge}) = 25.355 \pm 0.118 \) (90% C.L.)
then \( \tau_j(^{130}\text{Te}) = 24.786 \pm 0.161 \) (90% C.L.)
with correlation: \( r_{ij} = 0.447 \)
Range of $T_i$ at 90% C.L. by Klapdor $0\nu2\beta$ claim, compared with the 90% limits placed by other experiments. The comparison involves the NME, their errors and correlations, as estimated in our work.

**NOTE:** None of the existing limits reaches the range preferred by Klapdor’s claim.
Application 3: combination of several nuclei and degeneracy effects

Assumptions: N different nuclei with measured half-life \( \tau_i = \tau_i^0 \pm s_i \quad (i = 1, \ldots, N) \)
light Majorana neutrino exchange

We obtain N linear equations

LEAST SQUARE METHOD

\[
\mu = \log_{10}(m_{\beta\beta}/\text{eV})
\]

\[
\mu_i = \mu_i^0 \pm \delta
\]

Exercise: We consider: \(^{76}\text{Ge} \quad ^{82}\text{Se} \quad ^{130}\text{Te} \quad ^{136}\text{Xe}\) and
we fix \(\mu_0 = -0.7 \quad (m_{\beta\beta} = 0.2 \text{ MeV})\) and 20% experimental uncertainty for predicted half-lives.
The relevant output parameter is then the uncertainty \(\delta\).
1, 2, 3, 4 nuclei combinations

TABLE IV: Combination of any among the four hypothetical half-life data $T_i$ in Eq. (35) with experimental uncertainty $\delta T_i/T_i = 20\%$. Results are given in terms of the total 1σ error $\delta$ on the parameter $\mu = \log_{10}(m_{\beta\beta}/eV)$, including theoretical uncertainties without and with correlations. Bullets indicate the data included in the evaluation (from 1 to 4 data).

<table>
<thead>
<tr>
<th># of data</th>
<th>$^{76}$Ge</th>
<th>$^{82}$Se</th>
<th>$^{130}$Te</th>
<th>$^{136}$Xe</th>
<th>$\delta$ (w/o corr.)</th>
<th>$\delta$ (with corr.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td>0.128</td>
<td>0.128</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>*</td>
<td></td>
<td></td>
<td>0.141</td>
<td>0.141</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td>*</td>
<td></td>
<td>0.163</td>
<td>0.163</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.191</td>
<td>0.191</td>
</tr>
<tr>
<td>2</td>
<td>*</td>
<td></td>
<td>*</td>
<td></td>
<td>0.095</td>
<td>0.128</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>*</td>
<td></td>
<td></td>
<td>0.100</td>
<td>0.128</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td>*</td>
<td></td>
<td>0.106</td>
<td>0.127</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td>*</td>
<td>0.107</td>
<td>0.141</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.114</td>
<td>0.141</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.124</td>
<td>0.163</td>
</tr>
<tr>
<td>3</td>
<td>*</td>
<td>*</td>
<td></td>
<td></td>
<td>0.082</td>
<td>0.127</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>*</td>
<td></td>
<td></td>
<td>0.085</td>
<td>0.127</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td>*</td>
<td></td>
<td>0.089</td>
<td>0.127</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td>*</td>
<td>0.093</td>
<td>0.140</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.075</td>
<td>0.127</td>
</tr>
</tbody>
</table>

W/o correlations: error $\delta$ decreases, by increasing the data sample

With correlations: error $\delta$ cannot be much better than the smallest theoretical uncertainty. A degeneracy effect appear, induced by correlations (i.e., error on Majorana mass is degenerate with smallest NME error).
Application 4: probing nonstandard particle physics models

Refs: literature recently reviewed by Deppisch and Pas, hep-ph/0612165
Gehman and Elliott, hep-ph/0701099

Half-life for a given candidate nucleus:

\[ [T_{i}^{0\nu}]^{-1} = G_{i}^{l} |M_{i}^{l}|^{2} \]

Nuclear physics

lepton number violation param

Particle physics

“Standard scenario”: light Majorana neutrino exchange and QRPA

Other mechanism $0\nu2\beta$ decay:

- **Heavy $\nu$**: exchange of Majorana neutrinos with heavy masses (> 1 GeV)
- **SUSY $\pi$**: supersymmetric models with R-parity violation and pion exchange
- **SUSY $g$**: as above, but with gluino exchange
- **RHC $\eta$**: left-right symm models with RH leptonic currents coupled to LH hadronic currents
- **RHC $\lambda$**: as above but coupled to RH hadronic currents
- **KK+1**: Kaluza-Klein neutrino exchange with brane-shift param. $a = 10^{+1}$ GeV$^{-1}$
- **KK-1**: as above, but with $a = 10^{-1}$ GeV$^{-1}$
Each (non)standard case is assumed to be dominant (no interference)
**Null Hypothesis:** we focus on 4 candidate nuclei $^{76}\text{Ge}$  $^{82}\text{Se}$  $^{130}\text{Te}$  $^{136}\text{Xe}$

and we fix: $T(^{76}\text{Ge}) = 10^{26}$ y

Within the standard scenario, we obtain a set of predictions for log half-lives and their $1\sigma$ errors:

$$\tau_i = \tau_i^0 \pm s_i^0 \quad \text{(with correlations } \rho_{ij})$$

Then, we assume that several experiments will observe positive $0\nu2\beta$ signals, with half-life accuracy $dT_i/T_i = 20\%$

Statistical approach: $\chi^2$ test to compare the null hypothesis with mock data

Centered at predictions for several nonstandard mechanisms, rescaled to match a benchmark value of $T(^{76}\text{Ge})= 10^{26}$ y

See: arXiv: 0905.1832 for details
**Results:**

<table>
<thead>
<tr>
<th>Particle physics model</th>
<th>4 nuclei, %</th>
<th>3 nuclei, %</th>
<th>4 nuclei, % (no ρ_{ij})</th>
<th>3 nuclei, % (no ρ_{ij})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heavy ν</td>
<td>53.3</td>
<td>49.6 - 71.3</td>
<td>29.8</td>
<td>28.0 - 49.4</td>
</tr>
<tr>
<td>SUSY π</td>
<td>5.2</td>
<td>5.6 - 14.7</td>
<td>0.2</td>
<td>0.7 - 2.0</td>
</tr>
<tr>
<td>SUSY ̄{\gamma}</td>
<td>27.1</td>
<td>18.8 - 47.7</td>
<td>9.8</td>
<td>9.4 - 24.4</td>
</tr>
<tr>
<td>RHC η</td>
<td>10.5</td>
<td>17.9 - 24.1</td>
<td>4.5</td>
<td>7.7 - 14.9</td>
</tr>
<tr>
<td>RHC λ *</td>
<td>97.6</td>
<td>89.5 - 97.9</td>
<td>50.1</td>
<td>38.3 - 69.1</td>
</tr>
<tr>
<td>KK+1 *</td>
<td>99.9</td>
<td>35.4 - 99.9</td>
<td>61.8</td>
<td>15.7 - 77.0</td>
</tr>
<tr>
<td>KK−1</td>
<td>37.0</td>
<td>14.9 - 57.2</td>
<td>4.8</td>
<td>2.6 - 15.7</td>
</tr>
</tbody>
</table>

**Conclusion:**

- with 4 nuclei, at least 2 non standard mechanism (RHC λ and KK+1) can be distinguished from the standard one at > 95% C.L.
- RHC λ and KK+1 models can reach P > 95% C.L. even with some 3 nuclei combinations.
- correlations of theoretical errors are crucial for the statistical power of the test: w/o correlations, significant discrimination is never obtained.
NULL HYPOTHESIS

-if $\rho_{ij} = \delta_{ij}$ all the ellipses would cover a much larger fraction of the plane
- if $\rho_{ij} = 1$ all the ellipses would shrink to slanted segments: theoretical errors would exactly cancel and the analysis would be dominated by experimental errors
- detailed estimates of the theoretical covariance matrix is crucial.
Application 5: probing nuclear physics models

Half-life for a given candidate nucleus:

\[ [T_i^{0\nu}]^{-1} = G_i^l |M_i^l|^2 \lambda_{ij}^2 \]

Nuclear physics

lepton number violation param

Particle physics

different decay mechanism

“Standard scenario”: light Majorana neutrino exchange and QRPA

Alternative NME calculations:

- QRPA calc by Suhonen and Kortelainen (S&K) for fixed \( g_A = 1.00 \) and Jastrow s.r.c.
- QRPA (S&K) for \( g_A = 1.00 \) and UCOM s.r.c.
- QRPA (S&K) for \( g_A = 1.25 \) and Jastrow s.r.c.
- QRPA (S&K) for \( g_A = 1.25 \) and UCOM s.r.c.
- Shell Model calc for fixed \( g_A = 1.25 \) and Jastrow s.r.c.
- Shell Model with \( g_A = 1.25 \) and UCOM s.r.c.
**Null Hypothesis:** the same as before (standard Majorana mechanism and QRPA).

**Mock data:** centered on the state-of-the-art calculations by nuclear physics models (for light Majorana neutrino exchange), normalized to: \( T^{(76}\text{Ge}) = 10^{26} \text{ y} \).

**Results:**

<table>
<thead>
<tr>
<th>Nuclear physics model</th>
<th>4 nuclei, %</th>
<th>3 nuclei, %</th>
<th>4 nuclei, % ((\rho_{ij} = 0))</th>
<th>3 nuclei, % ((\rho_{ij} = 0))</th>
</tr>
</thead>
<tbody>
<tr>
<td>QRPA (S&amp;K), (g_A = 1.00), Jastrow</td>
<td>86.8</td>
<td>45.0 – 93.7</td>
<td>19.4</td>
<td>22.5 – 37.0</td>
</tr>
<tr>
<td>QRPA (S&amp;K), (g_A = 1.00), UCOM</td>
<td>87.9</td>
<td>43.0 – 93.1</td>
<td>20.5</td>
<td>23.6 – 36.5</td>
</tr>
<tr>
<td>QRPA (S&amp;K), (g_A = 1.25), Jastrow</td>
<td>79.9</td>
<td>30.8 – 90.1</td>
<td>12.9</td>
<td>10.0 – 29.8</td>
</tr>
<tr>
<td>QRPA (S&amp;K), (g_A = 1.25), UCOM</td>
<td>80.9</td>
<td>28.1 – 89.9</td>
<td>14.2</td>
<td>14.5 – 30.3</td>
</tr>
<tr>
<td>Shell Model, (g_A = 1.25), Jastrow</td>
<td>78.9</td>
<td>46.4 – 89.4</td>
<td>50.3</td>
<td>21.0 – 68.4</td>
</tr>
<tr>
<td>Shell Model, (g_A = 1.25), UCOM</td>
<td>81.1</td>
<td>52.5 – 90.7</td>
<td>53.6</td>
<td>23.5 – 71.3</td>
</tr>
</tbody>
</table>

- none of the six nuclear models can reject the null hypothesis at > 95% C.L.
- the six nuclear models are phenomenologically indistinguishable from our reference QRPA model.
- the two most deviant nuclear physics models are closer to standard predictions than the two most deviant particle physics models;
- in any given panel, the largest deviations between theory and data are generally opposite in the two fig., implying that nuclear physics variations do not mimic particle physics variations.

**Conclusion:** using prospective data in 4 promising nuclei, the discrimination in some particle mechanism is not spoiled by estimated uncertainties in state-of-the-art nuclear theory calculations.
Conclusion

- NME for $0\nu2\beta$ decay are affected by relatively large theoretical uncertainties.
- We have shown that, within a given set of nuclei, the correlations among NME errors are as important as their size.
- We have made a first attempt to quantify the covariance matrix of the NME. Its effects have been clarified through a series of examples. Some consequences:
  - Correlations are important in comparing signals or limits from different nuclei.
  - They may severely limit the accuracy in the reconstruction of $m_{\beta\beta}$ from any number of $0\nu2\beta$ observations in different nuclei, due to a degeneracy effect.
  - State-of-the-art calculations for light Majorana neutrino exchange would not be distinguishable within errors (at 95% CL), even using four different nuclei.
  - However, light Majorana neutrino exchange could be distinguished from at least two alternative, nonstandard mechanisms (RHC$\lambda$ and KK+1 scenarios) at > 95% CL, using 4 (or even 3) prospective data from different nuclei.
- Correlations appear to be crucial ingredients in this context.

THANK YOU FOR YOUR ATTENTION
Backup Slides
The $g_A$ and $g_{pp}$ parameter

Experimentally, the observed GT strength ($\propto g_A^2$) in nuclei is weaker than in vacuum ("quenching"): $g_A < 1.25$.

The $0\nu2\beta$ matrix element components $M_{X}^{0\nu}$ ($X = F, GT, T$) may also depend on model-specific parameters. In QRPA (quasiparticle random phase approximation) the most important is the so-called particle-particle strength parameter $g_{pp}$:

$$M_{X}^{0\nu}(g_{pp}, g_A) = M_{GT}^{0\nu}(g_{pp}) + M_{T}^{0\nu}(g_{pp}) - \frac{M_{F}^{0\nu}(g_{pp})}{g_A^2}$$

$g_A$ and $g_{pp}$ expected to be of $O(1)$, but precise values not fixed by first principles. May be different in different nuclei. Must be constrained by experimental data.